Optimists, pessimists and stock prices*

Kent Daniel† Alexander Klos‡ Simon Rottke§

June 8, 2024

Abstract

We review the academic findings from psychology and economics on disagreement, and specifically on the effect of disagreement on asset prices. We discuss measurement of disagreement, and how disagreement coupled with constraints on short selling can sideline pessimistic investors and result in overpricing. We review the literature on short selling in financial markets, paying particular attention to how and why some issues become "hard-to-borrow", what factors go into the determination of borrowing costs, and discuss the evolution of borrowing costs over the last several decades. We show how an examination of the prices and borrowing costs for constrained stocks can lead to an improved understanding of how disagreement in financial markets arises and is resolved, and finally discuss directions for future research.

Keywords: disagreement, share lending, short-selling, mispricing.
JEL codes: G02, G12.

1 Introduction

In 1906, 787 people estimated the weight of a fat ox in a competition run at the annual show of the West of England Fat Stock and Poultry Exhibition. Attendees of the event were probably among the most qualified people in the UK for this particular prediction task. Nevertheless, Galton (1907) reports a large dispersion in their estimates. The difference between the 95% and the 5% quantile was 1,293 − 1,074 = 219 pounds. The mean estimate of 1,197 pounds, however, was exactly equal to the actual weight of the ox (Wallis, 2014).

Treynor (1987) examines a jellybean jar experiment to motivate market efficiency in a setting without short-sale constraints. Participants had to estimate the number of beans in a jar. Similar to Galton (1907), the mean estimate was quite accurate, despite considerable disagreement.[1]

In the late nineties, Welch (2000) asked financial economists for their one-year equity risk premium estimate. Predictions ranged from −9.5% to 18%. If the optimists and pessimists in an arguably homogeneous

*We thank Patrick Bolton, Owen Lamont, Kari Sigurdsson, and seminar participants at École Polytechnique Fédérale de Lausanne and University of Amsterdam for helpful comments and suggestions.

†Columbia Business School, New York, NY, USA, 10027 and National Bureau of Economic Research, Cambridge, MA, USA, 02138; email: kd2371@columbia.edu

‡Kiel University, Kiel, Germany, 24098; email: alexander.klos@qber.uni-kiel.de

§University of Amsterdam, The Netherlands, 1018 TV; email: simon.rottke@uva.nl

[1] One interpretation of these disagreement experiments could be that the different estimates simply reflect different and incomplete information on the part of the study participants, and that once participants learn the estimates of the other participants they would revise their estimates to full-information values, and posterior estimates would be identical (as in Grossman, 1976). At least in repeated jellybean-jar classroom experiments that one of us has conducted with economics PhD students, this does not happen. The variance of second round estimates is slightly lower and, inconsistent with Milgrom & Stokey (1982), most participants maintain their divergent views even after learning others' estimates.

group — finance professors who are likely aware of the level of disagreement — strongly disagree on the expected excess return of the market, how strongly would we expect optimists and pessimists of a more heterogeneous group of people to agree on the expected returns of a single security?[2]

These examples suggest that differences in beliefs are substantial, even among experts. What does a high degree of disagreement imply for the price of an asset? According to Miller (1977), optimists will set the price if their demand is large enough to absorb the fixed number of shares outstanding. To the extent that buyers are too optimistic about the future cash flows of the asset, overpricing will result. The stronger the optimists' views, the greater the overpricing will be. If, however, pessimists can create "new" shares through short selling, Miller's basic argument will fail. In a frictionless world with unrestricted short selling, the asset price will equal the consensus view (a weighted average of the agents' beliefs) regardless of the disagreement about future cash flows. With short sale restrictions, either regulatory or due to market frictions, overpricing will be present, and mispricing will depend on the magnitude of disagreement and the nature of the short-sale frictions.

This paper surveys the literature that examines and pushes forward Miller's argument. Our focus is on equity markets, where the data of most studies comes from. We will cover the nature of frictions in the equity lending market, discuss empirical proxies for disagreement and short sale constraints, and discuss recent attempts to leverage Miller's idea to a dynamic setting. Our discussion partly draws on results from psychology.[3] Still, our review will be selective in that we will review only the work most relevant for understanding financial markets. Thus, the reader should not expect an overview of the extensive literature on disagreement in philosophy and psychology.

Notably, our setting, and the Miller setting in general, is distinct from alternative settings in which there is common knowledge (Aumann, 1976). Agents do not draw inferences from the observed disagreement. Thus the setting for our paper is also inconsistent with those of Milgrom & Stokey (1982) and especially Diamond & Verrecchia (1987), in which agents both understand the implications of short-sale constraints on price formation and incorporate information of others into their belief formation process. In the Diamond & Verrecchia (1987) setting, there is no upward bias in prices resulting from the presence of short-sale constraints. The reason we abandon the common knowledge assumption is the considerable evidence from the behavioral literature that agents do not properly weigh the beliefs of others, and the evidence we will cover here suggesting that short-sale constrained securities do exhibit an upward bias in prices.

With this review, we would like to convey to the reader an understanding that constrained stocks can be a laboratory for studying disagreement in financial markets. For constrained securities for which the borrowing fee is non-zero, there is necessarily a different price for buying and selling. To be concrete, consider a security where we see transactions at a price of \$100/share, and short sales taking place when the annualized borrowing fee is 50% (something not that uncommon in today's market). This is at least *prima facie* evidence of strong disagreement: we know that there are investors who are willing to pay \$100/share and not lend out their shares, and others who are willing to pay a 50% borrowing fee to short at the same price of \$100/share. Studying how prices and borrowing fees evolve over time for such securities can lead to a richer understanding of how disagreement arises and how it is eventually resolved.

[2]Note that there is still considerable disagreement present in the results of the second survey in Welch (2000), which was conducted "after the first write-up of this article was available" (p. 509).

[3]See Barberis (2018) for a broad overview of psychology-based asset pricing models.

2 A short selective review of stylized facts about disagreement

An individual's beliefs refer to their mental model of the world and of how that world will evolve over time. In formal probability theory (Billingsley, 1995), a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ has three elements: the sample space, the event space or σ-algebra, and a probability measure on $\mathcal{F}$. In this formal setting, the investors' "beliefs" refer to the $(\Omega, \mathcal{F}, \mathcal{P})$ they use to understand the world and to make forecasts. In psychology "generally speaking, beliefs refer to a person's subjective probability judgment s concerning some discriminable aspect of his world; they deal with the person's understanding of himself and his environment." (Fishbein & Ajzen, 1975, p. 131). We are specifically concerned with the individual's forecasts of future uncertain outcomes, and the probability that they assign to those outcomes. We will define an individual as being rational if, from a statistical perspective, the probabilities that they assign to different events in the event space are consistent with observed outcomes. In contrast, a person is biased if, statistically, the probabilities they assign to events do not line up with the outcome frequency. In our applications, the empirical manifestation of a biased belief will sometimes be a consistently incorrect prediction of some moment of the distribution of an outcome such as the mean or the variance of a firms' earnings.[4] In our setting in which individuals disagree, if the aggregated probabilities assigned to events, with an appropriate weighting function on the individuals, are rational and unbiased, we label this as a "wisdom of crowds" effect (see Surowiecki, 2004, and numerous others).

We will start our survey in this section with a few stylized facts about beliefs, disagreement and the development of disagreement over time. We highlight these facts because of their relevance for the Miller (1977) argument.

1. *Beliefs influence economic choices.* There is now a large literature that connects beliefs to economic actions. This literature looks at a broad spectrum of economic agents, such as retail investors (Giglio et al., 2021) or homeowners (Kuchler et al., 2023). Bachmann et al. (2023) provide a broad overview of this specific research, as well as the use of expectations data in general economics research.

2. *Some financial information is subject to interpretation.* Intuitively, there is good reason to believe that different people interpret the same piece of information differently. For example, NVIDIA's market capitalization has more than tripled during 2023. It seems natural that momentum and value traders think differently about NVIDIA's expected return over the following year. Consistent with this idea, Cookson & Niessner (2020) report that about half of the disagreement in their social media dataset comes from disagreement across investment approaches.

3. *New information can either decrease or increase disagreement.* A natural assumption is that new information decreases disagreement. An earnings announcement, for example, is a piece of new public information that presumably causes the beliefs of extreme optimists and extreme pessimists to converge. Consistent with this idea, Berkman et al. (2009) present evidence that values of common disagreement proxies decrease around earnings announcements on average.

   However, if investors differ in their interpretation of the new information or disagree on the consequences of that information, increased disagreement may result. Indeed, Daniel et al. (2023b) show that the dispersion of analysts' beliefs on average increases after an earnings announcement *if* disagreement is *low* initially.

[4] Mankiw, Reis & Wolfers (2003) and others examine biases and disagreement in inflation expectations, and Greenwood & Shleifer (2014) examine biases in expectations of stock returns.

4. *Disagreement is persistent.* Giglio et al. (2021) analyze a bi-monthly survey of wealthy Vanguard clients. The survey elicited clients' expectations of future stock returns, GDP, and bond returns. Giglio et al. (2021) observe large and strikingly persistent heterogeneity in beliefs. Optimists tend to stay optimists, and pessimists tend to stay pessimists. Consistent with this, Daniel et al. (2023a) find individual stock-level disagreement takes about five years to be resolved, on average.

5. *Some beliefs are biased.* For example, beliefs about future market returns show evidence of return extrapolation (Greenwood & Shleifer, 2014). Investor expectations of future stock market returns are positively correlated with past performance, while many popular models predict a negative correlation.

6. *Some sophisticated investors are miscalibrated.* Miscalibration of beliefs describes the fact that the range of possible outcomes is systematically underestimated. Ben-David et al. (2013) report evidence of miscalibration among corporate managers in a 10-year panel of the Duke CFO survey. Interestingly, managers who are miscalibrated regarding the overall stock market are also miscalibrated regarding their own firm.

Miller (1977)'s argument combined with these stylized facts suggest that disagreement combined with security lending market frictions will lead to mispricing. It is a necessary condition that beliefs transform into actions. The fact that disagreement fluctuates over time suggests that mispricing of constrained stocks changes over time. If disagreement is persistent, then mispricing should be persistent in the presence of a continuously binding short-sale constraint. If the beliefs of some market participants are biased and/or miscalibrated, then insights from the behavioral sciences about how biases and miscalibration evolve over time may be helpful for predictions about mispricing persistence.

One could argue that disagreement does not matter for market outcomes. Biased agents would lose money on average and would either be driven out of the market or learn to behave in a Bayesian way. Careful theoretical analysis, however, shows that biased investors can survive in a market environment (Long et al., 1991) and that biased beliefs can even arise as a consequence of successful trading (Gervais & Odean, 2001). A further argument is that strategies run by arbitrageurs with large amounts of money under their control will correct any mispricing. However, a large literature shows that arbitrageurs face constraints in profiting from mispricings, meaning that they will not always "undo" these mispricings (see Gromb & Vayanos, 2010, for a survey). Short-sale constraints are one important limit to arbitrage, and here we will argue that these constraints can facilitate large and persistent overpricing.

3 A description of the share lending market

The standard way to take a short position in a security is to borrow that security from an owner, committing to return an equivalent security when the owner requests it, and then to sell the borrowed security. In addition, there is generally a transfer of collateral from the borrower to the lender in the form of some safe security (e.g., cash or treasury securities) with a value equal to the market valuation of the borrowed securities plus some amount of margin. In the US, the amount of margin required is governed by SEC regulations and/or by the self-imposed requirements of the borrower.[5] The lender is required to pay interest on the collateral at some overnight risk-free rate—usually close to the Federal-Funds rate—minus some fee. As of December 2023, slightly more than 60% of US common stocks had an annualized indicative fee of

[5]Per SEC Regulation T, the Federal Reserve sets the minimum initial margin for stock loans by securities brokers and dealers in the United States to their clients, which is currently 50%.

less-than 1%. A stock with a fee less than or equal to 1% is referred to as a general-collateral or GC stock; a stock that has a fee greater than 1% is called *special*. The total interest a borrower earns on the stock loan collateral they post is labeled the *rebate rate*, and if the fee is sufficiently large, then the stock will have a negative rebate rate. For loans of US equity securities, stock loans are overnight but are usually renewed daily. However, either the borrower or the lender can terminate the loan at any time.

