September 20, 2015 Comments Welcome

JEP Symposium on Overconfidence

Overconfident Investors

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1 Introduction

Over the last several decades, the idea that psychological biases affect financial markets has gradually gained increasing, though not uniform, acceptance among finance scholars. Two key factors have driven this paradigm shift: increasing evidence of strong asset return predictability that is at odds with the efficient markets hypothesis; and the success of models grounded in empirical psychology in explaining this evidence, in generating new testable hypotheses.

Asset markets exhibit trading volumes that are high, and both individuals and asset managers trade aggressively, which causes them to incur large trading costs. In addition asset prices display patterns that, given their strength, are difficult to reconcile with rational-expectations based theories of price formation.

In this paper, we discuss the role of overconfidence as an explanation for these sets of patterns. Overconfidence means having mistaken valuations and believing in them too strongly. Based on these views overconfident agents will be very willing to trade with others who do not share their views. So we expect overconfidence to have major effects on trading volume and on asset prices.

By focusing on overconfidence as an important part of the explanation, we do not mean to suggest that overconfidence is the only behavioral phenomenon worth considering, nor that it should serve as an all-purpose explanation for all financial anomalies. But for several reasons overconfidence is likely to be a key factor in financial decision making: first, overconfidence is a widespread psychological phenomenon. For example, in an overview of financial decision-making, DeBondt and Thaler (1995) wrote: "Perhaps the most robust finding in the psychology of judgment is that people are overconfident." Although there have been further developments in the psychological and financial literature since that time, we believe that this statement on the whole still stands. Studies of both *overplacement* (overestimation of one's rank in the population on some positive dimension) in the psychological laboratory (Benoît, Dubra and Moore 2015) and the field (Merkle and Weber 2011) and of *overprecision* (overestimation of the accuracy of one's belief) in financial field settings demonstrate very strong overconfidence even by financial professionals (Ben-David, Graham and Harvey 2013). A belief in one's high merits naturally induces an expectation of successful future success, and indeed people tend to be overoptimistic about their life prospects (Weinstein 1980) and this optimism directly affects their financial decisions (Puri and Robinson 2007).[1]

[1]The psychology of overconfidence is covered in greater depth by [[**** ??? ******]] in this issue.)

Second, overconfidence is associated with a cluster of effects. In addition to overplacement and overprecision, people overestimate their ability to influence or control future events (*the illusion of control*). Overprecision includes overestimation of the ability to accurately forecast the future, which has obvious importance for financial markets. A mental process that can help support overconfident beliefs is *self-attribution bias*, in which people give credit their own talents and abilities for past successes, while blaming their failures on bad luck. Such a bias helps people persist in their overconfidence. Indeed, overconfidence is at least part of the plausible explanation for a broad set of economic phenomena associated with overly aggressive individual beliefs and decisions, such as disagreement with other informed individuals (agreeing to disagree), entrepreneurial failures, excess investment, excessive risk taking, gambling, stock market bubbles, excess trading volume and excess volatility. More speculatively, it suggests a possible source of boom-bust patterns in investment and business activity.[2]

On the other hand, it might seem that actors in financial markets would be less susceptible to overconfidence, because financial markets have measurable outcomes and extensive feedback, and so experience in such markets should teach participants not to be less overconfident in their professional work. However, overconfidence has also been documented among experts and professionals, including those in the finance profession. For example, overconfidence is observed among corporate financial officers (Ben-David, Graham and Harvey 2013) and among professional traders and investment bankers (Glaser, Weber and Langer 2013).

To evaluate the importance of overconfidence for financial markets, we proceed as follows. We start by reviewing some financial market anomalies at odds with rational agent asset pricing theories, emphasizing those results that investor overconfidence-based theories can help explain. In particular, we focus on the arguments that trading volumes are excessive, on evidence that security returns are predictable, and that corporate managers behave aggressively in their individual trading behavior and corporate policies. In the following section, we sketch a sequence of models of investor trading and security prices, with increasing complexity, and discuss the empirical implications of each of these models. Our hope is that this presentation will make clear to the reader which aspects of the model are important in delivering each set of empirical implications. We tie implications to the empirical results. We then offer some conclusions about how overconfidence contributes to our understanding

[2]A starting point to the literature on how overconfidence affects behavior (other than investor behavior, the topic of this essay) would include, among others, Camerer and Lovallo (1999), Johnson (2009), Malmendier and Tate (2005), Neale and Bazerman (1985), and Hoelzl and Rustichini (2005).

of financial markets.

2 Evidence on Trading Patterns and Return Predictability

The last several decades have seen a shift away from the fully rational paradigm of financial markets towards one in which investor behavior is influenced by psychological biases. Two of the strongest factors contributing to this evolution have been the plausibility of and evidence for the idea that people are imperfectly rational in systematic ways, and evidence on security market trading volumes and returns that is hard to reconcile with fully rational models. The result has been a blossoming of financial models that incorporate biases in investor (and manager) judgement and decision making.

Economists have long understood that economic decision makers are not fully rational – the question is whether financial markets behave *as if* participants were.[3] This is the notion of *market efficiency* spelled out in Fama (1970): if investors in frictionless markets compete with one another, then securities will be correctly priced based on all publicly available information. If any new information is released which suggesting that a firm is underpriced, then in rushing to take advantage of this information competing investors will quickly push the price of that security up to a level where the security is no longer a good deal. Thus, in an efficient market, a trading strategy based on existing (*ex ante*) information cannot be used to earn abnormal profits. If no such strategy exists, then a market is efficient – prices are set *as if* all investors were rational. In contrast, if such trading strategies do exist there is a *return anomaly*: such opportunities suggest either that rational agents are not fully exploiting available profit opportunities, or that risk aversion or market frictions constrain their ability to do so.

The evidence summarized in Fama (1970) suggested that financial security prices did indeed behave as if the important participants were rational, subsequent tests provided evidence suggesting that the Capital Asset Pricing Model (CAPM) – a theory compatible with efficient markets – was a fairly good description of the cross-section of equity returns (Black, Jensen and Scholes 1972, Fama and MacBeth 1973). Furthermore, Jensen (1968) showed that actively managed mutual funds, gross of fees, performed according to the CAPM: the managers neither beat the market index nor lose relative to it, presumably because there

[3] This view is nicely explained in the billiards player analogy in Friedman (1953).

are no good deals, and no bad deals, in a perfectly functioning market.

However, starting in the mid-1970s, as financial datasets grew and information processing technology improved, scholars started to uncover a number of *anomalies* that cast doubts on the fully rational paradigm. Although this paradigm was flexible enough to be fitted to some of the new stylized facts, doubts have grown. Several patterns in trading volume, average returns, and return volatility are at odds with once-standard theories based upon rational expectations, and have received overconfidence-based explanations.

Notable among these anomalies are patterns of return predictability. These indicate that it is possible, using publicly available information, to construct trading strategies that earn expected returns that exceed what would be expected based upon traditional benchmarks that reflect differences in the riskiness of different securities. Indeed, there are sophisticated investors, such as hedge funds, who are in the business of constructing such trading strategies. Such arbitrage behavior by smart investors helps correct mispricing and reduce excessive return predictability, but does not necessarily eliminate them. Excessive return predictability poses a severe challenge to the hypothesis that investors are rational, because suggests that investors are making serious mistakes: they are throwing away money buying overpriced securities that subsequently do poorly, and are missing out on buying underpriced securities that subsequently do well.

2.1 Disagreement, Speculative Trade, and Trading Volume

A financial trade requires that two parties agree to disagree, in the sense that at a given price one party believes it is a good idea to sell the asset while the other party believes it is a good idea to buy it. Theoretical research has shown that under surprisingly mild assumptions, rational individuals should not agree to disagree. Intuitively, if we start with the same prior beliefs, yet now we disagree, this suggests that at least one party has information that the other party should be taking more fully into account (Aumann 1976). In a similar spirit, rational investors should not place bets with each other; the fact that another investor is willing to take the opposite side of my trade should suggest to me that this investor knows something I do not know (Grossman 1976, Milgrom and Stokey 1982, Tirole 1982). So leading rational frictionless models of asset pricing imply that after a single round of trading everyone should hold the market portfolio rather than placing speculative bets against each other.

Of course, there are possible reasons for informed agents to trade other than disagreement, such as liquidity motives (such as sending a child to college), or to rebalance to achieve a more diversified portfolio (for example, after a shock to one's labor income or human capital). But such motives for trade are relatively limited, and do not seem to explain the magnitudes of trade, or the willingness of investors to incur the large transaction costs that they pay to make such large trades.

The total volume of trade in financial markets is vast. Over the period 1980-2014, the annualized average turnover for the 500 largest US stocks has averaged 223%, or just over \$100 billion per day. Over the year 2014, the total dollar trades in these top 500 stocks was \$29.5 trillion – nearly double the 2014 US GDP of \$17.4 Trillion.[4] Notional trade in foreign exchange is even larger. Froot and Thaler (1990) report that, as of 1989, average trading in the foreign exchange market was about \$430 billion per day as compared to daily US GDP of \$22 billion and daily trades in goods and services of \$11 billion.

Speculative trade can arise in rational models if investors in securities markets are periodically required to sell of buy securities as a result of liquidity shocks. Several models starting with Grossman and Stiglitz (1976) have shown that if there are random shocks to security supply (designed to capture the idea that there are investors whose need to cash out of their positions is unpredictable to others), this can add enough noise to make room for some speculative trading.

However, liquidity shocks alone don't seem to explain the magnitudes of trade that we observe, or the patterns in trading volume. Rather, several findings point to overconfidence as an explanation. Everyday experience suggests that there is considerable disagreement across individuals in the economy, each individual believing that he or she is correct. In overconfidence-based models, investors who are overconfident form judgments about the value of a security that put too much weight on their own views, and insufficient weight on the views of other investors (as reflected in the security's price). So overconfident investors expect high profits from trading on their opinions. The actual profits they realize may in fact be small, or negative if there are transactions costs or if overconfident trading induces mispricing.

The excessive trading of individual investors can be called *the active investing puzzle.* Individual investors trade individual stocks actively, and on average lose money by doing so. The more actively investors trade, the more they lose (Odean 1999, Barber and Odean 2000*a*). For example, Barber and Odean (2000*b*) find that some households trade much more

[4]Stock trading volumes come from Collin-Dufresne and Daniel (2014), who use CRSP data. 2014 US GDP is taken from the IMF.