In the US, as of May 28, 2024, there is a one-day settlement period (referred to as "T+1") between when shares of common stock are sold, and when the actual shares must be delivered. That is, the trade settles on the day after it was initiated. Similarly, if a stock loan is recalled, borrowers are required to deliver the borrowed shares to the lender within a day.

Short-sellers must *locate* shares in advance of their sale, meaning they must either borrow or have reasonable grounds for expecting that they will be able to borrow the shares they are selling. However, market makers can engage in *naked* short selling, meaning they are exempt from the pre-locate requirement. Of course, market makers must actually borrow (or buy) shares before the settlement date.[6]

3.1 Rehypothecation

In the US, exchanges monitor the amount of short interest, which is the number of shares that have been borrowed and sold. Shares can be lent, or rehypothecated, multiple times. Thus short interest can and sometimes does exceed the number of shares outstanding.

Also, every actual share certificate has some eventual owner who is, necessarily, not lending the share out. This owner has voting rights, and will receive all dividends from the corporation. Once the owner of a share lends it out, they are no longer the owner and lose their voting rights and their entitlement to receive dividends. Instead of the dividend payment, they receive a payment-in-lieu from the borrower.

One implication of the above is that 100% of shares outstanding are not lent out. Every share, even if it is lent out and then sold to someone else, eventually must end up in the hands of someone who does not lend it out. Even if that "someone else" rehypothecates it, it will then be sold to someone else again (and possibly again), until someone will not lend it out again. Consequently, 100% of the shares outstanding are necessarily foregoing the fees that could be earned by lending the shares.

3.2 Share lenders

Brokerage firm clients can either hold shares in *margin* or *non-margin* accounts. In the US, all shares held in margin accounts are held in street name,[7] and can be lent by the brokerage firm for the purpose of short selling. In contrast, shares held in non-margin accounts are not available for lending.

Historically, in the US, most brokerage firms did not pass on the lending fees to the brokerage account holders of the lent shares. This has changed in recent years, and at least several brokerage firms have started paying half of the fee to the margin account holders, and to other account holders who agree to lend their shares.[8]

Institutional investors are a large source of share loans in the US. Institutions are often willing to lend out their shares, and often employ a *lending agent* to assist them with these loans. In many cases, the custodian is the lending agent. The lending agent finds borrowers for the shares, and can provide indemnification if

[6] This is governed by SEC Reg SHO, rule 203(b)(1) and 203(b)(2), see https://www.federalregister.gov/d/04-17571/p-396, last accessed, May 20, 2024.

[7] See https://www.sec.gov/answers/street.htm, last accessed, May 20, 2024.

[8] For example, Interactive Brokers and Vanguard.

desired or required by the lender. In exchange, the lending agent receives a "split" of the revenue collected from the borrower. Rizova (2011, p. 11) reports that mutual fund lending agents receive between 14% and 50% of the fee.[9]

Once the insitution's shares are lent, typically to a prime broker, the prime broker will then rehypothecate those shares to their clients, who are often hedge funds. The prime broker charges a still higher fee to their clients. Thus in aggregate the fee received by the initial share lender (e.g., the mutual fund) may be far less than the fee paid by the eventual borrower of the shares (e.g., the hedge fund)

Mutual funds face a number of regulatory restrictions on share lending. In particular, mutual funds are not allowed to lend out more than one third of the value of their total assets at the portfolio level (meaning they can lend more than that in individual positions).[10]

Most share loans are not done through a centralized exchange. This was not the case historically: Jones & Lamont (2002) discuss the NYSE lending post which ceased operation in the 1930s. Recently there have been some attempts to centralize share lending (also see our Anecdote 1).

The supply of shares available for borrowing can become restricted, particularly if a number of shares are held by individuals in non-margin accounts. If the demand for borrowing exceeds the supply, then, rationing takes place via an increase in the fee that is charged. As we discuss later, there has been a large increase in the fraction of stocks that are "special" in the last several decades. For the most extreme specials, the annualized fee can exceed 1,000%. For an extreme special stock with an annualized fee of 1,000%, and assuming a federal funds rate of 5%, the borrower would earn interest on the funds they deposit with the lender at a rebate rate of $(5\% - 1000\%) = -995\%$ per year. Thus short selling such extreme negative rebate rate stocks is only profitable if the position is held for a short period of time, and if the stock price declines precipitously over the holding period.

Lending the shares of such high fee stocks is extremely lucrative, but as noted above, there are multiple frictions that make this less profitable for lenders than it immediately appears.

4 A static view on disagreement and financial markets

Section 2 argues that high disagreement is a prevalent feature of financial markets. This section presents a model that illustrates how a combination of disagreement and the constraints on short selling discussed in Section 3 affects security market prices. We will use the model as guidance in our literature review on the nature of the frictions in the lending market, empirical strategies, and unsettled research questions. Our endeavor here will be static; we will only briefly touch on how disagreement evolves over time. Dynamic issues will be mainly delegated to Section 5.

4.1 A static model of disagreement in financial markets

The following highly stylized two-period model illustrates how a combination of disagreement and borrowing constraints can affect security prices. In Section 5, we expand this to a multi-period model to investigate how dynamic considerations—information arrival and the resolution of uncertainty—change these conclusions. Throughout, we build on the dynamic model in Daniel et al. (2023b), but formulate it in this section in terms of a static model, similar to Blocher et al. (2013).

[9] See Johnson & Weitzner (2024) for more recent evidence on security lending agents.

[10] See https://www.sec.gov/investment/divisionsinvestmentsecurities-lending-open-closed-end-investment-companieshtm and https://www.sec.gov/divisions/investment/noaction/1997/brinsonfunds112597.pdf, last accessed, May 20, 2024.

The model has two types of agents: *passive investors* and *speculators*. In aggregate, passive investors hold all Q of the outstanding shares of the risky security. In addition we specify that the passive investors lend out a fraction λ of these shares even in the limit as the fee they receive for doing so approaches zero. They will lend out more shares if the fee they receive for doing so is large, as we discuss below.

The speculators are further divided into two groups, each with unit measure: *optmists* and *pessimists*. Each speculator acts as a price takers and has CARA utility. They have a common Arrow-Pratt measure of risk aversion γ. The speculators do not lend out the shares they purchase, and consequently do not receive any lending fees. However, if the speculators choose to take a short position in the risky asset, they are required to borrow it from the passive investors and pay the borrowing costs or, equivalently, the lending fee.

There are two periods indexed by $t \in \{1, 2\}$, a riskfree asset with a return of zero, and a single risky asset. At time $t = 2$, the risky asset pays a liquidating divided which is drawn from $\tilde{D} \sim \mathcal{N}(\mu, \sigma^2)$, where σ^2 is common knowledge. However, at time $t = 1$ the speculators disagree about the mean of this distribution: the optimists believe that the mean of the liquidating dividend is equal to $\mathbb{E}^O\{\tilde{D}\}$, while the pessimists believe that the mean is equal to $\mathbb{E}^P\{\tilde{D}\} < \mathbb{E}^O\{\tilde{D}\}$. We are agnostic about why they disagree (but see the previous section). The rational expectation in this model is $\mathbb{E}^R\{\tilde{D}\} = \mu$. The consensus view is equal to rational expectation if $\frac{1}{2}\left(\mathbb{E}^O\{\tilde{D}\} + \mathbb{E}^P\{\tilde{D}\}\right) = \mu$.

The passive investors lend out a fraction λ of the shares of the risky asset even if the lending fee they receive for doing so is zero. We also assume that the quantity they will lend increases with the lending fee they earn. Specifically we assume the quantity of shares supplied is:

$$X = \lambda Q + \frac{1}{\tau}c, \tag{1}$$

for non-negative borrowing costs $c \geq 0$, and where $X \leq Q$.[11] Alternatively, we can write the borrowing costs as a function of the shares supplied to the lending market as:

$$c = \tau\,(X - \lambda Q) \tag{2}$$

for $X \geq \lambda Q$, and $c = 0$ otherwise. Here τ is a measure of search costs, which we will discuss further below.

In this setup, notice that: (1) the speculators set the price of the risky asset; (2) the holdings of the optimists and pessimists must sum to zero (because the passive investors hold all outstanding shares); (3) the optimists always take a long position and the pessimists always take a short position in the asset; and (4) if the quantity borrowed and shorted by the pessimists exceeds λQ, then the borrowing costs c will be positive.

With this framework, we can solve for the equilibrium share price P at time 1. Given the CARA-normal setup and the other model assumptions, the demand of the optimists will be positive, and equal to:

$$d^O = \frac{\mathbb{E}^O\{\tilde{D}\} - P}{\gamma\sigma^2}. \tag{3}$$

[11] Our simple, stylized model does not permit rehypothecation, so the maximum number of shares lent is equal to the number held by the institutional investor, Q. In reality the ratio of short interest to shares outstanding will sometimes exceed 100% as a result of rehypothecation.

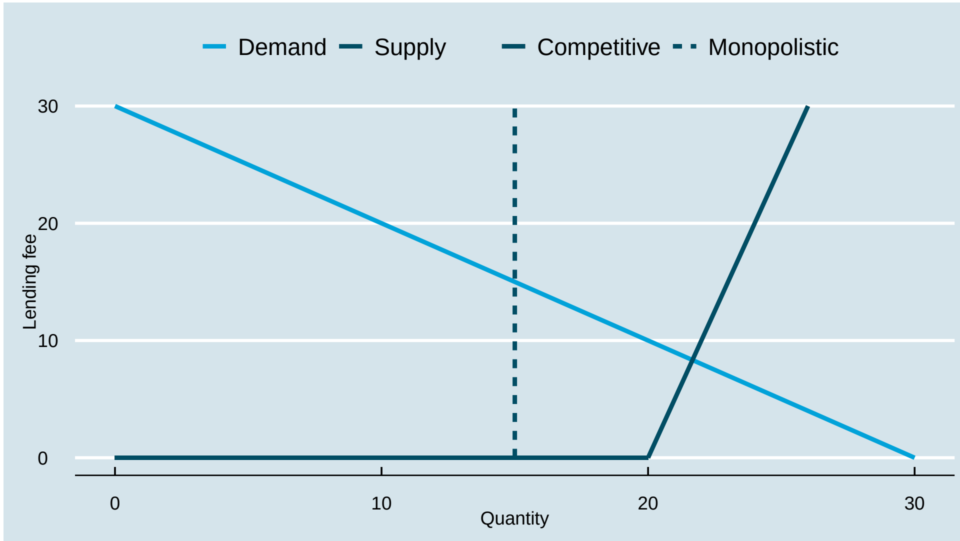

Figure 1: The figure illustrates demand and supply curves in the lending market. Parameters for the (solid) competitive-market curves are τ=5, λ=.2, σ=γ=1, Q=100, $\mathbb{E}^P\{\tilde{D}\}$=70, and $\mathbb{E}^O\{\tilde{D}\}$=130. The (dashed) monopolistic supply curve is discussed in Section 4.2.