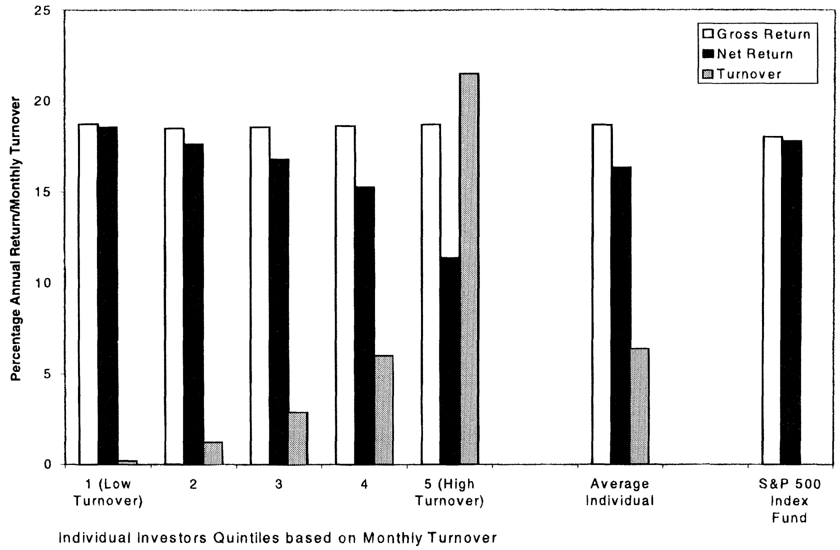


Figure 1. Monthly turnover and annual performance of individual investors. The white bar (black bar) represents the gross (net) annualized geometric mean return for February 1991 through January 1997 for individual investor quintiles based on monthly turnover, the average individual investor, and the S&P 500. The net return on the S&P 500 Index Fund is that earned by the Vanguard Index 500. The gray bar represents the monthly turnover.

Figure 1: **Barber and Odean (2000), Figure 1**

than others across about 78,000 clients of a large discount brokerage firm over the 1991-1996 time period. The turnover and gross- and net-returns to the clients in different turnover quintiles are summarized in Figure 1 from their paper.

The gray bars give the average monthly turnover of the accounts in each of quintiles. Strikingly, the average monthly turnover in the fifth quintile is above 20%/month. How do these investors do? The white bars give gross returns, and show that across quintiles, there is little variation in average gross returns.

While the gross returns of the different quintiles are about equal, the black bars show that the net returns are quite different. These investors pay large fees to trade (given their high volume of trade), which drives down the returns of the most active individual investors dramatically. The net returns of all quintiles except the lowest are lower than the net return from investing in an S&P 500 index fund.

Tests that aggregate across individual investors at the annual level also find that the stocks that individual investors buy tend to subsequently underperform. Investor losses can be astonishingly large; in the aggregate, Taiwanese investors' annual losses amount to 2.2% of Taiwan's gross domestic product and 2.8% of total personal income (Barber, Odean and Zhu 2009). In experimental markets as well, some investors overestimate the precision of their signals, are more subject to the winner's curse, and have inferior trading performance (Biais et al. 2005). Greater ease of trading gives investors free rein to harm themselves by more aggressive trading, as occurred with the rise of online trading (Barber and Odean 2002, Choi, Laibson and Metrick 2002).

Furthermore, individuals invest in active mutual funds instead of indexing for better net-of-fees performance. Indeed, an endogenous response to overconfidence about the ability to select active fund managers is for active funds to be established and to charge high fees (French 2008, Malkiel 2013)

Overconfidence provides a natural explanation for the active investing puzzle, as it causes investors to trade more aggressively even in the face of transactions costs or adverse expected payoffs (Odean 1998). In one of the rare studies of investor trading that directly measures overconfidence, Grinblatt and Keloharju (2009) associate the trading behavior of Finnish investors with the results of a psychometric test given to all Finnish males at age 19 or 20. The study finds that overconfident investors (as well as investors who are prone to sensation seeking) trade more often.

Also consistent with overconfidence as an explanation for the active investing puzzle, Kelley and Tetlock (2013) construct a structural model of market trading which includes informed rational investors as well as uninformed investors who trade either for hedging reasons, or to make an (overconfident) bet on perceived information. They estimate this model using a dataset on trades, prices and information releases for US traded firms, and conclude that, without overconfidence-based trading, volumes would be smaller by a factor of 100.

Motivated by psychological evidence that men are more overconfident than women in decision domains traditionally perceived as masculine, such as financial matters, Barber and Odean (2001) compare the trading behavior and performance of the two genders. Consistent with higher confidence, the average turnover for accounts opened by men is about 1.5 times higher than accounts opened by women, and as a result men pay 0.94%/year higher transaction costs. Even the gross (benchmark-adjusted) returns of the men in the sample are lower, though this difference is not statistically significant. As a result the net-of-fees returns of men are far lower.[5]

Other aspects of investor trading behavior are also consistent with overconfidence and the psychological processes that accompany it. Individual investors tend to trade more after they experience high stock returns. For example, early adopters of online trading tended

[5] In an experimental study, D'Acunto (2013) examines the extent to which gender stereotypes help to explain greater risk-taking by men. D'Acunto finds that when men's identity is either threatened or primed, men become less risk averse.

to make the switch after unusually good personal performance, and subsequently traded more actively (Barber and Odean 2002, Choi, Laibson and Metrick 2002). This may be why stock market trading volume increase after high returns, as has been documented in a large number of countries (Griffin, Nardari and Stulz 2007).

In many markets, trading volume is correlated with lagged returns, often over long horizons. For example, turnover in US common stocks was at levels of over 100% late in the bull market of the 1920s, fell through the 1930s and 40s to low levels, and then rose dramatically from the 1990s up through the financial crisis (Collin-Dufresne and Daniel 2014). Statman, Thorley and Vorkink (2006) find that U.S. market turnover is positively correlated with lagged monthly market returns, and that individual security turnover is positively associated with lagged market turnover after controlling for lagged own security turnover and returns.

These findings are consistent with bias in self-attribution, as modeled by Daniel, Hirshleifer and Subrahmanyam (1998) and Gervais and Odean (2001); investors who have experienced high returns attribute this to their high skill, and become more overconfident. Indeed, Statman, Thorley and Vorkink (2006) interpret their findings as largely consistent with a dynamic-overconfidence based explanation, as opposed to an alternative explanation based upon the disposition effect.

An important friction in many securities markets – especially less liquid ones – is that short-selling can be difficult or costly. As a result, pessimists about a stock find it harder to trade on their views than optimists. If some of the optimists do not adequately take into account that pessimists about the stock are sidelined by short-sale constraints, the optimists will overvalue the stock, resulting in equilibrium overpricing. So when overconfidence, as reflected in stubborn disagreement of investors about a security, is combined with short sales constraints, we expect the security to become overpriced (Miller 1977).

Motivated by this theory, Diether, Malloy and Scherbina (2002) measure disagreement by the dispersion in analysts' earnings forecast. Consistent with greater disagreement causing greater overpricing, firms with greater dispersion of analyst forecasts on average earn lower returns. This is usually interpreted as evidence that investor disagreement matters; overconfidence provides a natural explanation for *why* disagreement exists and matters.

Since high volatility creates greater scope for disagreement, this approach also suggests overpricing of more volatile stocks. Consistent with this, Ang *et al.* (2006, 2009) and Baker, Bradley and Wurgler (2011) show that high idiosyncratic-volatility stocks earn lower subsequent returns than low volatility stocks. This hypothesis is also consistent with the finding that high systematic risk (e.g., market beta) stocks typically earn too low a premium in equity markets, and in other markets (Black, Jensen and Scholes 1972, Frazzini and Pedersen 2014), relative to what would be predicted by the Capital Asset Pricing Model.

During the high-tech boom at the turn of the millennium, episodes of strong disagreement were documented in which, remarkably, the market value of a parent firm was sometimes substantially less than the value of its holdings in one of its publicly-traded divisions (Lamont and Thaler 2003). This reflected the fact that an optimistic set of investors were excited about a glamorous division, and the relatively pessimistic investors who were setting the price of the parent firm found it too costly or troublesome to short-sell the glamorous division to bring its price in line with that of the parent.

Also consistent with overvaluation induced by investor disagreement, stocks with tighter short-sale constraints have stronger return predictability anomalies (Nagel 2005), and greater long-short asymmetry in the accrual anomaly (Hirshleifer, Teoh and Yu 2011). Such asymmetry between the long and the short side of return anomalies is especially strong during optimistic periods, when overvaluation is most severe (Stambaugh, Yu and Yuan 2012).

Overconfident disagreement, combined with short sale constraints, can also cause dynamic patterns of increasing overpricing. Building on Harrison and Kreps (1978), Scheinkman and Xiong (2003) present a model in which overconfidence generates disagreements among agents regarding asset fundamentals. Owing to short sale constraints, investors buy stocks that they know to be overvalued in the hope of selling at even higher prices to more optimistic buyers. This magnifies the pricing effects of disagreement. Such bubbles should be more severe in markets with lower available supply of shares ('float') (Hong, Scheinkman and Xiong 2006), as confirmed for a bubble in Chinese warrants (Xiong and Yu 2011).

Although overconfidence causes serious problems, it is not all bad. Overconfidence can induce investors to investigate more, and/or to trade more aggressively based on their signals. This sometimes results in greater incorporation of information into price (Hirshleifer, Subrahmanyam and Titman 1994, Kyle and Wang 1997, Odean 1998, Hirshleifer and Luo 2001).

Furthermore, overconfidence encourages investors to participate in asset classes, such as the stock market or international investing, that they might otherwise neglect (owing to other irrationalities such as fear of the unfamiliar). This can potentially be a source of substantial welfare benefits. Empirically, a greater feeling of competence about investing is associated with more active trading and with greater willingness to invest in foreign stock markets (Graham, Harvey and Huang 2009).

2.2 Return Predictability

Here, we lay out the documented patterns in return predictability that are at odds with the efficient markets hypothesis and potentially attributable to overconfidence. In this section, we concentrate primarily on the nature and direction of the patterns as opposed to their magnitudes.

Of course, it is possible that the abnormal returns generated by "anomaly portfolios" are not anomalous at all. A strategy may earn high returns relative to some benchmark by virtue of exposure to some systematic risk factor that the benchmark does not capture. This suggests that documented patterns in returns might simply reflect mismeasurement of risk. In Section 2.3 we argue that the large premia earned by a combination of these anomaly-based strategies is too large to be explained plausibly by theories in which these premia arise in a frictionless setting based on rational consideration of risk.

2.2.1 Fundamental-scaled-price-based predictability

One of the earliest anomalies uncovered in academic research was the *size anomaly* (Banz 1981, Keim 1983) – the phenomenon that 'small' (*i.e.*, low-market-capitalization) firms earn higher returns than large firms. However, still stronger predictability is obtained when scaling the firm's market capitalization by a measure of the firm's fundamental value. Fama and French (1992) find that the *book-to-market* ratio – the book-value of equity, scaled by the firm's market capitalization – predicts returns. In particular, value firms (high book-to-price ratio firms) substantially outperform growth (low book-to-price) firms. However many other fundamental-to-price measures, including earnings-to-price, sales-to-price, and cash-flow-to-price ratios positively forecast future returns (Lakonishok, Shleifer and Vishny 1994).

Long-term reversal (DeBondt and Thaler 1985) can also be understood as fundamental-to-price ratio, in which the fundamental proxy is the long-ago price of the stock. Intuitively, a stock that is mispriced now probably did not share the same mispricing years ago. Daniel and Titman (2006) add an additional dimension to this point; if past long-term returns are decomposed into a component associated with public-information and an orthogonal component, reversal is only observed for the orthogonal component. The component of the past return associated with public information does not reverse.

2.2.2 Momentum and slow correction predictability

A common pattern in event studies is continuation of the event-date return, so that events that are on average good news experience high subsequent returns, and the opposite for bad news events (see, e.g., the summary in Hirshleifer (2001)). One notable example of this is *post-earnings announcement drift* or *earnings momentum*: firms which announce high earnings relative to forecasts, or whose price jumps up on an announcement date, earn high returns over the subsequent 3-6 months (Bernard and Thomas 1989, Bernard and Thomas 1990).