The demand of the pessimists will be negative, i.e., they will go short. Since the pessimists pay per-share borrowing costs of c, their demand is:

$$d^P = -\frac{P - \mathbb{E}^P\{\tilde{D}\} - c}{\gamma\sigma^2} \quad \text{for} \quad c \geq 0. \tag{4}$$

Market clearing requires that the demand of the optimists and pessimists sum to zero:

$$\frac{\mathbb{E}^O\{\tilde{D}\} - P}{\gamma\sigma^2} - \frac{P - \mathbb{E}^P\{\tilde{D}\} - c}{\gamma\sigma^2} = 0, \tag{5}$$

and that the quantity of shares borrowed (and shorted) is less than or equal to the quantity supplied by the passive investors. The borrowing costs are zero if $\frac{P-\mathbb{E}^P\{\tilde{D}\}}{\gamma\sigma^2} \leq \lambda Q$. The borrowing costs are positive if $\frac{P-\mathbb{E}^P\{\tilde{D}\}}{\gamma\sigma^2} > \lambda Q$. We say a stock is "constrained" if $c > 0$.

Market clearing in the share lending market requires that, for a constrained stock,

$$\frac{P - \mathbb{E}^P\{\tilde{D}\} - c}{\gamma\sigma^2} = \lambda Q + \frac{1}{\tau}c. \tag{6}$$

In other words, imposing market clearing in the lending and the stock market uniquely identifies both the equilibrium share price P and the equilibrium borrowing costs c (Blocher et al., 2013, Daniel et al., 2023b, Atmaz et al., 2024, Gârleanu et al., 2023, Sikorskaya, 2023). Intuitively, the share price equilibrates demand from optimists and supply from (short selling) pessimists, and the borrowing costs equilibrate demand for borrowing shares from short sellers and supply from the passive investors who lend shares.

The solid lines in Figure 1 illustrate the demand and supply curves in the share-lending market when the lenders behave competitively; we will discuss the (dashed) monopolistic supply curve in Section 4.2. In this figure, the demand to borrow shares at $c = 0$ is 30, and is above the free lending supply of $\lambda Q = 20$, so to equilibrate the supply and demand the borrowing costs c must rise to a level where supply and demand are equal. However, the demand to borrow shares is a function of the share price, and the share price is a function of the borrowing costs.

To see this, we can rearrange (5) to get the market clearing price of the stock as a function of the (equilibrium) borrowing costs:

$$P = \frac{1}{2}\left(\mathbb{E}^O\{\tilde{D}\} + \mathbb{E}^P\{\tilde{D}\}\right) + \frac{1}{2}c \tag{7}$$

In the special case where a "wisdom-of-crowds" effect holds—that is if a rational agent's expected dividend is equal to the average of the optimists' and pessimists' expectations $\mathbb{E}^R\{\tilde{D}\} = \frac{1}{2}\left(\mathbb{E}^O\{\tilde{D}\} + \mathbb{E}^P\{\tilde{D}\}\right)$—then Equation (7) implies P is "rational" if $c = 0$, but that overpricing will occur if $c > 0$.

Equation (6) shows that frictions in the lending market are crucial. In a perfectly functioning lending market, where new shares are easy to find (i.e., in the limit as $\tau \to 0$), competition between lenders will drive borrowing costs to zero, and the equilibrium price in (7) will be equal to the consensus expectation of the dividend.

However, when a stock is constrained, solving (6) for c yields

$$c = \frac{\tau}{\gamma\sigma^2 + \tau}\left(P - \mathbb{E}^P\{\tilde{D}\} - \lambda Q\gamma\sigma^2\right) \tag{8}$$

The equilibrium price is

$$P = \frac{\gamma\sigma^2 + \tau}{2\gamma\sigma^2 + \tau}\left(\mathbb{E}^O\{\tilde{D}\} + \mathbb{E}^P\{\tilde{D}\}\right) - \frac{\tau}{2\gamma\sigma^2 + \tau}\left(\mathbb{E}^P\{\tilde{D}\} + \lambda Q\gamma\sigma^2\right) \tag{9}$$

and the equilibrium per-share borrowing costs are

$$c = \frac{\tau}{2\gamma\sigma^2 + \tau}\left(\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\} - 2\lambda Q\gamma\sigma^2\right) \tag{10}$$

The revenues earned by the security lenders total $c\left(\lambda Q + \frac{1}{\tau}c\right)$.

Figure 2 illustrates the effect of disagreement on borrowing costs and equilibrium prices when the share lenders behave competitively (solid lines). Beyond a certain threshold (governed by the kink in the supply curve in the lending market, at λ), borrowing costs become non-zero and equilibrium prices start increasing with disagreement. The dashed lines, which show equilibrium prices and borrowing fees when lenders behave monopolistically, are discussed in Section 4.2.

The static model nests several interesting special cases. First, note that if shares for lending are easy to find ($\tau \to 0$, no "kink" in the supply curve), then the borrowing costs will be zero ($\lim_{\tau\to 0} c = 0$) and the asset price will be equal to $\lim_{\tau\to 0} P = \frac{1}{2}\left(\mathbb{E}^O\{\tilde{D}\} + \mathbb{E}^P\{\tilde{D}\}\right)$. If the wisdom-of-crowds effect holds (as defined earlier) then the price is equal to the dividend expected by the rational agent.

Second, in the limit as the search costs approach infinity ($\tau \to \infty$) and if disagreement is sufficiently high to push the borrowing costs above 0 (i.e., if $\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\} > 2\lambda Q\gamma\sigma^2$), then the pessimists will short all of the λQ shares available for borrowing, and the optimists will buy them. The risky asset price P and borrowing costs c must be set so that the optimists and pessimists will do so. The optimists must, in aggregate, hold the λQ shares, that is, $\lambda Q = \left(\mathbb{E}^O\{\tilde{D}\} - P\right)/\gamma\sigma^2$. The pessimists choose to short λQ shares, that is, $\lambda Q = \left(P - \mathbb{E}^P\{\tilde{D}\} - c\right)/\gamma\sigma^2$. Here the first equation determines the risky asset price: $P = \mathbb{E}^O\{\tilde{D}\} - \lambda Q\gamma\sigma^2$, and substituting this price into the second equation gives the borrowing costs: $c = \mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\} - 2\lambda Q\gamma\sigma^2$.

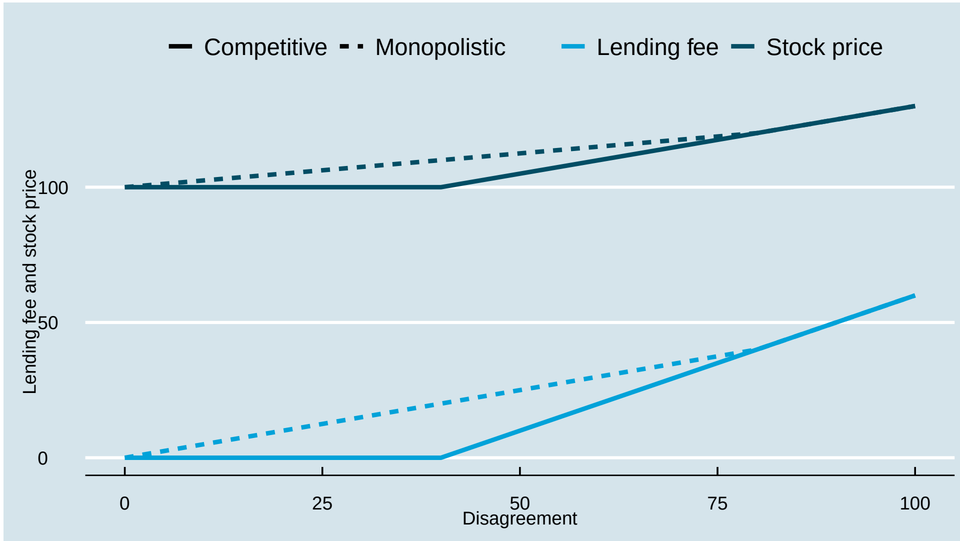

Figure 2: The figure plots equilibrium stock price and borrowing costs as a function of disagreement when share lenders compete (solid lines) and when the share lender behaves monopolistically (dashed line). Disagreement is defined as $\mathbb{E}^O\{\tilde{D}\}-\mathbb{E}^P\{\tilde{D}\}$, and the average expected payout next period is always 100. Further parameters are τ=∞, λ=.2, σ=γ=1, and Q=100.

Finally, in the limit where the number of shares available for borrowing approaches zero (i.e., as $\lambda \to 0$) and search costs approach infinity ($\tau \to \infty$), the above analysis shows that the price and borrowing costs approach $P = \mathbb{E}^O\{\tilde{D}\}$ and $c = \mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\}$. That is, the price is the optimists' expectation of $\tilde{D}$, and the borrowing costs c are the level of disagreement.

4.2 Frictions in the lending market

Short sellers must borrow shares before selling them. In Equation (1), we model the lending supply as $\lambda Q + \frac{1}{\tau}c$. This simplified framework captures the basic empirical facts of the share lending market. D'Avolio (2002) shows that the majority of stocks can be borrowed at a small fee. We capture this fact by allowing for free lending up to a fraction λ of the available shares Q. Free lending supply could come from index funds and other large institutional investors with large lending programs. Here, intense competition and very small marginal costs should lead to a willingness to lend out shares for a tiny fee that captures administrative costs. In the model, we assume that passive institutional investors with large holdings are lending out for free. Institutional ownership is a useful proxy for free lending supply when taking the model to the data.[12]

Once free lending supply is exhausted, brokers must search for additional shares to borrow (Duffie et al., 2002). Finding shares involves leg work: Identifying potential lenders among smaller or even individual investors, and convincing them to convert their accounts to margin accounts. The smaller and/or otherwise obscure the stock, the more difficult such an endeavor would be.

The model captures this idea by assuming that the search costs of finding a new share are constant and equal to τ. The equilibrium fee in Equation (1) is $c = \tau(X - \lambda Q)$ and reflects that these search costs must be paid by the short sellers for every share that exceeds free lending supply.

[12] Sikorskaya (2023) argues theoretically and shows empirically that small constrained stocks included in an index experience an increase in lending supply, but also an increase in demand and fee.

An interesting additional friction can arise from imperfect competition in the share lending market. We consider an extreme example within our model to illustrate the argument. Say the lender is a monopolist and able to hold back shares. Assume for simplicity, that it is impossible for potential borrowers to find additional shares ($\tau \to \infty$), and that disagreement is large enough so that shorting demand at zero costs will exceed free lending supply of λQ, that is: $\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\} > 2\lambda Q\gamma\sigma^2$. If the monopolistic lender chooses to lend out S shares instead of λQ, then her revenues are

$$cS = \left(\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\} - 2S\gamma\sigma^2\right) S \tag{11}$$

Maximizing (11) yields

$$S^* = \frac{\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\}}{4\gamma\sigma^2} \tag{12}$$

A monopolistic lender has an incentive to restrict supply whenever $S^* < \lambda Q$. If disagreement is low initially and the equilibrium lending fee would be zero in a competitive market $\left(\frac{P-\mathbb{E}^P\{\tilde{D}\}}{\gamma\sigma^2} \leq \lambda Q\right)$, then a single lender would generate revenues by withholding shares, and artificially creating a constrained stock.

The dashed vertical line in Figure 1 illustrates the behavior of a monopolistic lender in a numerical example. Instead of lending out the amount of shares implied by the competitive equilibrium represented by the solid lines, she holds back shares and only lends out S^* shares (15 in this example). Thereby, she maximizes her revenues, leading to higher borrowing costs and greater overpricing of the risky asset.

The comparative statics of this can be seen in Figure 2, focusing on the dashed lines. Borrowing costs (and the equilibrium stock price) increase with disagreement from zero onward. This means that stocks that would be unconstrained and fairly priced with a competitive lending market (for any level of disagreement left of the "kink") are now short-sale constrained and exhibit overpricing. If disagreement rises to a level that implies $S^* = \lambda Q$, the monopolist no longer has an incentive to hold back shares, and the monopolistic market generates the same outcomes as the competitive market.

There is not much literature on the role of intermediaries in the lending markets. In a recent contribution, Chen et al. (2024) discuss how intermediaries may inhibit competition, potentially (and paradoxically) even benefiting both lenders and short sellers (if the latter value secrecy).

Anecdote 1: Share lenders against prime-brokers in a legal case

In 2001, some of the major investment banks founded a platform called EquiLend to improve the securities lending workflow. In 2017 they were sued by pension funds and other investors, who accused them of "relegat[ing] the stock lending market to the stone age" by using their board positions on EquiLend to boycott startup platforms in order to keep monopoly control over the market and charge excessive lending fees since 2009. Up until August 2023, a subset of the banks has settled for a combined $580m in damages.

Case 1:17-cv-06221-KPF-SLC

4.3 How to test Miller's hypothesis?

The model developed in Section 4.1 allows us to think about empirical strategies in a structured way. Specifically, Equation (7) suggests that overpricing should be positively related to lending fees. Panel data of lending fees with broad cross-sectional and time-series coverage has recently become available—specifically the Markit lending fee data provides daily lending fees for almost every US common stock listed on major exchanges since 2010 (see Jones & Lamont, 2002, Jones, 2012, for studies using lending market data from 1926 to 1933). A number of papers do indeed use lending fee data, either from Markit or from alternative sources (Boehme et al., 2006, Cohen et al., 2007, Prado et al., 2016, Blocher & Ringgenberg, 2018, Cookson et al., 2022). However, since tests of asset pricing models often require longer time-series to achieve sufficiently high levels of statistical power, it is important to come up with good alternative proxies for borrowing constraints when direct measures of the borrowing costs are not available.

With a fixed number of outstanding shares, Miller's empirical prediction is that overpricing for a single stock is an increasing function of the dispersion in beliefs. Short interest is an observable equilibrium outcome that is undoubtedly strongly related to disagreement.

In our model, short interest is equal to the shorting demand of the pessimist. However, Equation (6) shows that short interest for a constrained stock does not only depend on the amount of disagreement ($\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\}$) but also on the free lending supply λQ, the riskiness of the asset σ^2, the risk aversion of speculators γ, and the search costs τ to find new shares to borrow. Each of these quantities is stock-specific, making cross-sectional analyses harder to interpret. In other words, stocks with low short interest may be stocks with low disagreement or stocks with strongly binding short-sale constraints. Stocks with high short interest may be constrained stocks with high disagreement or stocks with moderate disagreement and no short-sale constraints. Examining the relationship between disagreement proxies and stock returns is certainly interesting, but tests of Miller (1977) should appropriately control for binding short-sale constraints in the lending market.

A similar argument applies to looking at institutional holdings and future stock returns. The institutional ownership ratio, defined as the market value of the holdings scaled by the stock's market capitalization, is a reasonable proxy for the share of free lending supply λ. However, the static model shows that cross-sectional variation in institutional ownership should only matter for constrained stocks with high search costs. Here, the missing shorting demand and the unobserved search costs confound results.

The model suggests that the most convincing proxy for shorting costs is a suitable combination of short interest and institutional ownership. Recent studies use short interest divided by institutional ownership (SIRIO, Drechsler & Drechsler, 2016), or independent sorts on these two quantities (Asquith et al., 2005, Daniel et al., 2023a). Historically, however, short interest and institutional ownership have been used as individual proxies for short-sale constraints on their own. We include these studies in our review of qualitative results in the next section. From a quantitative perspective, results reported in papers that use only short interest or only institutional ownership should be viewed as lower bounds for the real economic importance of short-sale constraints.

4.4 Empirical evidence on Miller's hypothesis

Miller's hypothesis is overwhelmingly supported by empirical analyses. Stocks with high short interest have negative abnormal future returns (Figlewski, 1978, Asquith & Meulbroek, 1996, Desai et al., 2002). High disagreement predicts negative abnormal performance going forward (Diether et al., 2002, Goetzmann & Massa, 2005). Stocks which have experienced a reduction in the number of mutual fund owners, presumably leading to tighter short-sale constraints, underperform subsequently (Chen et al., 2002). Disagreement can predict increases in short interest and low returns around options introductions, consistent with the hypothesis that the option market somewhat mitigates short-sale constraints (Boehme et al., 2006). The returns of stocks with high disagreement are negative around earnings announcements (Berkman et al.,

2009) and EDGAR inclusions (Chang et al., 2022), suggesting resolution of disagreement. Studies using proprietary data from the lending market or studies that simultaneously proxy for supply and demand in the lending market allow a better identification of constrained stocks and their findings further support Miller's hypothesis (Jones & Lamont, 2002, Asquith et al., 2005, Cohen et al., 2007).

Using a high-frequency disagreement measure based on StockTwits, a social media site devoted to discussion of investment issues, and daily lending fee data, Cookson et al. (2022) find high returns on days with increased disagreement, particularly for stocks with a high ratio of short interest to lendable value. They also show that overpricing caused by disagreement attracts trading activities of activists and short sellers.

Miller's overpricing hypothesis is a leading explanation for the long-run underperformance of IPOs (Ritter & Welch, 2002). It is difficult to short a new firm, and optimists set the price at the end of the first trading day. As the firm ages, more shares become available in the lending market, and short-sale constraints become less binding. In experimental asset markets in the spirit of Smith et al. (1988), relaxing short-sale constraints, ceteris paribus, leads to lower prices (see, for example, Haruvy & Noussair, 2006). Last, short-sale constraints are positively related to the before-cost profitability of quantitative strategies designed to exploit mispricing (Nagel, 2005, Hirshleifer et al., 2011, Stambaugh et al., 2012, Drechsler & Drechsler, 2016, Engelberg et al., 2023, Muravyev et al., 2023).

4.5 Directions for further research

4.5.1 Why do short-sale constraints matter so much?

A natural question to ask is how severe short-sale constraints are in reality. One of the earliest papers examining this question with lending fee data was D'Avolio (2002), who used proprietary data on lending fees. He found that, on average over his sample period of April 2000–September 2001, 91% of stocks had annualized borrowing costs of less than 1%.

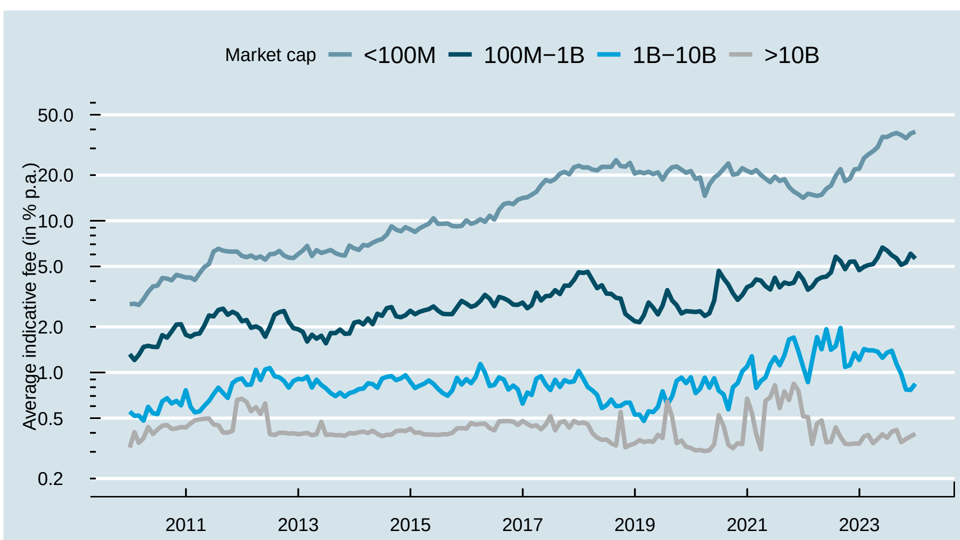

Figure 3: The figure shows the time-series of equally-weighted daily indicative fees as reported by Markit by different size groups over time.

This has changed. While financial markets have become more efficient on many dimensions, this appears not to be the case in the stock lending market. Borrowing costs for the very largest stocks have remained about the same. However, for low market capitalization stocks (<100M USD) traded on major US exchanges, Daniel et al. (2024) report an increase in the average indicative lending fee across firms from around 2.8% in 2010 to almost 40% in 2023. Even for larger stocks, the fee has more than quadrupled, from roughly 1.3% in 2010 to around 5.6% in 2023 for stocks between 100M and 1B USD market capitalization, and increased by more than 50% for stocks between 1B and 10B USD. While D'Avolio (2002) found around 9% of stocks were "hard-to-borrow" by his definition (fee >1% p.a., which is also often cited by practitioners as the threshold for "specialness"), by 2010 that share was already up to 12%, and it has since increased to almost 40% of all stocks in 2023.

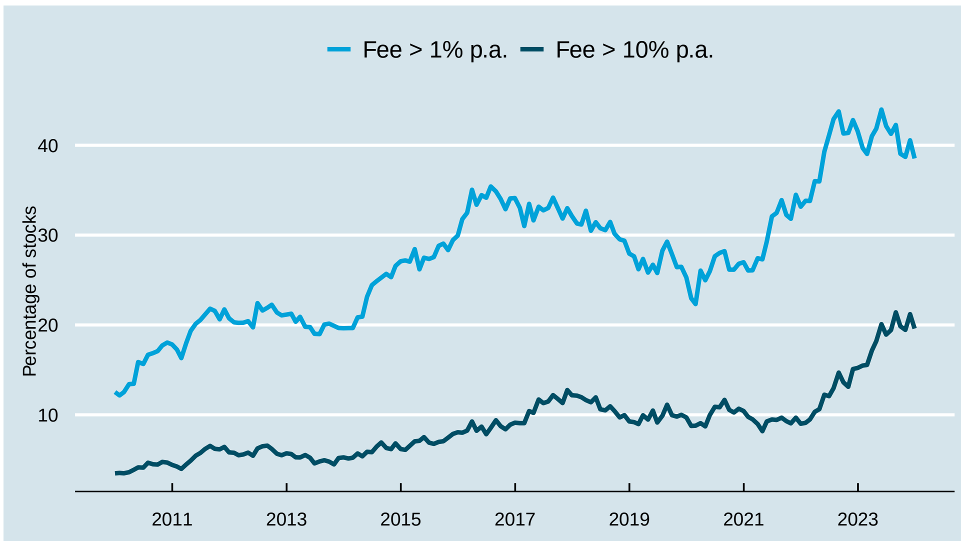

Figure 4: The figure plots the fraction of US common stocks with a CRSP sharecode of 1,2, or 3 with borrowing costs larger than 1% or 10% per annum based on the indicative fee as reported by Markit, as a function of time.

This means that, today, about 40% of US stocks are "hard-to-borrow" and therefore potentially subject to the Miller mechanism illustrated earlier. The picture does not look much better internationally (Japan being the exception), where similar trends in lending fees can be observed (Daniel et al., 2024). The trend implies a potentially less efficient price discovery process (Bris et al., 2007, Saffi & Sigurdsson, 2010, Boehmer & Wu, 2012, Beber & Pagano, 2013). It could also be related to the fact that many quantitative strategies appear to still generate positive excess returns out-of-sample before costs (Jensen et al., 2023), while they seem to be unprofitable after accounting for short-selling costs (Muravyev et al., 2022).