Price momentum is the tendency for returns over the past 3-12 months to continue in the same direction in the future 3-12, in many asset classes. The overconfidence explanation for momentum involves a pattern of continuing overreaction and slow correction.

Price momentum in the U.S. stock market has several key features. First, it is predominantly associated with lagged price changes that can be attributed to public information releases. In contrast, price changes which cannot be associated with news tend to exhibit reversal rather than continuation (Chan 2003, Tetlock 2011). Second, momentum effects are weak for value stocks, but strong for growth stocks (Daniel and Titman 1999). Third, momentum strategies generate especially strong returns in calm periods when the past return on the market is high (Cooper, Gutierrez and Hameed 2004, Daniel and Moskowitz 2014), but exhibit strong negative skewness and earn lower returns in turbulent (high volatility) bear markets (Daniel and Moskowitz 2014, Daniel, Jagannathan and Kim 2015).

Notably, Asness, Moskowitz and Pedersen (2013) document strong value and momentum anomalies in non-US data, and in other asset classes (currencies, commodity futures and government bonds). In section 3, we discuss how a dynamic overconfidence model can potentially explain, in an integrated framework both the value and momentum effects, and some of the additional implications of these models.

Moskowitz (2015) shows that the same momentum and value/reversal patterns observed in other asset classes are also present in sports betting venues. Sports betting markets are a useful testbed for overconfidence based theories because the outcomes of these contests are unlikely to be dependent on macroeconomic outcomes which may proxy for the marginal utility of a representative agent (see the next section for more on this). Moskowitz argues that the presence of value and momentum effects in sports betting markets is consistent with delayed overreaction theories of asset pricing such as the overconfidence-based theories discussed in Section 3. Finally, consistent with the model in Daniel, Hirshleifer and Subrahmanyam (2001), he finds that higher ambiguity predict stronger momentum and value returns, consistent with what is observed in financial markets.

2.2.3 Underreaction to or Neglect of Cash-Flow-Relevant Information

Many items reported in financial statements can be useful in forecasting firms' future earnings. Investors do not appear to make full use of such information. One prominent example is accruals. Operating accruals are the accounting adjustments made to a firm's cash flows to obtain earnings, a standard measure of profitability. Such adjustments include sales transactions whose payments have not yet arrived, and expense transactions for which actual payments have not yet been made. Sloan (1996) shows that market prices don't fully reflect the extent to which earnings arise from cash flows or accruals.

The issuance of new securities or repurchase of existing securities are major corporate events that contain information relevant for future cash flows. Repurchases tend to be followed by high returns (Ikenberry, Lakonishok and Vermaelen 1995), and seasoned equity and debt issues in many countries by negative abnormal returns (Loughran and Ritter 1995, Spiess and Affleck-Graves 1995, Henderson, Jegadeesh and Weisbach 2006). Daniel and Titman (2006) and Pontiff and Woodgate (2008) develop more comprehensive measures of share issuance over a given time period, and show that lagged measure of issuance strongly forecast returns. At the aggregate level as well, the share of equity issues in total new equity and debt issues has been a negative predictor of U.S. market returns (Baker and Wurgler 2000).

2.3 Return Predictability – Magnitudes

The return patterns documented in the preceding section might reflect rational risk premia rather than mistakes or biases on the part of investors. Here, we summarize evidence on the risk and rewards of strategies based upon these effects to see if this explanation is plausible. We argue that the evidence presents a daunting challenge to rational asset pricing models. A portfolio which simultaneously exploits several the patterns of return predictability documented in the preceding section generates an exceptionally high reward-to-risk ratio. Using insights from Hansen and Jagannathan (1997), accommodating these premia within any frictionless rational expectations model would require extreme (and we will argue, unrealistic) variation in investor marginal utility across states of the world.

We start with a set of anomaly portfolios designed to capture the patterns described in the preceding section. We begin with the "small minus big" portfolio, or SMB, proposed by Fama and French (1993), which captures the difference in average returns between small and large market-capitalization firms. This portfolio, at the beginning of each month, takes a long position in $1 worth of low-market-capitalization stocks, financed by taking a short position in $1 worth of large-market capitalization stocks. So historically, investors should have been able capture the returns of this *zero-investment* or *$1-long/$1-short* portfolio with minimal transaction costs (despite the need to sell short). This portfolio has been used in numerous academic studies, and yearly, monthly and daily returns from 1926 on are available on Ken French's website.

In addition to the SMB portfolio, we use a set of other zero-investment portfolios that capture the other anomaly premia:

1 The "High minus Low" or HML portfolio is formed to exploit the persistently higher returns of stocks with high book-to-market ratios and those with low book-to-market ratios. The portfolio involves buying value stocks – stocks with ratios of book-value of equity to market-value of equity in the top 30% of all stocks on the New York Stock Exchange – and sells growth stocks, with book-to-market ratios in the bottom 30%.

2 The "Up minus Down" or UMD portfolio is a price momentum portfolio (Carhart (1997), Fama and French (1993)). It is formed by buying stocks that rose in price in the previous time period (often 12 months, and, for reasons we will not go into, skipping the most recent month) and taking a short position in stocks that declined in price in the previous time period. Thus, it is based on an underlying momentum in stock behavior.

3 The ISU or "ISsUance" portfolio, or ISU buys a value-weighted portfolio of firms which, over the preceding 3 years, repurchased stock, and shorts a portfolio of stocks that issued new equity, based on the Daniel and Titman (2006) measure.

4 The ACR or "ACcRual" portfolio goes long a portfolio of firms which had the lowest the ratio of accruals to earnings over the past year, and goes short on the firms which had the highest accruals.

5 The BAB or "Betting-Against-Beta" portfolio is constructed following the description in Frazzini and Pedersen (2013). The long side of the portfolio is a leveraged portfolio of low-beta stocks. The portfolio takes a short position in high-beta stocks.

Table 1: **Anomaly-Based Strategy Sharpe Ratios**

This table presents the realized *ex-post* optimal strategy Sharpe-ratios from 1963:07-2014:05 for a set of long-short portfolios based on a set of anomalies taken from the finance literature: Mkt-Rf, SMB, HML are the three Fama and French (1993) portfolios; UMD is the Carhart (1997) price momentum portfolio; ISU and ACR are the Daniel and Titman (2006) issuance and accrual portfolios respectively; BAB is the Frazzini and Pedersen (2013) Betting-Against-Beta portfolio; and finally IVOL is the Ang et al. (2006) idiosyncratic-volatility portfolio.

Portfolio Weights (%)								Sharpe
Mkt-Rf	SMB	HML	UMD	ISU	ACR	BAB	IVOL	Ratio
100.0	–	–	–	–	–	–	–	0.39
34.9	18.7	46.4	–	–	–	–	–	0.76
25.8	10.5	33.0	30.7	–	–	–	–	1.07
8.0	4.5	33.9	17.7	26.8	9.1	–	–	1.37
7.7	12.4	13.8	4.5	18.0	10.2	9.5	24.0	1.78

6 Finally, the IVOL or "Idiosyncratic-VOLatility" portfolio each month takes a long position in the set of firms that had the lowest idiosyncratic volatility of daily returns over the preceding one month, and shorts the highest idiosyncratic volatility stocks, measured following the procedure specified in Ang et al. (2006).

The *Sharpe ratio* of a portfolio is the ratio of its reward to its risk. We define it here to be the ratio of the annualized excess return on the portfolio to the annualized return standard deviation of the portfolio. To summarize how an investor might optimally exploit these anomalies, it is useful to examine the Sharpe ratios achieved by combining the anomaly portfolios into super-portfolios.

Table 1 presents Sharpe ratios for portfolios consisting of the US Market portfolio – specifically the Center for Research in Security Prices value-weighted index return – along with various mixtures of the seven candidate anomaly portfolios.[6] Each row of Table 1 represents a different combination of the set of anomaly portfolios designed to achieve a high Sharpe ratio. The first eight columns show the weights on each of the anomaly portfolios, and the number in the ninth (and last) column gives the annualized Sharpe ratio of the overall portfolio that combines them.[7]

Thus, the first row of the table shows that during this sample period, a portfolio that

[6]Mkt-Rf is the notation used by Fama and French (1993) for the excess return of the CRSP value weighted index, relative to the 1-month US treasury-bill return in the same month.

[7]The component portfolios are normalized so that each of has the same volatility over the 1963:07-2014:05 sample period. Thus, the weights given in the table are proportional to the volatility of that component.

was 100% invested in the market index (Mkt-RF) experienced an annualized Sharpe ratio of 0.39. Specifically, the annualized return, net of the one-month Treasury-bill rate was 6%, and the annualized volatility was 15.5%. The second row shows how much an investor could have improved on the market Sharpe Ratio by also investing in the size-based SMB and value-based HML portfolios. The optimal combination of these three portfolios results in a Sharpe ratio of 0.76, nearly double that of the market on its own. The next few lines of the table show that the ability to invest in the momentum factor brings the Sharpe ratio up to 1.07, and the ability to invest in the issuance and accrual portfolios brings it up further to 1.37. Finally, if the investor had been free to invest in any of these eight portfolios, and knew beforehand the distribution of returns over this period, that investor could have earned a Sharpe ratio of 1.78, more than *four times* higher than that of the market.[8]

The existence of this high Sharpe ratio anomaly portfolio, which has a low correlation both with market portfolio returns and with aggregate consumption growth, poses a challenge to frictionless, rational asset pricing models. Any asset-pricing model—whether rational or behavioral—needs to explain why investors are apparently passing up these very high-return, low volatility investments. In any rational-expectations setting, asset premia arise only when the asset's returns are risky, meaning that they are high when the investor is rich, (i.e., when marginal utility of wealth is low) and are low when the investor is poor (and marginal utility is high). To explain the such a large risk premium, marginal utility must be quite variable. The Hansen and Jagannathan (1991) bound shows that, to explain the existence of a portfolio with a Sharpe-ratio of 1.8, requires that the annualized standard deviation in marginal utility growth be greater than 180%. Both casual observation and macroeconomic data suggest that marginal utility growth doesnft vary this much. For example, the annualized volatility of aggregate US consumption growth is about 1.8%.

Also, the macroeconomics profession is still wrestling with the *equity premium puzzle*—the finding that the Sharpe ratio of the equity market portfolio, which is about 0.4 (annualized), is so high relative to the low volatility of consumption growth (Hansen and Singleton 1983, Mehra and Prescott 1985, Weil 1989). Thus, the far higher Sharpe ratio associated with this anomaly portfolio is even harder to reconcile with a rational investor model.

[8]The numbers presented in this table are the Sharpe ratios for the optimal portfolios, calculated as if investors knew up front the realized distribution of returns. Our main conclusions still apply if we correct for the bias involved with assuming investor foreknowledge of distributions: for example, an equal-weighted combination of the eight portfolio earns a annualized Sharpe-ratio of 1.54. Also, Asness, Moskowitz and Pedersen (2013) document that a 50/50 combination of *just* value and momentum portfolios, but diversified across different regions and asset classes produces an annualized Sharpe ratio of 1.59.