This trend raises questions. What could have driven the increase in fees for small firms? Is the lending market competitive or could there be monopolistic tendencies (see Chen et al., 2024, as well as our box on a recent legal case) and have such tendencies worsened over time? Why does supply not react to such large fees? Any long-investor should be happy to receive fees of 10% or more as additional income for simply lending out stock. What keeps them from making their shares available to the market? What do these recent trends imply for the decision to lend rather than sell (Evans et al., 2017)? Is there a connection to recent trends in the distribution of market capitalization moving to fewer much larger firms (Gao et al., 2013, Doidge et al., 2017, Autor et al., 2020, Schlingemann & Stulz, 2022), to frictions related to passive indexing or institutional lending regulation (e.g. Sikorskaya, 2023), or to price discrimination in the securities lending market (Chague et al., 2017, Barbosa et al., 2020)?

4.5.2 Understanding supply and demand in the lending market

Estimating demand and supply curves is hard and a classic topic in economics (Wright, 1928). Estimating demand and supply curves in the lending market is arguably harder than in most other markets. Supply and demand of different stocks are probably different, and supply and demand of the same stock likely changes quickly over time.

A standard response is to find a set of instruments that influences demand but not supply. Kolasinski et al. (2013) use short-term technical trading indicators as instruments for short-term demand and fit a quadratic supply curve. Beneish et al. (2015) examine the empirical relationship between common indicators of overvaluation and lending supply. Chen et al. (2024) present evidence consistent with big lenders holding back inventory and analyze the economic consequences.

However, financial economists have not yet fully understood the drivers of supply and demand in the lending market (just as for supply and demand in equity markets, see, for example Koijen & Yogo, 2019). For example, what causes large disagreement shocks? Answering this and related questions is a promising avenue for further research.

4.5.3 Are short sellers always right?

In the context of the static model presented in Section 4.1, we introduced the concept of the wisdom-of-crowds (Surowiecki, 2004). In our simple model, wisdom-of-crowds implies that optimists and pessimists are equally wrong in their assessment of firm value but their beliefs' aggregation leads to rational prices for unconstrained securities (see Da & Huang, 2020, for empirical evidence on the wisdom-of-crowds effect in asset markets). However, empirical analysis of constrained stocks' realized returns goes strongly against the wisdom-of-crowds hypothesis. Specifically, some evidence suggests that short sellers tend to possess superior information and earn zero or slightly positive risk-adjusted returns, even after accounting for stock borrowing costs (Boehmer et al., 2008, Engelberg et al., 2012, Jones et al., 2016, Boehmer et al., 2020). Consistent with this, "hard-to-borrow" stocks earn negative alphas that are roughly equal in magnitude to the fee, meaning that the most optimistic investors who buy these stocks and do not lend them earn negative and highly statistically significant alphas. Summarizing this evidence, Reed (2013) states:

> From an academic perspective, one of the most robust findings of the literature is the fact that short sellers are generally informed traders, meaning short sales predict negative future returns (p. 255).

This evidence could still be consistent with a wisdom-of-crowds effect if only the most sophisticated investors participate in short selling and these investors are informed and unbiased, but a much broader set of investors with biased views, and who (irrationally) agree to disagree, take long but not short positions in securities markets.

Moreover, value, momentum, and other anomalies, even for stocks which are unconstrained, suggest biases on the part of the average investor. Also, consistent with the hypothesis that even the most sophisticated investors are not fully rational, there is an emerging literature documenting suboptimal investment behavior among institutional investors (Edelen et al., 2016, von Beschwitz et al., 2021, Akepanidtaworn et al., 2023). One may argue that short sellers are a particularly educated subgroup of institutional investors, but at least in the data set of von Beschwitz & Massa (2020) short sellers exhibit a disposition effect. Overall, while the literature has successfully established that short sellers are a more sophisticated group of investors, the extent to which they are able to fully eliminate the effect of behavioral biases on their trading decisions is not yet clear.

In the next section, we present a model in which both optimists and pessimists suffer from the same biases (Daniel et al., 2023a,b). As we show, in unconstrained stocks, the biases of optimists and pessimists largely cancel out, but not completely. The crowd is therefore not wise. The result is momentum and long-term reversal for unconstrained stocks. However, when stocks become constrained, the larger biases of the optimists (and pessimists) are reflected in prices and far greater return predictability results. As we discuss, this is consistent with empirical evidence.

5 The dynamics of disagreement

In the previous sections we have mainly taken a static perspective. In this section we investigate the influence of disagreement on stock prices in a dynamic perspective where agents can learn and disagreement can evolve. If a high-disagreement stock is overpriced, how will mispricing evolve over time?

An important question is how biased beliefs can persist in the first place. A lot of information is available to investors, and there are plenty of opportunities to learn.

In economics, a large literature on motivated beliefs has evolved over the previous twenty years (see Bénabou & Tirole, 2016, for a review). In short, there are things that people want to believe. If they want to believe that they are skilled or that their initially formed beliefs are correct, they will be incentivized to downplay information contradicting this view. Instead, they will focus on information supporting their priors. This idea is reminiscent of confirmation or confirmatory bias (Wason, 1960, Klayman & Ha, 1987) studied in the psychology literature. Rabin & Schrag (1999) define confirmatory bias as follows (p. 38): "A person suffers from confirmatory bias if he tends to misinterpret ambiguous evidence as confirming his current hypotheses about the world." It will take time to convince such a person that he got it wrong in the first place. These ideas are consistent with the persistence of retail investors' beliefs reported in Giglio et al. (2021) and the persistence of analyst disagreement documented in Daniel et al. (2023a). They are also consistent with self-attribution bias which has been applied in the behavioral finance literature (Daniel et al., 1998).

Direct evidence on motivated beliefs over time comes from the lab and the field. Zimmermann (2020) shows in laboratory experiments that the effect of positive feedback is persistent, while negative feedback only has a transitory effect on beliefs about one's own relative performance in an IQ task. After a month, negative feedback is less accurately recalled than positive feedback. Gödker et al. (2024) provide experimental evidence that investors tend to over-remember positive investment outcomes and suppress memories of negative ones, leading them to become overconfident. Huffman et al. (2022) study food store managers that are partly paid based on past performance. These managers are overconfident about their own performance. Furthermore, they have overly positive memories of their past performance. Biased recollection of positive performance and overconfidence are positively correlated.

Perhaps surprisingly, having more information than others may even lead to higher degrees of overconfidence. In Oskamp (1965), respondents received information for a prediction task in chunks. Participants felt more competent with more information, although their prediction accuracy remained unchanged.

5.1 Evidence from equity markets

In Daniel et al. (2023a) we explore the predictable returns of constrained stocks following shocks to their market values. The motivation of this examination is to study the evolution of disagreement. Consistent with the simple model in Section 4.1, we argue that market prices for the constrained stocks we examine serve as proxies for the optimistic investors' estimation of the stocks' values (*i.e., the optimists' expectation of the discounted cashflows*), and that borrowing costs and other proxies for disagreement serve as proxies for the difference between the valuations of optimistic and pessimistic investors. We explore the predictable returns over a long horizon as a way of determining how the resolution of disagreement occurs.

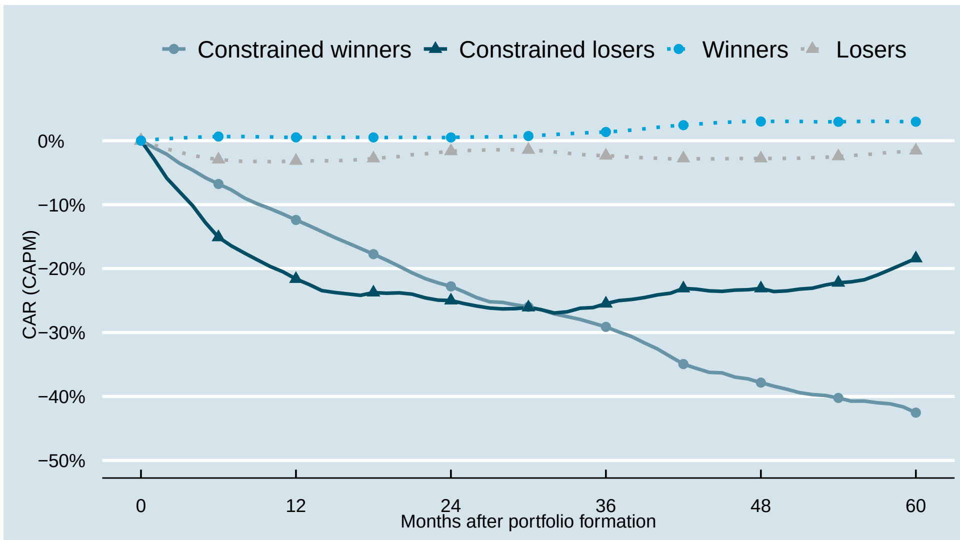

Figure 5: The solid/dotted lines present CARs for value-weighted portfolios of constrained/unconstrained firms that, over the period from $t-12$ to $t-1$ month relative to the formation date t, earned a cumulative return that put them in in the top ("winner") or bottom ("loser") 30% of firms. This plot is taken from Daniel et al. (2023a), page 2434.

The dotted lines in Figure 5 are cumulative abnormal returns (CARs) for *unconstrained* past winners and past losers, over the period from 1927:01–2020:06. These value-weighted portfolios, formed each month, are based on rankings on the cumulative return from 12 months before to 1 month before the portfolio formation date. The "winner" portfolio contains the top 30% of firms, and the "loser" portfolio the bottom 30%. Roughly speaking, what is plotted is the average cumulative abnormal return that would have been earned over horizons between 1 and 60 months.

Daniel et al. (1998) show that the momentum (Jegadeesh & Titman, 1993) and long-term reversal (DeBondt & Thaler, 1985, 1987) effects imply a hump-shaped impulse response function: that is, a positive price shock should be followed by positive returns for about one year, and then negative returns for about 3–5 years, and vice-versa for losers. Given the plot's scaling, it is difficult to see here, but both unconstrained winners and unconstrained losers exhibit this hump-shaped CAR.

The solid lines in Figure 5 show the corresponding CAR plot functions for *constrained* past winners and losers, over the period from 1980:05–2020:06 where we have access to data that allows us to identify constrained firms using a combination of short interest and institutional ownership. A comparison with the CARs for unconstrained firms shows that the patterns are dramatically different for constrained stocks. Both the constrained past winners and the constrained past losers earn strongly negative abnormal returns following the portfolio formation date. The striking difference between the winners and losers is the horizon over which these negative abnormal returns persist: for the past losers, the negative returns persist for one year: starting one year post-formation, the portfolio returns are economically small and are not statistically different from zero. In contrast, for the past winners the returns are also strongly negative and highly statistically significant in years 2–5 following portfolio formation.

Note also that the magnitudes of these forecastable returns are large: the constrained past losers lose about 20% in the first year post-formation, and the constrained past winners lose just over 50% of their value in the 5 years post-formation, relative to a market benchmark.

While statistical inference associated with CARs can be problematic (Barber & Lyon, 1996, Lyon et al., 1999), in Daniel et al. (2023a) we confirm that these results are robust and highly statistically significant using a set of time-series regressions on monthly rebalanced buy-and-hold portfolio returns. Specifically, we show that in the first year following portfolio formation, the constrained winner and loser portfolios earn monthly abnormal returns of -137 and -95 bps ($t = -3.95, -4.07$). However, in years 2-5 post-formation, the monthly abnormal return is -62 bps ($t = -4.93$) for the past winners, but +26 bps ($t = 1.03$) for the past losers, and that difference in monthly alpha of 88 bps is highly significant ($t = 4.33$).[13]

In Daniel et al. (2023a) we also show that measures of disagreement between optimists and pessimists—including the indicative fee from Markit and analyst disagreement for the part of the sample were these data are available—are large and exhibit the same level of persistence for constrained past losers and winners, suggesting that the difference in the persistence of predictable returns does not result from differences in the resolution of disagreement for winners and losers.