If a strictly rational investor model has a difficult time explaining these pricing anomalies, perhaps an answer can be found in trading frictions that make it costly for investors to trade to exploit perceived profit opportunities. However, the magnitude of such frictions, as captured by bid-ask spreads, is too small to address the puzzle. For moderate sized trades in large firms, such as those used to construct the zero-investment portfolios described here, such spreads are relatively small.

Alternatively, maybe these results arise from data mining, and if one looked at different time periods, or a limited set of these portfolios, or weighted the portfolios equally rather than (after the fact) optimally, then the pricing anomalies would disappear. One can tinker with different time periods, or different portfolios, or different weights. But the opportunities presented by these anomaly portfolios remains.

What other theories can explain the patterns in Table 1? To rephrase the question, is it possible that the beliefs of investors are biased in ways that induce the seven pricing anomalies listed earlier? The overconfidence-based models in the literature suggest that the answer is yes. In these models, investors continue to optimize, but do so based on incorrect beliefs about the state probabilities. Under this explanation, investors think that the state probabilities are such that the expected returns of the anomaly portfolios are not abnormally high, despite the evidence in Table 1.[9]

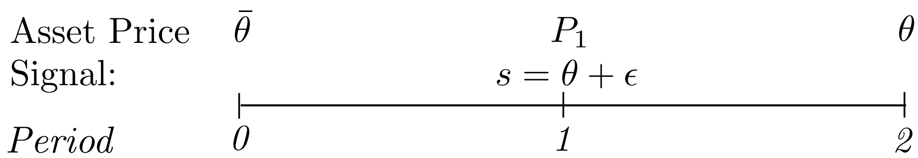
How might overconfidence generate the anomalies that underlie Table 1, so that overconfident investors do not believe that these portfolios outperform? In the next subsection, we catalog some of the patterns over time that underlie the strategies used in constructing Table 1. Then in the next major section we lay out overconfidence models that can potentially explain these patterns.

3 Overconfidence Based Models of Asset Price Formation

In the frictionless rational expectations framework, investors process information perfectly. This implies that asset prices are always equal to rationally discounted expected cashflows, where discount rates are equal to rational expectations of returns. Investors earn returns that are, on average, exactly what they expect.

[9]There could still be rational investors as well, who do correctly perceive the high available Sharpe ratios. If such investors are relatively small in number, and capital constrained, their trading to exploit the profit opportunities does not eliminate them.

Figure 2: **Model 1: Basic One-Signal Model – Timeline**



This figure illustrates the timeline for the basic three-date, one-signal model discussed in Section 3.1. At time 0, the investor knows only the prior distribution from which the time 2 final payoff is drawn. At time 1 the investor observes a noisy signal s. At $t = 2$ the payoff θ is revealed.

As discussed in the previous section, so-called zero investment portfolios constructed to reveal anomalies have produced high Sharpe ratios—high average excess returns with low volatility—and which have low correlation with macroeconomic shocks that might plausibly represent risk. Thus researchers have turned to behavioral models in an attempt to explain these patterns. The behavioral models rely on either non-standard preferences, or biased beliefs.

In models with non-standard preferences, investors still correctly expect that high excess returns are achievable with these anomaly-based portfolios. In these models investors choose not to invest more in these portfolios because they find certain kinds of risk extraordinarily painful to bear. In contrast, biased belief models posit that investors make mistakes in the way that they form beliefs about asset payoffs, and specifically, that these investors incorrectly weight some of the information presented to them in forming their expectations. Overconfidence-based models fall into this category.

We now provide a sequence of models that illustrate some key insights of the overconfidence-based approach. The first model is a bare-bones setting which captures the fact that an overconfident investor overreacts to a signal that is perceived as private, resulting in overreaction and correction, consistent with evidence of long-run return reversals discussed earlier. We then present models that show how refinements to this basic model, grounded in the psychological evidence on overconfidence, can plausibly generate several of the other anomalies described above.

3.1 Model 1: One Signal

To develop some intuition for how investor over- or under-confidence causes price overreactions and corrections, consider the static overconfidence model, which involves a three-date, one-signal example. The timeline is given in Figure 2. For the moment, we assume that the overconfident representative investor in the model is risk neutral. There are three dates, $t \in \{0, 1, 2\}$, and two securities: a riskfree asset with a riskfree rate of zero, and a security which will pay an uncertain liquidating dividend θ at time 2. The prior distribution for θ is known. At $t = 1$, the investor receives a private signal of the form $s = \theta + \epsilon$. Due to overprecision this investor believes that the precision of the signal is greater than it actually is. The date 1 price in this setting is a weighted average of the prior expectation and the signal, with relative weights proportional to the investor's perceived precisions of the prior and of the signal.

We are interested in whether the asset return is forecastable. If the investor is rational then the price P_1 is equal to the rational expectation of the payoff $\mathbb{E}[\theta]$, and in this case the price change from date 1 to date 2 is unforecastable—that is, it is not correlated with any price change from time 0 to time 1, nor with the signal received in time 1. The information the signal provides for fundamental value is correctly impounded into price at date 1, and the market is efficient.

However, if the investor overestimates the precision of her signal, she will overreact to the signal. Thus, a positive signal will cause the date 1 price to be too high, resulting in too high a price change between dates 0 and 1. This will result in return reversal—on average the price then falls back between dates 1 and 2. In contrast, if the investor underestimates the precision of the signal, the date 1 price will underreact, and the subsequent price change will on average again be positive: return momentum.

What might cause the investor's estimate of the signal precision to differ from the true precision? One answer that has been offered is investor overconfidence. In the simplest version of the Daniel, Hirshleifer, and Subrahmanyam model, the representative investor observes only a private signal and is overconfident about that signal, resulting in price reversal. Alternatively, in Eyster, Rabin and Vayanos (2013), investors make a different error; they fail to infer fully the private signals received by other investors from the price. Effectively, as a representative investor underestimates the precision of the aggregate private signal, and prices underreact to new information. Their model implies price momentum, but because this is a result of pure underreaction to information, there is no reversal.

Neither pure underreaction nor pure overreaction, as reflected in Model 1, fully captures the return predictability evidence discussed earlier, in which there is momentum at shorter horizons and reversal at longer horizons. In addition, Model 1 does not allow for public information signals prior to the terminal date, and therefore does not allow consideration of whether returns can be predicted based on public information such as the news of a new

Figure 3: **Model 2: Separate Public and Private Signals – Timeline**

Asset Price	$\bar{\theta}$	P_1	P_2	θ
Signal:		$s_I = \theta + \epsilon_I$	$s_B = \theta + \epsilon_B$	
Period	*0*	*1*	*2*	*3*

This figure presents the timeline for the four-date, two-signal model discussed in Section 3.2. Now, the investor observes distinct pri$V$ate and pu$B$lic signals s_V and s_B at $t = 1$ and $t = 2$, respectively. At $t = 3$ the asset payoff θ is revealed.

equity issue by the firm. In Model 2 we introduce public signals, which allows consideration of this issue.

3.2 Model 2: Public and Private Signals

To capture these patterns, we need to move to a richer model. In Model 2, we introduce separate public and private signals. This model is the "static-overconfidence" model of Daniel, Hirshleifer and Subrahmanyam. The timeline for Model 2 is given in Figure 3. There are now four dates, and two signals: s_V is a pri**V**ate signal, and s_B is pu**B**lic. As in Model 1, the investor is overconfident, and therefore overestimates the precision of the private signal at time 1. However, she correctly estimates the precisions of the public signal and the prior.

This approach delivers several additional features. First, as in Model 1, the market overreacts to the private signal, and therefore the price change from time 0 to time 1 will be in the opposite direction of the price change from time 1 to time 3. In addition, the market underreacts to the public signal: that is, price changes are autocorrelated, $cov(R_{2,3}, R_{1,2}) > 0$.

Given this positive return autocorrelation, it is tempting to jump to the seemingly obvious conclusion that the public signal s_B will be a positive return predictor. Surprisingly, this turns out to be incorrect. Intuitively, consider the rationally updated expectation of the fundamental θ conditional on the public signal. We want to see if, on average, the date 2 price differs from this expectation. If so, the public signal can be used to predict the subsequent return.

For example, assuming that the precisions of the prior and public signal are equal, the prior is 0, and the public signal is 100, then the rational updated expectation of the payoff will be 50. On average, the unbiased private signal will 50 (the expected fundamental plus mean zero noise). So even though the private signal is overweighted relative to the public signal in market price, there is no mispricing *on average*, conditional on the public signal. On average the private signal has zero effect on the expectation, which is a weighted average of 50 with 50. So on average there is no conditional mispricing.[10]

So to explain the evidence of underreaction to corporate announcement documented earlier, a further refinement is needed. Suppose that a good or bad news public signal is an event chosen by the firm or some other party in opposition to the private signal. For example, perhaps the firm announces a new equity issue—a bad news event—when the firm is overvalued (i.e., it received a positive private signal). There is evidence that firms that issue equity are indeed overvalued (Loughran and Ritter 1995, Dong, Hirshleifer and Teoh 2012). In a similar way, there is evidence suggesting that firms engage in repurchase—a good news event—in response to undervaluation (Ikenberry, Lakonishok and Vermaelen 1995). We call such public signals 'selective.'

To the extent that public signals are selectively undertaken in opposition to preexisting mispricing, such signals will show return continuation, wherein the long-run return after the event is on average of the same sign as the initial market reaction to the event. This implication is consistent with the strong performance of the ISU (issuance) portfolio described earlier.

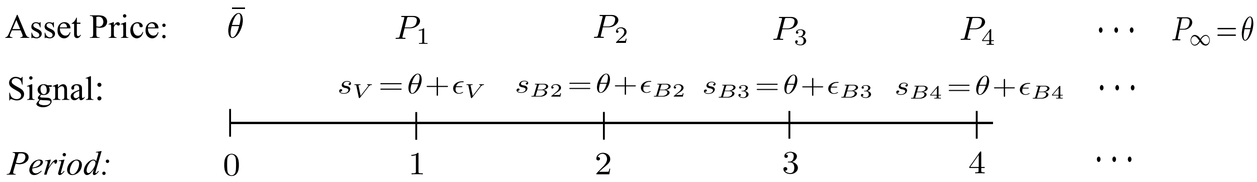
However, Model 2 still does not deliver the key empirical predictions that there will be both medium term price momentum (Jegadeesh and Titman 1993) and long term reversal. To deliver these implications we need to consider the psychology of how overconfidence changes over time as people receive feedback from their environments.

3.3 Model 3: Dynamic Overconfidence

The psychological motivation for Model 3 is the fact that confidence changes over time as people receive feedback about their judgements and decisions. When people learn that their recent forecasts were accurate, they tend to revise their confidence upward, and when they learn that they were wrong they tend to revise it downward. However, this process is not symmetric owing to self-attribution bias. As mentioned earlier, this is the tendency of people to treat successes as mainly a reflection of their own skills and failures as mainly a matter of bad luck—the "heads I win, tails it's chance" fallacy (Langer and Roth (1975)). Self-attribution bias explains how overconfidence can persist over time, despite the opportunities

[10]So knowing the public signal does not allow one to forecast the future return from time 2 to time 3. For the interested reader, a formal proof of this assertion is given as Appendix B.

Figure 4: **Model 3: Dynamic Overconfidence Model – Timeline**



This figure illustrates the dynamic overconfidence model timeline. At time 1, the informed investors receive a private signal s_I. At each subsequent time (2, 3, ...), the investor receives additional public signals, with uncorrelated noise terms.

people have to learn about their abilities through experience.

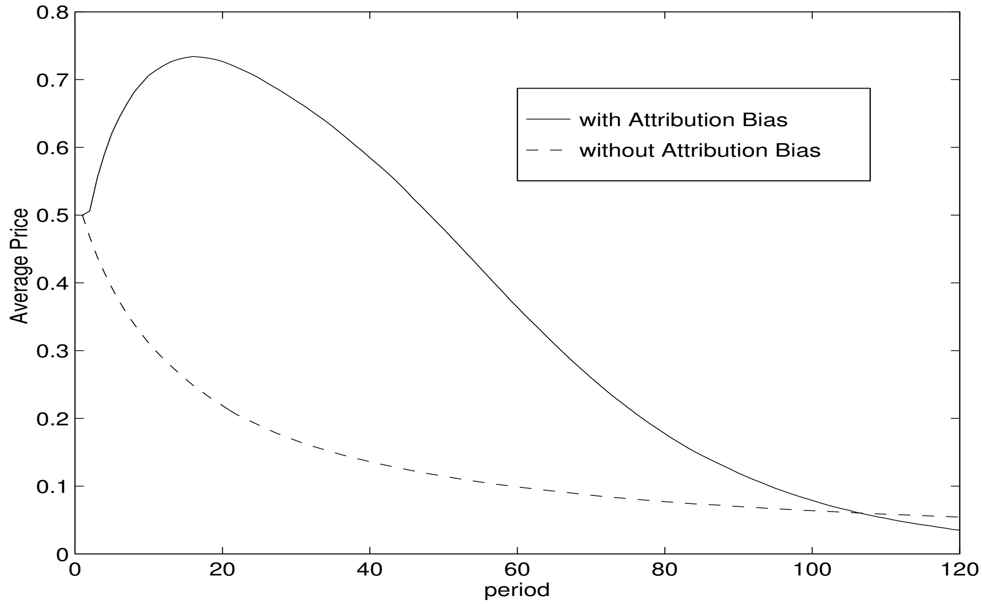
Incorporating the dynamics of overconfidence into our price formation model allows us to derive more realistic predictions for patterns of return continuation and reversal at short versus long horizons. To do so, we need to give investors opportunities to update their estimate of their private signal precision. Thus we adopt the structure illustrated in Figure 4, with the change that there are now an unlimited number of public signals arriving at times 2, 3, 4, ...

Consistent with findings from the psychology literature, we specify that the investor's estimate of private signal precision shifts through time as a function of whether the investor's private signal proves to be consistent with subsequently arriving public signals. Our specification for confidence updating is *ad hoc*, but is roughly consistent with the psychology literature. In particular, investors update their estimates of their signal accuracy based on their historical forecast success, but in a biased way.

Think of the "cumulative public signal" as the average of all previous public signals. In the context of this model, if the arrival of the next public signal pushes the cumulative public signal in the direction of the investor's private signal—then the investor becomes more confident in her private signal. In the terms used earlier, overconfidence in the precision of the signal rises. In contrast, if the new public signal pushes the price away from the investor's valuation, the investor loses confidence, and her estimate of the private signal falls. Self-attribution bias is captured in this model by having the degree of overconfidence increase more with a confirming outcome than it decreases with a disconfirming outcome.

The investor's perceived precision evolves over time based on public signal arrival. The updating rule is that when the arrival of the next public signal pushes the cumulative public signal (and market price) in the direction of the investor's private signal, then the investor

Figure 5: **Response to a unit private signal – static and dynamic overconfidence models.**



This figure illustrates the impulse response to a private signal $s_V = 1$ at time 1 when the true security value is $\theta = 0$. In this simulation the prior and private-signal precisions are equal. The dashed line illustrates the impulse response in the static overconfidence setting. The solid line is the impulse response in the dynamic overconfidence setting. (from Daniel, Hirshleifer and Subrahmanyam (1998)).

becomes more confident in her private signal. So her estimated signal precision increases by a factor of $1+\overline{k}$. In contrast, if the new public signal pushes the price away from the investor's valuation, the investor loses confidence, and her estimate of τ_V falls by a factor of $(1 - \underline{k})$. Biased self-attribution is captured by the assumption that $\overline{k} > \underline{k}$: the investor's estimated precision increases more with a good outcome than it decreases with a bad outcome.

Figure 5 illustrates the impulse response to to a private signal of $s_V = 1$ at time 1, when the prior of $\bar{\theta} = 0$, the true security value is $\theta = 0$, and the prior and private signal precisions are equal. The dashed line illustrates the price path with static overconfidence (*i.e.*, Model 2). Here, because of the equal precisions, the price at time 1 is 0.5—the average of $\bar{\theta}$ and s_V. However, starting at $t = 2$, with the arrival of the first public signal, the price on average starts to decline, as the average public signal is equal to $\theta = 0$. The price then asymptotically approaches the true security value of $\theta = 0$.

The solid line in Figure 5 illustrates the average price path with dynamic overconfidence (*i.e.*, Model 3). As in the static overconfidence setting, the price initially moves to $P_1 = 0.5$. However, now the investor, in response to the (noisy) public signal, the investor will on average update too favorably about his skill as reflected in the precision of his private signal.

As this updating process continues, on average the market will overreact in a continuing way to the original signal. Eventually, however, as further public news arrives, the overpricing must on average correct. So there is a hump-shaped impulse response function. Similarly, on the down-side there is a trough-shaped impulse response function—the mirror image of the solid line in Figure 5.

This shape implies momentum at short lags and reversal at long lags. For example, focusing on the hump-shape (the long side), in the overreaction phase there will tend to be positive returns followed by positive returns. During the correction phase there will be negative returns followed by negative returns. A similar point applies on the short side.

In contrast, with a long enough lag, a positive return on the left side of the hump (the overreaction phase) will tend to be followed by a negative return as the market will by then be in the correction phase. Again, a similar point follows in mirror image on the downside. In sum, a model with self-attribution and dynamic shifts in confidence implies positive short-lag autocorrelations and negative long-lag autocorrelations and is therefore consistent with evidence of momentum and long-run reversal discussed earlier. It is also consistent with the strong performance of the UMD (Up Minus Down) momentum-based portfolio described earlier.

3.4 Models with Both Rational and Overconfident Investors

In the models covered so far, prices are set by the overconfident investors. How would these conclusions change were we to introduce a mass of rational investors into these models? These investors would act as arbitrageurs, pushing prices toward fundamental values.

Daniel, Hirshleifer and Subrahmanyam (2001) explore such a setting as an extension of the three-date static-overconfidence model explored at the beginning of this section. In this generalized approach, the market has a continuum of risk-averse investors, who start off identical to each other. There are N securities, and the joint distribution of their fundamental payoffs is common knowledge. At time 1 investors receive different private signals. Some receive signals about what we call factor realizations—common influences that affect the returns of all securities—while other receive signals about what we call residual payoff components—the pieces of security payoffs that are not explained by common factors.

Investors are overconfident about the signal they receive: they believe that the precision of that signal is higher than it is actually is. However, an investor who does not receiving a signal correctly assesses the signal's precision. Thus, the investors who do not receive a signal—but instead infer the signal as it manifests itself through prices—act as arbitrageurs with respect to the mispricing induced by this signal.

In this setting, the magnitude of the mispricing caused by overconfidence is lower with arbitrageurs than without. The arbitrageurs eliminate *some of* the mispricing, but not all, because they are risk averse. More broadly, this setting, yields a number of implications for the relationships between risk and return.

First is that across securities, size and fundamental/price ratios are predictors of future security returns. Size is a negative predictor, because a firm that is large in market value will on average be large in part because it is overvalued. This ability of size to predict returns can explain the performance of the SMB (Small Minus Big) portfolio described earlier.

For a similar reasons, fundamental/price ratios (such as the earnings-to-price or book-value to price ratio) are positive return predictors. However, the model implies that scaling of price by a fundamental measure can improve return predictability (if the fundamental proxy is not too noisy), because a firm can have high price for fundamental reasons, not just because of mispricing. This ability of fundamental-to-price to predict returns can explain the performance of the HML (High Minus Low) book-to-market-based portfolio described earlier, and why the HML portfolio tends to perform even better than the SMB portfolio.

Another key implication of this setting is that the amount of mispricing will be constrained by the factor structure, which affects how risky it is to arbitrage mispricing. When all investors are overconfident, relatively extreme mispricing is feasible. However, when there are arbitrageurs with rational perceptions, high Sharpe ratios become an attractive opportunity to exploit. Such exploitation acts as a constraint on possible mispricing.[11]

In particular, in the limit as the number of securities in the market becomes arbitrarily large, it is possible to form portfolios that hedge away factor risk and exploit any mispricing of residual payoff components. Such portfolios are virtually riskfree. This implies that owing to arbitrage activity, there will be almost no security-specific mispricing (with the possible exception of a small number of securities).

In contrast, to arbitrage the mispricing of a factor, an investor must bear substantial factor risk. This implies that substantial factor mispricing can persist in equilibrium. However, this contrast between almost perfect arbitrage of idiosyncratic mispricing but not of factor mispricing comes in part from the stylized assumption that markets are perfectly liquid. For illiquid stocks, arbitrage is more costly, so such stocks can have idiosyncratic mispricing as

[11]More precisely, the flow of wealth from irrational to rational investors becomes arbitrarily large, which clearly is not sustainable.

well.

In this setting, regressing across stocks on β (the classic risk measure of the Capital Asset Pricing Model) as well as the fundamental-to-price ratio generally helps disentangle risk premium versus mispricing effects. However, if overconfidence about signals is extreme and the fundamental is measured perfectly, even though β is priced, it has no *incremental* power to predict future returns. Intuitively, the fundamental-to-price ratio captures both standard risk effects and mispricing effects—both drive market price down relative to expected future cash flows.

So in the limiting case in which the firm-specific signal the overconfident investors receive is pure noise, and the fundamental proxy is perfect (the best rational forecast of future cash flows), β does not provide any additional useful information to predict returns. The fundamental/price ratios will eliminate β in a multiple regression when forecasting the cross-section of future returns. This implication is consistent with empirical studies mentioned earlier in which book-to-market eliminates beta in predicting returns.

Finally, this model has excessive disagreement, because investors insist on relying too heavily on the signals they possess, and trade against rational arbitrageurs who do not possess those signals and do not overweight the signals' precision. It follows immediately that there is excessively large volume of trade. Indeed, in the current setting, if all investors were rational there would be no speculative trade. So overconfidence helps explain the remarkably high volumes of trade in liquid securities.

We have discussed how the models in this section can explain the strong performance of the first four trading strategies summarized in Table 1. We close this section by discussing whether overconfidence can help explain the performance of the remaining three trading strategies: ACR (long on low-accrual firms, short on high accrual firms), BAB (long on low-beta stocks, short on high-beta stocks), and IVOL (long on stocks with low idiosyncratic volatility, short on stocks with high idiosyncratic volatility).