What can explain the difference in the persistence for the constrained past winners and losers? We argue that these differences result from persistent over-optimism on the part of the agents who take long positions in the constrained past winners, and strong but far less persistent optimism on the part of those who hold the constrained past losers. The following dynamic disagreement model generates this set of belief patterns, and the belief distortions that also result in momentum and long-term reversal effects for unconstrained stocks.

5.2 A dynamic model

The single-period model we presented in Section 4.1 built on the intuition of Miller (1977), and showed that a combination of disagreement and short-sale constraints can lead to overpricing. Here, we extend that model to a multi-period setting with the goal of studying the evolution of prices, expected returns, and disagreement/borrowing costs.

The specific dynamic model we present here delivers implications consistent with the price and disagreement dynamics for constrained and unconstrained common stocks that we presented in Section 5.1. Here, we present the model setup, the basic findings, and some intuition for these findings. Daniel et al. (2023a) and the corresponding online appendix present the full derivation of the model.

In this dynamic model, there are two assets: a risk free asset which earns a return of zero each period and a risky asset which pays an uncertain liquidating dividend $\tilde{D}_T$ at time T. To capture the information dynamics that drive the dynamics of return predictability, we follow Hong & Stein (1999) and specify that the liquidating dividend is a sum of dividend innovations each period.

[13] These abnormal returns are relative to a Fama & French (1993)-Carhart (1997) four-factor benchmark. The t-statistics are Newey & West (1987), with automatic lag selection (Newey & West, 1994).

In contrast to the Hong & Stein (1999) specification, where the innovations are mean zero, in our specification the innovations are drawn from a (time-invariant) *i.i.d.* distribution $\tilde{\epsilon}_t \sim \mathcal{N}(\mu_\epsilon, \sigma^2)$. Importantly, the agents in our model do not directly observe μ_ϵ, but do have a common prior distribution for μ_ϵ at time $t = -1$, $\mu_\epsilon \sim \mathcal{N}\left(0, \zeta^2\right)$, which they update over time as they observe the realized dividend innovations (ϵ_t's). All agents are Bayesian, but do not optimally use all information available to them.

The motivation for this specification is the following: given symmetric information at $t = -1$, all agents agree on the firm value in period $t = -1$. However, because after this point they see different parts of the information set and process this information differently, they will start to disagree about the firm's value over time. Their disagreement will result in different posterior distributions for μ_ϵ. One group will become more optimistic, meaning they think that the firm will generate higher average cashflows going forward, and the second group will be more pessimistic. Our objective in writing the model this way is to develop an understanding of how this disagreement will evolve over time, and how this disagreement will affect price dynamics.

Given our modeling assumptions, each agent's posterior distribution for μ_ϵ will be normal, but the distributions will have different means and variances. Specifically, for an agent from subgroup i, we denote the mean and variance of their posterior distribution over μ_ϵ, after observing the new information at time t, as $\mu_\epsilon \sim \mathcal{N}\left(\hat{\alpha}_{i,t}, \hat{\eta}^2_{i,t}\right)$. What kind of information different agents see and how they update their priors will define the subgroup of an agent, and will be specified next.

5.2.1 Agents

The agents in our model are either active or passive. In aggregate, the passive investors hold exactly the total outstanding supply of shares, and their demand is not a function of the share price. Passive investors can either be institutions or individuals. Institutions are willing to lend out shares at zero cost, while individuals are not. The fraction of institutional holdings corresponds to λ, the share of free lending supply.

This means that the two types of active agents must hold zero shares in aggregate. For simplicity, we assume that all active agents maximize utility over their period $t + 1$ wealth in each period t. Their utility is exponential. Agents within a type are homogeneous, i.e., their risk aversion coefficients are identical, and the belief formation process follows the same rules.

All active agents must locate and borrow the shares they sell short. As in the static model in Section 4.1, locating shares beyond the free lending supply (λ) offered by institutions incurs search costs, and all borrowers must pay the same equilibrium borrowing costs c_t. Active agents who buy shares do not lend out these shares.

The first type of active agent overreacts to new information. In Daniel et al. (2023b), we model these agents as *overconfident-informed* (OC). Their overconfidence is paired with access to information, in that they immediately observe the innovations $\tilde{\epsilon}_t$ at time t, presumably as a result of effort on their part. Consistent with Daniel et al. (1998, DHS) and Gervais & Odean (2001), they overestimate the precision of their views on the value of the risky security.

The second type of active agent underreacts to new information. We model these agents as *newswatchers* (NW), following Hong & Stein (1999, HS). As in HS, the NW do not have access to the innovations $\tilde{\epsilon}_t$ at time t (when the OC access this information). Rather, the information slowly diffuses through the population of NW as in HS. Finally, again following HS, the NW ignore the information content of prices; they deviate from full rationality in that they do not infer the signals of the other agents from prices.

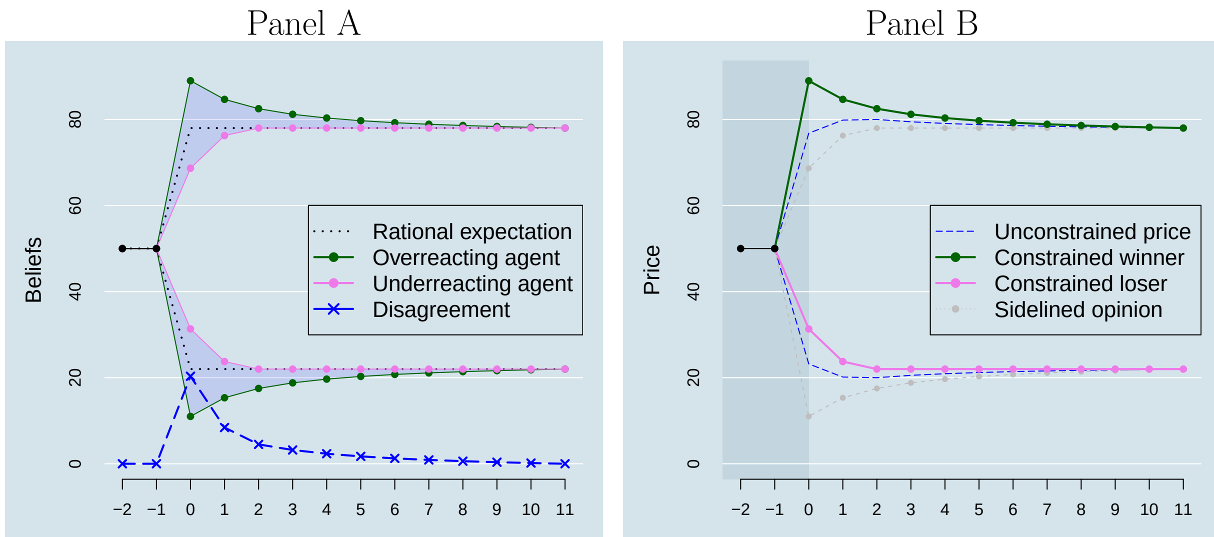

Figure 6: Panel A plots belief (i.e., the expectation of the single liquidating dividend of the risky asset) paths for overreacting (green) and underreacting (pink) agents, for positive and negative information shocks (at time $t = 0$). The dotted lines represent rational expectation beliefs for those same shocks. The dashed blue line (labeled disagreement) plots the difference between the overreacting and underreacting agents' beliefs. Panel B plots the resulting prices in unconstrained (dashed blue) and fully short-sale constrained (solid green and pink) markets. The dashed gray line represents the opinions of the sidelined agents. This plot is taken from Daniel et al. (2023a), page 2437.

5.2.2 Dynamic Model Implication

Consider first the upper three lines in Panel A of Figure 6. These illustrate the evolution of beliefs of the OC and NW (over- and underreacting) agents following a positive shock to firm value at $t = 0$.

At $t = -1$, before the release of the first dividend innovation, all active agents share a common prior on the distribution of $\tilde{D}_T$, which is 50 in the example. Following a positive innovation at $t = 0$, fully observed only by the OC, their view (ie., $\mathbb{E}^{OC}_{t=0}[\tilde{D}_T]$) jumps above the rational expected value. This is because, as a result of their overconfidence, they place too much weight on the positive innovation they observe at $t = 0$. In contrast, the underreacting NW see only part of the innovation, and as a result stay close to their prior. However, as the positive signal $\tilde{\epsilon}_0$ diffuses through the population of NW, they revise upward their estimates of the final dividend.

Here, two model parameters are required for calibration: the speed of the diffusion of information through the population of NW and the degree of overconfidence in the population of OC. These parameters determine how quickly the OC- and NW-beliefs converge to rationality. We parameterize the model so that NW diffusion takes about one year, and so that the resolution of the OC's biased beliefs takes about 5 years. In the case of an equal-magnitude positive/negative shock, the belief dynamics for the two types of agents will be symmetric: the overreacting agents overreact to the new information, and the underreacting underreact.

The right panel of Figure 6 plots the prices that result from these belief dynamics in constrained and unconstrained markets. First, consider a setting with a sufficient number of passive institutional investors so that borrowing costs are zero. In this case, prices will be approximately an average of the beliefs of the two types of active agents. In the model, this leads to the hump-shaped impulse response function labeled "unconstrained price".

However, when the number of passive institutional investors is small, then the price will (approximately) equal beliefs of the optimists; the pessimistic type will be sidelined. Following a positive shock, the overreacting investors will set the price, and following a negative shock the underreacting investors will set the price.

These implications are consistent with the price patterns documented in Section 5.1: the hump-shaped impulse response function for unconstrained stocks (i.e., momentum followed by long-horizon reversal), and the negative returns for constrained stocks which are highly persistent for past winners and less persistent for past losers.

Thus, the Daniel et al. (2023a) model can capture the return dynamics of constrained and unconstrained stocks. However as we discuss in Sections 5.3 and 5.4 it cannot explain other notable features of security markets—high trading volume and short squeezes—that possibly result from disagreement.

5.3 Trading volume and disagreement

Hong & Stein (2007) survey previous and present several new empirical analyses showing that price changes tend to be accompanied by high trading volume. One of these analyses is Ofek & Richardson's (2003) investigation of internet stocks during the DotCom era. Ofek & Richardson (2003) present evidence that heterogeneous beliefs paired with binding short-sale constraints can explain internet stocks' rise and fall.

While these results are consistent with the models discussed here, the models fail to explain the persistently high trading volume in internet stocks during the DotCom era. If the optimists stay optimists and the pessimists stay pessimists over a certain period, then there is no reason to trade across groups.

A potential route to make the model more realistic is introducing further heterogeneity within active trader types. If optimists set the price and all the pessimists are sidelined, then trading among the optimists has to generate the empirically observed high trading volume.

In a related context, Barberis et al. (2018) introduce *wavering* in a return extrapolation model that explains prices and volume in price bubbles. The main idea is to model asset demands of biased agents as weighted averages of a fundamental and an extrapolative component. The fundamental demand results from maximizing a utility function defined over the next period's wealth, assuming statistically correct beliefs about future cash flows. The extrapolative demand is based on past price changes. Barberis et al. (2018) assume that there are many biased agents, each with her own weight. If these weights independently vary over time, that is, if they waver, there will be a lot of trading within the group of biased agents in times of divergent fundamental and extrapolative demands. As a result, the model produces a high degree of trading volume around the bubble's peak, where pessimists (the fundamental traders in their model) are sidelined. To the best of our knowledge, the literature currently lacks an application of wavering outside of return extrapolation models.

Heterogeneity among biased active traders in a model with short-sale constraints may also give rise to a resale option. A trader may buy an asset today because she believes that there will be a chance to sell the asset at a higher price to another agent in the future, realizing a capital gain (Harrison & Kreps, 1978). Scheinkman & Xiong (2003) show in a continuous-time model that this trading motive can create speculative bubbles that are accompanied by high trading volume.