Taking these in reverse order, we begin with IVOL and BAB. Together these are patterns of underperformance of risky stocks—risky both in the sense of highly sensitive to market realizations, and high idiosyncratic volatility. We have discussed evidence that investor disagreement is associated with overpricing and low returns, and the model of Miller (1977) wherein owing to short-sales constraints, irrational optimists dominate price setting. Overconfidence provides a natural explanation for the irrational tendency for investors to be too insistent in disagreeing, and for optimists to fail to fully adjust for the fact that there are pessimists who have been sidelined by short-sale constraints. High risk firms have greater scope for overconfidence and disagreement, so we expect this source of overpricing to be greatest for high risk firms. So overconfidence provides a natural explanation for the idiosyncratic volatility and betting against beta effects.

Finally, the accrual anomaly is usually understood as arising from limited investor attention. A firm's earnings is the sum of its cash flow and accrual components. The cash flow component of earnings is a much more favorable indicator than the accrual component of high future profits (Sloan 1996). Investors who do not delve into earnings to separately evaluate these components will tend to extrapolate based on current earnings to forecast future profitability. It follows that they will overvalue firms with high accruals and undervalue firms with low accruals.

In our view, overconfidence is an important part of understanding return anomalies that are usually attributed solely to limited investor attention. Limited attention has a much bigger effect on price if investors fail to recognize that the information they are neglecting is important. Such neglect is natural when an investor is overconfident about the quality of the investor's current knowledge. A similar point is made by Kahneman (2011), who discusses the tendency of people to be overconfident about fast heuristic judgements ("System 1"). For example, if an investor neglects accruals and decides that a stock looks cheap, but understands that accruals are providing useful information to other investors, the investor will be hesitant to buy heavily. But if the investor is overconfident that the signals she has chosen to attend to are the most important ones, she will tend to underweight the information implicit in price, and will trade too aggressively. So limited attention anomalies may to a large extent also be symptoms of overconfidence.

4 Cursedness: A Related Approach to Asset Pricing

We make no attempt at a systematic review of all behavioral approaches to investment here, but one alternative, cursedness, is notable for its potential overlap with the overconfidence approach.[12] Indeed, Eyster, Rabin and Vayanos (2013) point out that cursedness can potentially explain several financial economic phenomena that are often understood in terms

[12]Other behavioral approaches include representativeness and conservatism (Edwards (1968), Kahneman and Tversky (1972), Barberis, Shleifer and Vishny (1998)); realization utility (Barberis and Xiong (2012)); mental accounting and prospect theory (Kahneman and Tversky (1979), Thaler (1985), Barberis and Huang (2001), Grinblatt and Han (2005)); limited attention (Kahneman (1973), Hirshleifer and Teoh (2003), Peng and Xiong (2006)); and anchoring (Tversky and Kahneman (1974), George and Hwang (2004)).

of overconfidence.

In cursedness, a game theoretic equilibrium concept developed in Eyster and Rabin (2005), individuals underweight the information implicit in the actions of others. An example is provided by the winner's curse—the phenomenon that those who win a sealed-bid auction often have submitted too high a bid, since the very fact of winning is an indication that others do not value the object as highly. The conclusion that if I win, others have information that is more adverse than mine—is a subtle inference which bidders do not necessarily make. A sophisticated bidder who understands the winner's curse will then tend to bid more conservatively to adjust for the danger of overbidding, or at times choose not bid at all and thus receive a safe outcome of zero.

An overconfident individual who overweights his own signal will, accordingly, also underweight the information implicit in the actions of others, so the overconfidence and cursedness approaches yield overlapping implications. One view of cursedness is that it is a consequence of overconfidence about the value of one's intuitive reasoning, resulting in a failure to be modest about the results of that reasoning.

However, the cursedness approach does have some distinct implications as compared with the usual implementation of overconfidence in financial models. Since an overconfident individual overestimates the precision of a private signal signal, the behavior of that individual is too aggressive even when others have no signals. In contrast, cursedness has no effect unless others have signals that the cursed individual might fail to take into account. Similarly, even when an individual possesses no private information signal, eliminating overconfidence, cursedness causes over-aggressiveness owing to neglect of the information of others.

These distinctions matter for a key argument of Eyster, Rabin and Vayanos (2013) in favor of cursedness over overconfidence as an explanation for overly aggressive trading. According to this argument, an overconfident investor should still trade little, because the investor should recognize that one personal signal is minor relative to the aggregated signals of millions of other investors, some of whom are highly expert. In contrast, a cursed investor ignores those other signals, and hence trades too readily.

However, an investor could be overconfident about the uniqueness of his signal, not just its quality. Suppose, for example, that security value $x = x_1 + x_2$, that the investor believes he has a signal about x_1, and that millions of others are observing signals about x_2. Even with moderate levels of overconfidence about signal precision, such an investor may trade quite aggressively, despite being fully aware that there are many other informed players in the market.

Furthermore, we believe that cursedness does not go far to explain the phenomenon of aggressive trading by poorly informed individual investors. The cursedness argument for such trading is that such investors neglect the information of others, and therefore believe that their own trades have positive expected value. However, a poorly informed investor who is only cursed, not overconfident, understands perfectly well that the expected profitability of such trades is quite small. But in practice, making such trades is costly, owing to brokerage fees, time costs, and risk (including loss of diversification). These frictions or a modest degree risk aversion will therefore easily deter aggressive trading by investors who are cursed but understand that they are ill-informed.

Many financial economists now believe that the great bulk of individual investors—those who are not insiders, financial professionals, or remarkable amateurs—have little or no useful private information that would allow them to trade profitably in individual stocks. These are exactly the kind of ill-informed investors who—even if they suffer from cursedness—should recognize that they have negative net benefits from aggressive trading, after costs and adjusting for risk.

Finally, the empirical evidence summarized in Section 2 documents short-term return momentum and long-term return reversal in numerous markets. The model of cursedness in Eyster, Rabin and Vayanos (2013) explains momentum as a pure underreaction phenomenon. As such, it explains momentum but not long-run reversal. The overconfidence approach, in contrast, explains momentum and reversal jointly as parts of a phenomenon of continuing overreaction and sluggish correction (Daniel, Hirshleifer and Subrahmanyam 1998).

In summary, we believe that the psychology underlying cursedness is real, and that cursedness is a rich approach for understanding economic phenomena. We do not, however, see cursedness, at least taken in isolation, as offering an explanation for the key patterns presented here—excessive trading, short term momentum and long term reversal—that have motivated the use of overconfidence in models of securities markets.

5 Conclusion

This essay has two main themes: (1) There are anomalies in financial markets—unprofitable active trading, and patterns of return predictability, that are puzzling from the perspective of traditional purely rational models; and (2) models of overconfidence, and of the dynamic psychological processes that underlie overconfidence, can plausibly explain why these patterns exist and persist.

Given the powerful force of arbitrage, some economists may feel more comfortable with explanations for return predictability based on perfect rationality. To those readers, we would point out that the empirical anomalies are more or less agreed upon by the leading fans of the efficient markets hypothesis and those with a more behavioral bent. For example, the data underlying the three- and five-factor models of Eugene Fama and Kenneth French suggest that portfolios can be built that provide high returns can be achieved with relatively low volatility. The main disagreement is not over the empirical reality of the anomalies described in this paper, but about what components must be added to an asset pricing model to describe them.

More broadly, overconfidence also promises to help integrate behavioral financial theory, because it provides a plausible foundation for other far-reaching theories. For example, some authors have emphasized the importance of investor disagreement in understanding financial markets (Hong and Stein 2007). Overconfidence provides a natural explanation for why investors who process the same public information end up disagreeing so much. Limited investor attention has also recently been offered as an explanation for various empirical patterns in trading and prices. Overconfidence explains why investors who neglect important information would nevertheless trade aggressively, so that such neglect can influence price. So overconfidence offers a microfoundation for other important building blocks of behavioral models.

Appendices

A An Alternative Formulation of Overconfidence

As an extension of the overconfidence model in Section 3.2, in this section we explore an alternative formulation of overconfidence first proposed in Scheinkman and Xiong (2003, SX).[13] In contrast with the specification of the model presented in the main text, in which the overconfident investor perceives that the investor's private signal has lower variance than it actually does, in the SX specification the overconfident investor thinks that the invetor is observing a signal that is highly correlated with innovations in firm value, when in reality the signal is only loosely correlated with firm value innovations.[14]

This formulation has the advantage of generating momentum effects without biased-self-attribution. However, this formulation embeds multiple assumptions about the agent's information processing. In standard modeling of overconfidence,[15] as presented in Section 3, the agent receives signal which are unbiased, but imprecise, and the overconfident agent overestimates the signal precision. In this alternative formulation, the signal the agent receives is biased toward the prior, or to put it differently, it is more strongly aligned with old information than the individual realizes, and it is this bias that generates the momentum effect. While *overprecision* is well documented in the psychology literature, we are not aware of psychological evidence for the idea that people underestimate the degree to which their signals are aligned with with old information (after taking into account any overprecision).

To illustrate this alternative formulation of overconfidence, we construct a simple model like that in Section 3.2, but with a structure that captures the SX structure. As in the model in Section 3.2, true asset value θ is drawn from a common knowledge prior distribution:

$$\theta = \bar{\theta} + \epsilon_0, \tag{1}$$

where $\epsilon_0 \sim \mathcal{N}(0, 1/\tau_0)$ The timeline for the model is as follows: at date 0, the agent knows only the prior distribution, and the price $P_0 = \bar{\theta}$. At time 1 the agent observes distinct hard

[13]This specification of is also used in Alti and Tetlock (2013) and Kelley and Tetlock (2013).

[14]See pp. 1189-1190, Scheinkman and Xiong (2003)

[15]By 'standard' we mean the signal as truth plus noise, as used in the models of Kyle and Wang (1997), Odean (1998), Daniel, Hirshleifer and Subrahmanyam (1998), Daniel, Hirshleifer and Subrahmanyam (2001), Hirshleifer and Luo (2001), and others.

(h) and soft (s) signals of the form:[16]

$$\begin{aligned}
\sigma_h &= \theta + \epsilon_h \\
\sigma_s &= \bar{\theta} + \eta\epsilon_0 + \sqrt{1-\eta^2}\ \epsilon_s.
\end{aligned} \tag{2}$$

At time 2, θ is revealed and $P_2 = \theta$.

ϵ_h and ϵ_s are mean zero, normally distributed with precisions τ_s, and τ_h.[17] However, SX model the investor's overconfidence as leading her to believe that the private/soft signal is:

$$\sigma_s = \bar{\theta} + \eta_C\epsilon_0 + \sqrt{1-\eta_C^2}\ \epsilon_s, \tag{3}$$

where $\eta_C > \eta$.

To see how this is distinct from the standard overconfidence setting, note that equations (2) and (3) can be rewritten as:

$$\begin{aligned}
\sigma_s &= \eta\theta + (1-\eta)\bar{\theta} + \sqrt{1-\eta^2}\ \epsilon_s \\
\sigma_s &= \eta_C\theta + (1-\eta_C)\bar{\theta} + \sqrt{1-\eta_C^2}\ \epsilon_s
\end{aligned}$$

In the setting in Section 3.2, overconfident investors underestimate the variance of ϵ_s. In contrast, in this setting an "overconfident" investor (with $\eta_C > \eta$) not only underestimates the signal variance, but also overestimates the extent to which σ_s is pushed away from the prior $\bar{\theta}$ and towards the true value of θ. As a result, when the agent's overconfidence is of this form, P_1 will be pushed away from θ and towards $\bar{\theta}$.