5.4 Short squeezes

The models in Sections 4.1 and 5.2 show that disagreement and limited lending can lead to overpricing. A key question is, of course, why optimists who have a long position in a security would not lend out that security. As discussed elsewhere, when the share lending market is well developed, an atomistic investor who is not behaving strategically will always find it optimal to lend out her shares and collect the lending fees. Of course, there are a lot of caveats here: most importantly, agents have to behave in a non-strategic manner and competitive intermediaries must facilitate stock loans in a frictionless manner.

There are many examples of situations where agents choose not to lend their shares out to potential short sellers. The motivation for this may be strategic, and specifically to facilitate a short squeeze.

Lamont (2012, p. 21) discusses short squeezes, and specifically what he calls "urge events" where firm management urges their shareholders to stop lending their shares to short sellers. He finds that these events do not raise prices of their shares, on average, though he also argues that these attempts at a coordinated share withdrawal may delay price declines. However, he finds that the longer-term abnormal returns following these urge events are "abysmal".[14]

We have noted elsewhere that, generally, only small market capitalization stocks with low institutional ownership become constrained. A counterexample to this is the case of Volkswagen. In 2008, Porsche announced that they would attempt to acquire Volkswagen and moreover announced actions that would result in the cessation of lending to short sellers, forcing them to buy to cover their short positions. Allen et al. (2021) argue that this announcement led to buying in anticipation of the withdrawal of share lending. As a result, Volkswagen temporarily became the world's largest company by equity market capitalization.

Anecdote 2: Short squeeze narratives

According to the SEC, a short squeeze "refers to the pressure on short sellers to cover their positions as a result of sharp price increases or difficulty in borrowing the security the sellers are short. The rush by short sellers to cover produces additional upward pressure on the price of the stock, which then can cause an even greater squeeze" (see https://www.sec.gov/investor/pubs/regsho.htm, last accessed, May 20, 2024).

Selective historical short squeezes have been entertaining enough to generate interest beyond financial economists and the financial press. The most salient example in recent years is probably the GameStop case, which is not only a major test case for new theories in the academic literature (Pedersen, 2022, Atmaz et al., 2024, Gârleanu et al., 2023), but also inspired a 2022 Netflix documentary "Eat the Rich: The GameStop Saga", and a major motion picture "Dumb Money," released by Sony in 2023.

A widely recognized historical example is Piggly Wiggly in the 1920s. According to the New York Times Bestseller *Business Adventures* (Brooks, 2019), Piggly Wiggly founder Clarence Saunders decided "to beat the Wall Street professionals at their own game" and started a debt-financed buying campaign in response to a group of market participants short selling Piggly Wiggly shares. As of Monday, March 19, 1923, Saunders acquired 198,872 of the 200,000 outstanding shares. Saunders then recalled the shares he owned, putting pressure on the short sellers who were unable to buy them at the open market. As a response, the Governing Committee of Exchange suspended trading in Piggly

[14] See Schultz (2023) for a more recent discussion of short squeezes.

Wiggly stocks and extended the delivery deadline. Short sellers and their brokers located shares over the counter, which eventually drove Saunders into bankruptcy.

5.5 Directions for further research

Dynamic models with disagreeing agents have become increasingly popular (see Harrison & Kreps, 1978, Scheinkman & Xiong, 2003, 2004, Hong et al., 2006, Pedersen, 2022, Hugonnier & Prieto, 2024, for an incomplete list). In recent years, many authors simultaneously model security and lending markets (Blocher et al., 2013, Daniel et al., 2023b, Chen et al., 2024, Atmaz et al., 2024, Gârleanu et al., 2023, Sikorskaya, 2023). The models have been applied to quite different empirical applications. Chen et al. (2024) examine market power in the lending market, Daniel et al. (2023a) describe long-term returns of constrained stocks, Sikorskaya (2023) studies prices and lending supply around index inclusions, while the GameStop case is examined empirically in Atmaz et al. (2024) and Gârleanu et al. (2023). Each model captures at least one mechanism that is absent in the other models. For each of these models, it would be interesting to see if additional empirical phenomena could be understood over and above the original application.

Short squeezes are a fascinating interaction between the share-lending and the share-exchange market. An open question is whether beliefs can become completely detached from fundamentals. Perhaps investors purchasing a stock like GameStop do so not because of any view on the cashflows the security will eventually distribute, but rather because they believe that short sellers and others who hold and buy the stock will push prices further away from fundamentals, which will allow them to close out their position at a higher price. In his discussion of the Piggly Wiggly short squeeze, Brooks (2019) states that at one point during the episode, "...the real worth of the company was irrelevant; the point was the game." A model that employs an exaggerated speculative motive (as in Scheinkman & Xiong, 2003) may be a direction to take in modeling such behavior.

6 Conclusion

We survey the literature surrounding Miller's (1977) idea that disagreement paired with short-sale constraints results in overpricing. There is now ample evidence that constrained high-disagreement stocks have low returns going forward.

We document that the share lending market has become far less efficient in the last several decades, in that: (1) a much greater fraction of exchange traded common stocks are "special" (i.e., have annualized borrow costs exceeding 1%), and (2) the average borrowing costs on those special stocks are far higher than they were several decades ago.

Finally, we examine the dynamic response of the prices of constrained stocks to positive and negative price shocks from both a theoretical and empirical perspective. Empirically, we find that a portfolio of constrained winners loses about 50% relative to the market over a 5-year horizon. The alphas of this portfolio are negative and statistically significant in each of the five years following the price shock. This suggests that the mispricing of constrained winners is, on average, resolved over a five-year period. In contrast, the negative alphas associated with a portfolio of constrained losers persist only about 1 year. Our heterogeneous agent model in Daniel et al. (2023a) captures both of these empirical patterns, and explains the behavior of unconstrained stocks in response to price shocks.

We conclude with three remarks on the broader literature on heterogeneous agents, beliefs, and bubbles. First, the literature surveyed here shows that, under a binding short-sale friction, high-disagreement stocks underperform in a dramatic way. In extreme cases, these stocks experience large price run-ups, followed by large price declines, and exhibited other classic "bubble" features.

Second, Hong & Stein (2007) argue that disagreement models are the natural candidates to think about trading volume and prices simultaneously. The literature we survey here adds another argument in favor of heterogeneous agent models. Constrained stocks underperform and their asset price dynamics differ significantly from unconstrained ones. Disagreement models in which prices of constrained stocks disproportionately reflect the beliefs of optimists provide a more natural framework to think about these empirical facts than representative agent models.

Finally, the theories reviewed here suggest that prices of highly constrained stocks reflect the beliefs of optimists, and that lending fees of these stocks reflect the magnitude of the disagreement between optimists and pessimists. While the increasing frictions we document in the share lending market are bad from a welfare perspective, constrained stocks provide a valuable testing ground for theories of belief formation and disagreement in asset markets.

References

Akepanidtaworn K, Di Mascio R, Imas A, Schmidt L. 2023. Selling fast and buying slow: Heuristics and trading performance of institutional investors. *Journal of Finance* 78(6):3055–3098

Allen F, Haas MD, Nowak E, Tengulov A. 2021. Market efficiency and limits to arbitrage: Evidence from the Volkswagen short squeeze. *Journal of Financial Economics* 142(1):166–194

Asquith KP, Meulbroek L. 1996. An empirical investigation of short interest. Working Paper

Asquith P, Pathak PA, Ritter JR. 2005. Short interest, institutional ownership, and stock returns. *Journal of Financial Economics* 78(2):243–276

Atmaz A, Basak S, Ruan F. 2024. Dynamic equilibrium with costly short-selling and lending market. *Review of Financial Studies* 37(2):444–506

Aumann RJ. 1976. Agreeing to disagree. *The Annals of Statistics* 4(6):1236–1239

Autor D, Dorn D, Katz LF, Patterson C, Van Reenen J. 2020. The fall of the labor share and the rise of superstar firms. *Quarterly Journal of Economics* 135(2):645–709

Bachmann R, Topa G, van der Klaauw W. 2023. Handbook of economic expectations. Academic Press

Barber BM, Lyon JD. 1996. Detecting abnormal operating performance: The empirical power and specification of test statistics. *Journal of Financial Economics* 41(3):359–399

Barberis N. 2018. Psychology-based models of asset prices and trading volume. In *Handbook of Behavioral Economics: Applications and Foundations 1*, eds. BD Bernheim, S DellaVigna, D Laibson, vol. 1, chap. 2. Elsevier, 79–175

Barberis N, Greenwood R, Jin L, Shleifer A. 2018. Extrapolation and bubbles. *Journal of Financial Economics* 129(2):203–227

Barbosa F, Bonomo M, De Mello J, Mota L. 2020. Short-selling restrictions and returns: A natural experiment. Working Paper

Beber A, Pagano M. 2013. Short-selling bans around the world: Evidence from the 2007–09 crisis. *Journal of Finance* 68(1):343–381

Ben-David I, Graham JR, Harvey CR. 2013. Managerial miscalibration. *Quarterly Journal of Economics* 128(4):1547–1584

Bénabou R, Tirole J. 2016. Mindful economics: The production, consumption, and value of beliefs. *Journal of Economic Perspectives* 30(3):141–164

Beneish M, Lee C, Nichols D. 2015. In short supply: Short-sellers and stock returns. *Journal of Accounting and Economics* 60(2):33–57

Berkman H, Dimitrov V, Jain PC, Koch PD, Tice S. 2009. Sell on the news: Differences of opinion, short-sales constraints, and returns around earnings announcements. *Journal of Financial Economics* 92(3):376–399

Billingsley P. 1995. Probability and measure. New York, NY: John Wiley & Sons, 3rd ed.

Blocher J, Reed AV, Van Wesep ED. 2013. Connecting two markets: An equilibrium framework for shorts, longs, and stock loans. *Journal of Financial Economics* 108(2):302–322

Blocher J, Ringgenberg MC. 2018. Stock options, stock loans, and the law of one price. Working Paper

Boehme RD, Danielsen BR, Sorescu SM. 2006. Short sale constraints, differences of opinion, and overvaluation. *Journal of Financial and Quantitative Analysis* 41(2):455–487

Boehmer E, Jones CM, Wu JJ, Zhang X. 2020. What do short sellers know? *Review of Finance* 24(6):1203–1235

Boehmer E, Jones CM, Zhang X. 2008. Which shorts are informed? *Journal of Finance* 63(2):491–527

Boehmer E, Wu JJ. 2012. Short selling and the price discovery process. *Review of Financial Studies* 26(2):287–322

Bris A, Goetzmann WN, Zhu N. 2007. Efficiency and the bear: Short sales and markets around the world. *Journal of Finance* 62(3):1029–1079

Brooks J. 2019. Business adventures: Twelve classic tales from the world of wall street. London: John Murray Press

Carhart MM. 1997. On persistence in mutual fund performance. *Journal of Finance* 52(1):57–82

Chague F, De-Losso R, Genaro AD, Giovannetti B. 2017. Well-connected short-sellers pay lower loan fees: A market-wide analysis. *Journal of Financial Economics* 123(3):646–670

Chang YC, Hsiao PJ, Ljungqvist A, Tseng K. 2022. Testing disagreement models. *Journal of Finance* 77(4):2239–2285

Chen J, Hong H, Stein JC. 2002. Breadth of ownership and stock returns. *Journal of Financial Economics* 66(2-3):171–205

Chen S, Kaniel R, Opp CC. 2024. Market power in the securities lending market. Working Paper

Cohen L, Diether KB, Malloy CJ. 2007. Supply and demand shifts in the shorting market. *Journal of Finance* 62(5):2061–2096