To better illustrate the importance of this assumption, consider an extreme setting where $\eta = 0$, implying $\sigma_s = \bar{\theta} + \epsilon_s$—so that the informed investor's signal is equal to the mean of the prior distribution plus pure noise. However, assume that the investor is severely overconfident, meaning that $\eta_C = 1$, implying that the investor believes that her signal is unbiased, and infinite precision — *i.e.*, $\sigma_s = \theta$. Thus, $P_1 = \bar{\theta}$, while the rational expected time 2 price, conditional on the hard signal σ_h, is:

$$\mathbb{E}^R[P_2|\sigma_h] = \mathbb{E}^R[\theta|\sigma_h] = \left(\frac{\tau_0}{\tau_0+\tau_h}\right)\bar{\theta} + \left(\frac{\tau_h}{\tau_0+\tau_h}\right)\sigma_h.$$

[16] Alti and Tetlock (2013) label these signals as hard and soft, rather than as public and private.

[17] In the Section 3.2 specification, the public and private signals were revealed at times 1 and 2 respectively. Here, they arrive simultaneously at time 1.

Thus,

$$\mathbb{E}^R[r_{12}|\sigma_h] = \left(\frac{\tau_h}{\tau_0 + \tau_h}\right)(\sigma_h - \bar{\theta}).$$

Thus, in a setting where the investor both underestimates the noise variance in the private signal, and underestimates the extent to which the signal is shrunk towards the prior, there will be underreaction to public information signals, and a form of public-signal linked momentum will result. However, if all signals are unbiased—i.e., the true value θ plus noise—then additional model structure is necessary to generate the observed underreaction to public information, and price momentum.

How consistent is are the psychological evidence on overconfidence with these two possible specifications? The SX specification assumes a combination of overestimating signal precision (as in the standard overconfidence approach) and a distinct second bias of believing (above and beyond the effects of any misperception of signal precision) that the realized signal is closer to the true value than it really is. We view the psychological underpinning of overconfidence (that people think they are good at generating high quality signals) and the psychological evidence of overprecision as more supportive of the first bias than the second.

B A proof that in the Model 2 setting, $\mathbb{E}[R_{2,3}|s_B] = 0$

From the equation:

$$P_2 = \frac{1}{\tau_0 + \hat{\tau}_V + \tau_B}\left(\tau_0\bar{\theta} + \hat{\tau}_V s_V + \tau_B s_B\right),$$

$$\mathbb{E}[P_2|s_B] = \frac{1}{\tau_0 + \hat{\tau}_V + \tau_B}\left(\tau_0\bar{\theta} + \tau_B s_B + \hat{\tau}_V \mathbb{E}[s_V|s_B]\right). \tag{4}$$

However,

$$\mathbb{E}[s_V|s_B] = \mathbb{E}[\theta|s_B] = \frac{1}{\tau_0 + \tau_B}\left(\tau_0\bar{\theta} + \tau_B s_B\right).$$

Substituting this into equation (4) yields:

$$\mathbb{E}[P_2|s_B] = \frac{1}{\tau_0 + \hat{\tau}_V + \tau_B}\left(\tau_0\bar{\theta} + \tau_B s_B\right)\left(1 + \frac{\hat{\tau}_V}{\tau_0 + \tau_B}\right).$$

References

Alti, Aydogan and Paul C Tetlock. 2013. "Biased Beliefs, Asset Prices, and Investment: A Structural Approach." *The Journal of Finance*, 69(1): 325–361.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing and Xiaoyan Zhang. 2009. "High idiosyncratic volatility and low returns: International and further U.S. evidence." *Journal of Financial Economics*, 91(1): 1–23.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing and Xioayan Zhang. 2006. "The cross-section of volatility and expected returns." *The Journal of Finance*, 61(1): 259–299.

Asness, Clifford S., Tobias J. Moskowitz and Lasse Heje Pedersen. 2013. "Value and Momentum Everywhere." *Journal of Finance*, 68(3): 929–985.

Aumann, Robert J. 1976. "Agreeing to Disagree." *Annals of Statistics*, 4(6): 1236–1239.

Baker, Malcolm and Jeffrey Wurgler. 2000. "The Equity Share in New Issues and Aggregate Stock Returns." *Journal of Finance*, 55(5): 2219–2257.

Baker, Malcolm P., Brendan Bradley and Jeffrey Wurgler. 2011. "Benchmarks as Limits to Arbitrage: Understanding the Low Volatility Anomaly." *Financial Analysts Journal*, 67(1): 40–54.

Banz, Rolf W. 1981. "The Relationship between Return and the Market Value of Common Stocks." *Journal of Financial and Quantitative Analysis*, 14: 421–441.

Barber, Brad and Terrance Odean. 2000*a*. "Trading is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors." *Journal of Finance*, 55(2): 773–806.

Barber, Brad and Terrance Odean. 2000*b*. "Trading is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors." *Journal of Finance*, 55(2): 773–806.

Barber, Brad and Terrance Odean. 2001. "Boys will be Boys: Gender, Overconfidence, and Common Stock Investment." *Quarterly Journal of Economics*, 116(1): 261–292.

Barber, Brad and Terrance Odean. 2002. "Online Investors: Do the Slow Die First?" *Review of Financial Studies*, 15(2): 455–488.

Barber, Brad, Terrance Odean and Ning Zhu. 2009. "Do Retail Trades Move Markets?" *Review of Financial Studies*, 22(1): 151–186.

Barberis, Nicholas, Andrei Shleifer and Robert Vishny. 1998. "A Model of Investor Sentiment." *Journal of Financial Economics*, 49(3): 307–343.

Barberis, Nicholas and Ming Huang. 2001. "Mental Accounting, Loss Aversion, and Individual Stock Returns." *Journal of Finance*, 56(4): 1247–1292.

Barberis, Nicholas and Wei Xiong. 2012. "Realization Utility." *Journal of Financial Economics*, 104(2): 251–271.

Ben-David, Itzhak, John R. Graham and Campbell R. Harvey. 2013. "Managerial Miscalibration." *Quarterly Journal of Economics*, 128(4): 1547–1584.

Benoît, Jean-Pierre, Juan Dubra and Don A. Moore. 2015. "Does the Better-Than-Average Effect Show that People are Overconfident?: Two Experiments." *Journal of the European Economic Association*, 13(2): 293–329.

Bernard, Victor L. and Jacob K. Thomas. 1989. "Post-Earnings-Announcement Drift: Delayed Price Response or Risk Premium?" *Journal of Accounting Research*, 27: 1–36.

Bernard, Victor L. and Jacob K. Thomas. 1990. "Evidence that Stock Prices Do Not Fully Reflect the Implications of Current Earnings for Future Earnings." *Journal of Accounting and Economics*, 13: 305–340.

Biais, Bruno, Denis Hilton, Karine Mazurier and Sebastien Pouget. 2005. "Judgmental Overconfidence, self-monitoring and trading performance in an experimental financial market." *Review of Economic Studies*, 72: 287–312.

Black, Fischer, Michael Jensen and Myron Scholes. 1972. "The Capital Asset Pricing Model: Some Empirical Tests." In *Studies in the Theory of Capital Markets.* , ed. Michael C. Jensen, 79–121. New York:Praeger.

Camerer, Colin and Dan Lovallo. 1999. "Overconfidence and excess entry: An experimental approach." *American economic review*, 306–318.

Carhart, Mark M. 1997. "On Persistence in Mutual Fund Performance." *Journal of Finance*, 52(1): 57–82.

Chan, Wesley S. 2003. "Stock price reaction to news and no-news: drift and reversal after headlines." *Journal of Financial Economics*, 70(2): 223–260.

Choi, James J., David Laibson and Andrew Metrick. 2002. "How does the Internet affect trading? Evidence from investor behavior in 401(k) plans." *Journal of Financial Economics*, 64(3): 397–421.

Collin-Dufresne, Pierre and Kent Daniel. 2014. "Liquidity and Return Reversals." Columbia GSB working paper.

Cooper, Michael J., Roberto C. Gutierrez and Allaudeen Hameed. 2004. "Market States and Momentum." *Journal of Finance*, 59(3): 1345–1365.

D'Acunto, Francesco. 2013. "Identity, Overconfidence, and Investment Decisions." Working paper, University of California, Berkeley.

Daniel, Kent D., David Hirshleifer and Avanidhar Subrahmanyam. 1998. "Investor Psychology and Security Market Under- and Over-reactions." *Journal of Finance*, 53(6): 1839–1886.

Daniel, Kent D., David Hirshleifer and Avanidhar Subrahmanyam. 2001. "Overconfidence, Arbitrage, and Equilibrium Asset Pricing." *Journal of Finance*, 56(3): 921–965.

Daniel, Kent D., Ravi Jagannathan and Soohun Kim. 2015. "Tail Risk in Momentum Strategy Returns." Kellogg School working paper.

Daniel, Kent D. and Sheridan Titman. 1999. "Market Efficiency in an Irrational World." *Financial Analysts' Journal*, 55(6): 28–40.

Daniel, Kent D. and Sheridan Titman. 2006. "Market Reactions to Tangible and Intangible Information." *The Journal of Finance*, 61(4): 1605–1643.

Daniel, Kent D. and Tobias J. Moskowitz. 2014. "Momentum Crashes." Columbia Business School working paper.

DeBondt, Werner F. M. and Richard H. Thaler. 1985. "Does the Stock Market Overreact?" *Journal of Finance*, 40(3): 793–808.

DeBondt, Werner F. M. and Richard H. Thaler. 1995. "Financial Decision-Making in Markets and Firms: A Behavioral Perspective." In *Finance, Handbooks in Operations Research and Management Science*. Vol. 9, , ed. Robert A. Jarrow, Voijslav Maksimovic and William T. Ziemba, Chapter 13, 385–410. Amsterdam:North Holland.

Diether, Karl B., Christopher J. Malloy and Anna Scherbina. 2002. "Differences of Opinion and the Cross Section of Stock Returns." *Journal of Finance*, 57: 2113–2141. 5.

Dong, Ming, David A. Hirshleifer and Siew Hong Teoh. 2012. "Overvalued Equity and Financing Decisions." *Review of Financial Studies*, 25(12): 3645–3683.

Edwards, W. 1968. "Conservatism in human information processing." In *Formal Representation of Human Judgment*. , ed. Benjamin Kleinmuntz, 17–52. New York:John Wiley & Sons.

Eyster, Erik and Matthew Rabin. 2005. "Cursed Equilibrium." *Econometrica*, 73(5): 1623–1672.

Eyster, Erik, Matthew Rabin and Dimitri Vayanos. 2013. "Financial Markets where Traders Neglect the Informational Content of Prices." University of California, Berkeley working paper.

Fama, Eugene F. 1970. "Efficient Capital Markets: A Review of Theory and Empirical Work." *Journal of Finance*, 25(2): 383–417.

Fama, Eugene F. and James MacBeth. 1973. "Risk, Return and Equilibrium: Empirical Tests." *Journal of Political Economy*, 81: 607–636.

Fama, Eugene F. and Kenneth R. French. 1992. "The Cross-Section of Expected Stock Returns." *Journal of Finance*, 47(2): 427–465.

Fama, Eugene F. and Kenneth R. French. 1993. "Common risk factors in the returns on stocks and bonds." *Journal of Financial Economics*, 33: 3–56.

Frazzini, Andrea and Lasse Heje Pedersen. 2014. "Betting Against Beta." *Journal of Financial Economics*, 111(1): 1–25.

Frazzini, Andrea and Lasse H. Pedersen. 2013. "Betting Against Beta." *Journal of Financial Economics*, 111(1): 1–25.

French, Kenneth R. 2008. "Presidential Address: The Cost of Active Investing." *Journal of Finance*, 63(4): 1537–1573.

Friedman, Milton. 1953. "The Methodology of Positive Economics." In *Essays in Postive Economics*. 3–43. Chicago:University of Chicago Press.

Froot, Kenneth A. and Richard H. Thaler. 1990. "Anomalies: Foreign Exchange." *Journal of Economic Perspectives*, 4(3): 179–192.

George, Thomas and Chuan-Yang Hwang. 2004. "The 52-Week High and Momentum Investing." *Journal of Finance*, 59(5): 2145–2176.

Gervais, Simon and Terrance Odean. 2001. "Learning to be Overconfident." *Review of Financial Studies*, 14(1): 1–27.

Glaser, Markus, Martin Weber and Thomas Langer. 2013. "True Overconfidence in Interval Estimates: Evidence Based on a New Measure of Miscalibration." *Journal of Behavioral Decision Making*, 26(5): 405–417.

Graham, John R., Campbell R. Harvey and Hai Huang. 2009. "Investor Competence, Trading Frequency, and Home Bias." *Management Science*, 55(7): 1094–1106.

Griffin, John M., Federico Nardari and Rene M. Stulz. 2007. "Do Investors Trade More When Stocks Have Performed Well? Evidence from 46 Countries." *Review of Financial Studies*, 20(3): 905–951.

Grinblatt, Mark and Bing Han. 2005. "Prospect theory, mental accounting, and momentum." *Journal of Financial Economics*, 78(2): 311–339.

Grinblatt, Mark and Matti Keloharju. 2009. "Sensation Seeking, Overconfidence, and Trading Activity." *Journal of Finance*, 64(2): 549–578.

Grossman, Sanford J. 1976. "On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information." *Journal of Finance*, 31(2): 573–585.

Grossman, Sanford J. and Joseph E. Stiglitz. 1976. "Information and Competitive Price Systems." *American Economic Review*, 66(2): 246–253.

Hansen, Lars P. and Kenneth J. Singleton. 1983. "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns." *Journal of Political Economy*, 91: 249–265.

Hansen, Lars P. and Ravi Jagannathan. 1991. "Implications of Security Market Data for Models of Dynamic Economies." *Journal of Political Economy*, 99(2): 225–262.

Hansen, Lars P. and Ravi Jagannathan. 1997. "Assessing Specification Errors in Stochastic Discount Factor Models." *Journal of Finance*, 52(2): 557–590.

Harrison, John M. and David M. Kreps. 1978. "Speculative investor behavior in a stock market with heterogeneous expectations." *Quarterly Journal of Economics*, 93: 323–336.

Henderson, Brian J., Narasimhan Jegadeesh and Michael S. Weisbach. 2006. "World Markets for Raising New Capital." *Journal of Financial Economics*, 82(1): 63–101.

Hirshleifer, David. 2001. "Investor Psychology and Asset Pricing." *Journal of Finance*, 64(4): 1533–1597.

Hirshleifer, David, Avanidhar Subrahmanyam and Sheridan Titman. 1994. "Security Analysis and Trading Patterns when Some Investors Receive Information Before Others." *Journal of Finance*, 49(5): 1665–1698.

Hirshleifer, David and Guo Ying Luo. 2001. "On the Survival of Overconfident Traders in a Competitive Security Market." *Journal of Financial Markets*, 4(1): 73–84.

Hirshleifer, David and Siew Hong Teoh. 2003. "Limited Attention, Information Disclosure, and Financial Reporting." *Journal of Accounting & Economics*, 36(1-3): 337–386.

Hirshleifer, David, Siew Hong Teoh and Jeff Jiewei Yu. 2011. "Short Arbitrage, Return Asymmetry and the Accrual Anomaly." *Review of Financial Studies*, 24(7): 2429–2461.

Hoelzl, Erik and Aldo Rustichini. 2005. "Overconfident: Do You Put Your Money On It?*." *The Economic Journal*, 115(503): 305–318.

Hong, Harrison G., José A. Scheinkman and Wei Xiong. 2006. "Asset Float and Speculative Bubbles." *Journal of Finance*, 61(3): 1073–1117.

Hong, Harrison and Jeremy C. Stein. 2007. "Disagreement and the Stock Market." *Journal of Economic Perspectives*, 21(2): 109–128.

Ikenberry, David, Josef Lakonishok and Theo Vermaelen. 1995. "Market Underreaction to Open Market Share Repurchases." *Journal of Financial Economics*, 39(2-3): 181–208.

Jegadeesh, Narasimhan and Sheridan Titman. 1993. "Returns to buying winners and selling losers: Implications for stock market efficiency." *Journal of Finance*, 48(1): 65–91.

Jensen, Michael C. 1968. "The Performance of Mutual Funds in the Period 1945-1964." *Journal of Finance*, 23(2): 389–416.

Johnson, Dominic D.P. 2009. *Overconfidence and war.* Harvard University Press.

Kahneman, Daniel. 1973. *Attention and Effort.* Englewood Cliffs, New Jersey:Prentice-Hall.

Kahneman, Daniel. 2011. *Thinking, Fast and Slow.* New York, NY:Farrar, Straus and Giroux.

Kahneman, Daniel and Amos Tversky. 1972. "Subjective probability: A judgment of representativeness." *Cognitive Psychology*, 3(3): 430–454.

Kahneman, Daniel and Amos Tversky. 1979. "Prospect Theory: An analysis of decision under risk." *Econometrica*, 47: 263–291.

Keim, Donald B. 1983. "Size Related Anomalies and Stock Return Seasonality: Further Evidence." *Journal of Financial Economics*, 12: 13–32.

Kelley, Eric K. and Paul C. Tetlock. 2013. "Why Do Investors Trade?" Working paper, University of Arizona.

Kyle, Albert and F. Albert Wang. 1997. "Speculation Duopoly With Agreement to Disagree: Can Overconfidence Survive the Market Test?" *Journal of Finance*, 52(5): 2073–2090.

Lakonishok, Josef, Andrei Shleifer and Robert W. Vishny. 1994. "Contrarian investment, extrapolation and risk." *Journal of Finance*, 49: 1541–1578.

Lamont, Owen A. and Richard H. Thaler. 2003. "Can the Market Add and Subtract? Mispricing in Tech Stock Carve-Outs." *Journal of Political Economy*, 111(2): 227–268.

Langer, Ellen J. and Jane Roth. 1975. "Heads I win, tails it's chance: The illusion of control as a function of the sequence of outcomes in a purely chance task." *Journal of Personality and Social Psychology*, 32(6): 951–955.

Loughran, Tim and Jay Ritter. 1995. "The New Issues Puzzle." *Journal of Finance*, 50(1): 23–52.

Malkiel, Burton G. 2013. "Asset Management Fees and the Growth of Finance." *Journal of Economic Perspectives*, 27(2): 97–108.

Malmendier, Ulrike and Geoffrey Tate. 2005. "CEO Overconfidence and Corporate Investment." *Journal of Finance*, 60(6): 2661–2700.

Mehra, Rajnish and Edward Prescott. 1985. "The Equity Premium: A Puzzle." *Journal of Monetary Economics*, 15(2): 145–161.

Merkle, Christoph and Martin Weber. 2011. "True Overconfidence: The Inability of Rational Information Processing to Account for Apparent Overconfidence." *Organizational Behavior and Human Decision Processes*, 116(2): 262–271.

Milgrom, Paul and Nancy Stokey. 1982. "Information, Trade and Common Knowledge." *Journal of Economic Theory*, 26(1): 17–27.

Miller, Edward. 1977. "Risk, Uncertainty, and Divergence of Opinion." *Journal of Finance*, 32(4): 1151–1168.

Moskowitz, Tobias J. 2015. "Asset Pricing and Sports Betting." Chicago Booth Research Paper 15-26.

Nagel, Stefan. 2005. "Short sales, institutional investors and the cross-section of stock returns." *Journal of Financial Economics*, 78(2): 277–309.

Neale, Margaret A. and Max H. Bazerman. 1985. "The effects of framing and negotiator overconfidence on bargaining behaviors and outcomes." *Academy of Management Journal*, 28(1): 34–49.

Odean, Terrance. 1998. "Volume, Volatility, Price and Profit When All Traders are Above Average." *Journal of Finance*, 53(6): 1887–1934.

Odean, Terrance. 1999. "Do Investors Trade too Much?" *American Economic Review*, 89(5): 1279–1298.

Peng, Lin and Wei Xiong. 2006. "Investor attention, overconfidence and category learning." *Journal of Financial Economics*, 80(3): 563–602.

Pontiff, Jeffrey and Artemiza Woodgate. 2008. "Share Issuance and Cross-Sectional Returns." *The Journal of Finance*, 63(2): 921–945.

Puri, Manju and David T. Robinson. 2007. "Optimism and Economic Choice." *Journal of Financial Economics*, 86(1): 71–99.

Scheinkman, Jose A. and Wei Xiong. 2003. "Overconfidence, Short-Sale Constraints, and Bubbles." *Journal of Political Economy*, 111: 1183–1219.

Sloan, Richard. 1996. "Do stock prices fully reflect information in accruals and cash flows about future earnings?" *Accounting Review*, 71(3): 289–315.

Spiess, D. Katherine and John Affleck-Graves. 1995. "Underperformance in Long-Run Stock Returns Following Seasoned Equity Offerings." *Journal of Financial Economics*, 38: 243–268.

Stambaugh, Robert F., Jianfeng Yu and Yu Yuan. 2012. "The short of it: Investor sentiment and anomalies." *Journal of Financial Economics*, 104(2): 288–302.

Statman, Meir, Stephen Thorley and Keith Vorkink. 2006. "Investor Overconfidence and Trading Volume." *Review of Financial Studies*, 19(4): 1531–1565.

Tetlock, Paul C. 2011. "All the News That's Fit to Reprint: Do Investors React to Stale Information?" *Review of Financial Studies*, 24(5): 1481–1512.

Thaler, Richard H. 1985. "Mental Accounting and Consumer Choice." *Marketing Science*, 4(3): 199–214.

Tirole, Jean. 1982. "On the possibility of speculation under rational expectations." *Econometrica*, 50(5): 1163–1181.

Tversky, Amos and Daniel Kahneman. 1974. "Judgment under Uncertainty: Heuristics and Biases." *Science*, 185(4157): 1124–1131.

Weil, Philippe. 1989. "The Equity Premium Puzzle and the Risk-Free Rate Puzzle." *Journal of Monetary Economics*, 24: 401–421.

Weinstein, Neil D. 1980. "Unrealistic Optimism about Future Life Events." *Journal of Personality and Social Psychology*, 39(5): 806–820.

Xiong, Wei and Jialin Yu. 2011. "The Chinese Warrants Bubble." *American Economic Review*, 101(6): 2723–2753.