Cookson JA, Fos V, Niessner M. 2022. Does disagreement facilitate informed trading? Working Paper

Cookson JA, Niessner M. 2020. Why don't we agree? Evidence from a social network of investors. *Journal of Finance* 75(1):173–228

Da Z, Huang X. 2020. Harnessing the wisdom of crowds. *Management Science* 66(5):1847–1867

Daniel K, Klos A, Rottke S. 2023a. The dynamics of disagreement. *Review of Financial Studies* 36(6):2431–2467

Daniel K, Klos A, Rottke S. 2023b. Online-appendix: The dynamics of disagreement. Supplementary material

Daniel K, Klos A, Rottke S. 2024. Inefficiencies in the stock lending market. Work in progress

Daniel KD, Hirshleifer D, Subrahmanyam A. 1998. Investor psychology and security market under- and over-reactions. *Journal of Finance* 53(6):1839–1886

D'Avolio G. 2002. The market for borrowing stock. *Journal of Financial Economics* 66(2-3):271–306

DeBondt WFM, Thaler RH. 1985. Does the stock market overreact? *Journal of Finance* 40(3):793–808

DeBondt WFM, Thaler RH. 1987. Further evidence on investor overreaction and stock market seasonality. *Journal of Finance* 42(3):557–581

Desai H, Ramesh K, Thiagarajan SR, Balachandran BV. 2002. An investigation of the informational role of short interest in the nasdaq market. *Journal of Finance* 57(5):2263–2287

Diamond DW, Verrecchia RE. 1987. Constraints on short selling and asset price adjustment to private information. *Journal of Financial Economics* 18:277–311

Diether KB, Malloy CJ, Scherbina A. 2002. Differences of opinion and the cross-section of stock returns. *Journal of Finance* 57(5):2113–2141

Doidge C, Karolyi GA, Stulz RM. 2017. The U.S. listing gap. *Journal of Financial Economics* 123(3):464–487

Drechsler I, Drechsler QF. 2016. The shorting premium and asset pricing anomalies. Working Paper

Duffie D, Gârleanu N, Pedersen LH. 2002. Securities lending, shorting, and pricing. *Journal of Financial Economics* 66(2):307–339

Edelen RM, Ince OS, Kadlec GB. 2016. Institutional investors and stock return anomalies. *Journal of Financial Economics* 119(3):472–488

Engelberg JE, Evans RB, Leonard G, Reed AV, Ringgenberg MC. 2023. The loan fee anomaly: A short seller's best ideas. Working Paper

Engelberg JE, Reed AV, Ringgenberg MC. 2012. How are shorts informed? Short sellers, news, and information processing. *Journal of Financial Economics* 105(2):260–278

Evans R, Ferreira MA, Porras Prado M. 2017. Fund performance and equity lending: Why lend what you can sell? *Review of Finance* 21(3):1093–1121

Fama EF, French KR. 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33:3–56

Figlewski S. 1978. Market 'efficiency' in a market with heterogeneous information. *Journal of Political Economy* 86(4):581–597

Fishbein M, Ajzen I. 1975. Belief, attitude, intention, and behavior: An introduction to theory and research. Reading, Massachusetts: Addison-Wesley

Galton F. 1907. Vox populi. *Nature* 75(1949):450–451

Gao X, Ritter JR, Zhu Z. 2013. Where have all the IPOs gone? *Journal of Financial and Quantitative Analysis* 48(6):1663–1692

Gârleanu N, Panageas S, Zheng G. 2023. A long and a short leg make for a wobbly equilibrium. Working Paper

Gervais S, Odean T. 2001. Learning to be overconfident. *Review of Financial Studies* 14(1):1–27

Giglio S, Maggiori M, Stroebel J, Utkus S. 2021. Five facts about beliefs and portfolios. *American Economic Review* 111(5):1481–1522

Gödker K, Jiao P, Smeets P. 2024. Investor memory. Working Paper

Goetzmann WN, Massa M. 2005. Dispersion of opinion and stock returns. *Journal of Financial Markets* 8(3):324–349

Greenwood R, Shleifer A. 2014. Expectations of returns and expected returns. *Review of Financial Studies* 27(3):714–746

Gromb D, Vayanos D. 2010. Limits of arbitrage: The state of the theory. *Annual Review of Financial Economics* 2:251–275

Grossman SJ. 1976. On the efficiency of competitive stock markets where trades have diverse information. *Journal of Finance* 31(2):573–585

Harrison JM, Kreps DM. 1978. Speculative investor behavior in a stock market with heterogeneous expectations. *Quarterly Journal of Economics* 92(2):323–336

Haruvy E, Noussair CN. 2006. The effect of short selling on bubbles and crashes in experimental spot asset markets. *Journal of Finance* 61(3):1119–1157

Hirshleifer D, Teoh SH, Yu JJ. 2011. Short arbitrage, return asymmetry, and the accrual anomaly. *Review of Financial Studies* 24(7):2429–2461

Hong H, Scheinkman J, Xiong W. 2006. Asset float and speculative bubbles. *Journal of Finance* 61(3):1073–1117

Hong H, Stein JC. 1999. A unified theory of underreaction, momentum trading and overreaction in asset markets. *Journal of Finance* 54(6):2143–2184

Hong H, Stein JC. 2007. Disagreement and the stock market. *Journal of Economic Perspectives* 21(2):109–128

Huffman D, Raymond C, Shvets J. 2022. Persistent overconfidence and biased memory: Evidence from managers. *American Economic Review* 112(10):3141–75

Hugonnier J, Prieto R. 2024. Asset pricing with costly short sales. *Management Science* forthcoming

Jegadeesh N, Titman S. 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance* 48(1):65–91

Jensen TI, Kelly BT, Pedersen LH. 2023. Is there a replication crisis in finance? *Journal of Finance* Forthcoming

Johnson TL, Weitzner G. 2024. Distortions caused by lending fee retention. *Management Science* forthcoming

Jones CM. 2012. Shorting restrictions: Revisiting the 1930s. *Financial Review* 47:1–35

Jones CM, Lamont OA. 2002. Short-sale constraints and stock returns. *Journal of Financial Economics* 66(2-3):207–239

Jones CM, Reed AV, Waller W. 2016. Revealing shorts: An examination of large short position disclosures. *Review of Financial Studies* 29(12):3278–3320

Klayman J, Ha Y. 1987. Confirmation, disconfirmation and information in hypothesis testing. *Psychological Review* 94:211–228

Koijen RS, Yogo M. 2019. A demand system approach to asset pricing. *Journal of Political Economy* 127(4):1475–1515

Kolasinski AC, Reed AV, Ringgenberg MC. 2013. A multiple lender approach to understanding supply and search in the equity lending market. *Journal of Finance* 68(2):559–595

Kuchler T, Piazzesi M, Stroebel J. 2023. Housing market expectations. In *Handbook of Economic Expectations*, eds. R Bachmann, G Topa, W van der Klaauw. Academic Press, 163–191

Lamont OA. 2012. Go down fighting: Short sellers vs. firms. *Review of Asset Pricing Studies* 2(1):1–30

Long JBD, Shleifer A, Summers LH, Waldmann RJ. 1991. The survival of noise traders in financial markets. *Journal of Business* 64(1):1–19

Lyon JD, Barber BM, Tsai CL. 1999. Improved methods for tests of long-run abnormal stock returns. *Journal of Finance* 54(1):165–201

Mankiw NG, Reis R, Wolfers J. 2003. Disagreement about inflation expectations. *NBER Macroeconomics Annual* 18:209–248

Milgrom P, Stokey N. 1982. Information, trade and common knowledge. *Journal of Economic Theory* 26(1):17–27

Miller EM. 1977. Risk, uncertainty and divergence of opinion. *Journal of Finance* 32(4):1151–1168

Muravyev D, Pearson ND, Pollet JM. 2022. Is there a risk premium in the stock lending market? Evidence from equity options. *Journal of Finance* 77(3):1787–1828

Muravyev D, Pearson ND, Pollet JM. 2023. Anomalies and their short-sale costs. Working Paper

Nagel S. 2005. Short sales, institutional investors and the cross-section of stock returns. *Journal of Financial Economics* 78(2):277–309

Newey WK, West KD. 1987. A simple, positive, semi-definite, heteroskedasticity and autocorrelation consistent covaiance matrix. *Econometrica* 55(3):703–708

Newey WK, West KD. 1994. Automatic lag selection in covariance matrix estimation. *Review of Economic Studies* 61(4):631–653

Ofek E, Richardson M. 2003. Dotcom mania: The rise and fall of internet stock prices. *Journal of Finance* 58(3):1113–1138

Oskamp S. 1965. Overconfidence in case-study judgments. *Journal of Consulting Psychology* 29(3):261–265

Pedersen LH. 2022. Game on: Social networks and markets. *Journal of Financial Economics* 146(3):1097–1119

Prado MP, Saffi PAC, Sturgess J. 2016. Ownership structure, limits to arbitrage, and stock returns: Evidence from equity lending markets. *Review of Financial Studies* 29(12):3211–3244

Rabin M, Schrag JL. 1999. First impressions matter: A model of confirmatory bias. *Quarterly Journal of Economics* 114(1):37–82

Reed AV. 2013. Short selling. *Annual Review of Financial Economics* 5(1):245–258

Ritter JR, Welch I. 2002. A review of IPO activity, pricing, and allocations. *Journal of Finance* 57(4):1795–1828

Rizova S. 2011. Securities lending by mutual funds. Working paper

Saffi PAC, Sigurdsson K. 2010. Price efficiency and short selling. *Review of Financial Studies* 24(3):821–852

Scheinkman JA, Xiong W. 2003. Overconfidence and speculative bubbles. *Journal of Political Economy* 111(6):1183–1219

Scheinkman JA, Xiong W. 2004. Heterogeneous beliefs, speculation and trading in financial markets. *Paris-Princeton Lectures on Mathematical Finance 2003* :217–250

Schlingemann FP, Stulz RM. 2022. Have exchange-listed firms become less important for the economy? *Journal of Financial Economics* 143(2):927–958

Schultz P. 2023. Short squeezes and their consequences. *Journal of Financial and Quantitative Analysis* forthcoming

Sikorskaya T. 2023. Institutional investors, securities lending and short-selling constraints. Working Paper

Smith VL, Suchanek GL, Williams AW. 1988. Bubbles, crashes, and endogenous expectations in experimental spot asset markets. *Econometrica* 56(5):1119–1151

Stambaugh RF, Yu J, Yuan Y. 2012. The short of it: Investor sentiment and anomalies. *Journal of Financial Economics* 104:288–302

Surowiecki J. 2004. The wisdom of crowds: Why the many are smarter than the few and how collective wisdom shapes business, economies, societies and nations. New York: Random House

Treynor JL. 1987. Market efficiency and the bean jar experiment. *Financial Analysts Journal* 43(3):50–53

von Beschwitz B, Lunghi S, Schmidt D. 2021. Fundamental arbitrage under the microscope: Evidence from detailed hedge fund transaction data. *Review of Asset Pricing Studies* 12(1):199–242

von Beschwitz B, Massa M. 2020. Biased short: Short sellers' disposition effect and limits to arbitrage. *Journal of Financial Markets* 49:100512

Wallis KF. 2014. Revisiting Francis Galton's forecasting competition. *Statistical Science* 29(3):420–424

Wason PC. 1960. On the failure to eliminate hypotheses in a conceptual task. *Quarterly Journal of Experimental Psychology* 12(3):129–140

Welch I. 2000. Views of financial economists on the equity premium and on professional controversies. *Journal of Business* 73(4):501–537

Wright P. 1928. The tariff on animal and vegetable oils. Macmillan

Zimmermann F. 2020. The dynamics of motivated beliefs. *American Economic Review* 110(2):337–61