Optimists, pessimists and stock prices

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Abstract

We review the academic findings from psychology and economics on disagreement, and specifically on the effect of disagreement on asset prices. We discuss measurement of disagreement, and how disagreement coupled with constraints on short selling can sideline pessimistic investors and result in overpricing. We review the literature on the short-selling in financial markets, paying particular attention to how and why some issues become "hard-to-borrow", what factors go into the determination of borrowing-costs, and discuss the evolution of borrow costs over the last several decades. We show how an examination of the prices and borrow costs for constrained stocks can lead to an improved undestanding of how disagreement in financial markets arises and is resolved, and finally discuss directions for future research.

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1 Introduction

More than 100 years ago, 787 people estimated the weight of a fat ox in a competition run at the annual show of the West of England Fat Stock and Poultry Exhibition. Attendees of the event probably belong to the most qualified persons for this prediction task. Nevertheless, Galton (1907) reports a large dispersion in their estimates. The difference between the 95% and the 5% quantile was $1{,}293 - 1{,}074 = 219$ pounds. The mean estimate of 1,197 pounds, however, was exactly equal to the actual weight of the ox (Wallis, 2014).

Treynor (1987) examines a jellybean jar experiment to motivate market efficiency in a setting without short-sale constraints. Participants had to estimate the number of beans in a jar. Similar to Galton (1907), the mean estimate was pretty accurate, despite considerable disagreement.[1]

In the late nineties, Welch (2000) asked financial economists for their one-year equity risk premium estimate. Predictions ranged from -9.5% to 18%. If optimists and pessimists in an arguably homogeneous group of people — finance professors — strongly disagree on the expected excess return of the market, how strongly would we expect optimists and pessimists of a heterogeneous group of people to disagree on the expected returns of a single security?

These examples suggest that differences in beliefs are substantial, even among experts and even though mean or median views may well be accurate. What does a high degree of disagreement imply for the price of an asset? According to Miller (1977), optimists will set the price if their demand is large enough to absorb the fixed number of shares outstanding. To the extent that buyers are too optimistic about the future cash flows of the asset, overpricing results, the more so, the stronger the optimists' view deviate from the rational expectation. If, however, pessimists can to create "new" shares through short selling, asset prices are smaller than they would be otherwise. In a frictionless world with unrestricted short selling and where the consensus view is equal to rational expectation, all assets will be fairly priced, regardless of the disagreement about future cash flows. With short sale restrictions, either regulatory or due to market frictions, overpricing will be present, and mispricing will depend on the magnitude of disagreement and the nature of the short-sale frictions.

This paper surveys the literature that examines and pushes forward Miller's argument. Our focus is on equity markets, where the data of most studies comes from. We will cover the nature of frictions in the equity lending market, discuss empirical proxies for disagreement and short sale constraints, and discuss recent attempts to leverage Miller's idea to a dynamic setting. Our discussion partly draws on results

[1] Note that one interpretation of these disagreement experiments could be that the different estimates simply reflect different and incomplete information on the part of the study participants, and that once participants learn the estimates of the other participants they would revise their estimates to full-information values, and posterior estimates would be identical (as in Grossman, 1976). At least in jellybean jar classroom experiments that one of us has conducted with economics PhD students, this does not happen. The variance of second round estimates is slightly lower and, inconsistent with Milgrom & Stokey (1982), most participants maintain their divergent views even after learning others' estimates.

from psychology.[2] Still, our review will be selective as we cover only results that are most relevant for the discussions in the financial literature. The reader should not expect an overview of the extensive literature on disagreement in philosophy and psychology. We will also omit strands of the financial literature where disagreement, in contrast to Miller, could not lead to overpricing.[3]

A key idea that we want to leave the reader of this review with is that constrained stocks can be a laboratory for studing disagreement in financial markets. For constrained securities for which the cost of borrowing is non-zero, there is necessarily a different price for buying and selling. To be concrete, consider a security where we see transactions at price of $100/share, and short sales taking place when annualized borrow cost are 50% (something not that uncommon at the current time). This is at least *prima-facie* evidence of strong disagreement: we know that there are investors who are willing to pay $100 and not lend out their shares, and others who are willing to pay a 50% borrow cost. Studying how prices and borrow costs evolve over time for such securities can lead to a richer understanding of how disagreement arises and how it is eventually resolved.

2 A selective review of stylized facts about disagreement

We will start with a few stylized facts about disagreement and the development of disagreement over time. We highlight these facts because of their relevance for Miller (1977).

1. *Beliefs influence economic choices.* There is now a large literature that connects beliefs to economic actions. This literature looks at a broad spectrum of economic agents, such as retail investors (Giglio et al., 2021) or homeowners (Kuchler et al., 2023). Bachmann et al. (2023) provide a broad overview of this research as well as economic expectations in general.

2. *Information shocks matter for the dispersion of beliefs.* Important information should matter for investors' beliefs. Glaser & Weber (2005) surveyed retail investors before and after September 11, 2001. Mean beliefs differed dramatically.

   Intuitively, not all beliefs are affected in the same way. Consistent with this idea, republicans and democrats adjusted their portfolios differently after the 2016 presidential election (Meeuwis et al., 2022). Republicans increased and democrats decreased their portfolio risk. Political convictions also matter for credit risk analysts' belief (Kempf & Tsoutsoura, 2021).

[2] See Barberis (2018) for a broad overview of psychology-based asset pricing models.
[3] See Diamond & Verrecchia (1987), Varian (1985), Carlin et al. (2014) for prominent examples.

If information affects investors differently, then some shocks will increase disagreement, while others will decrease disagreement. Mankiw et al. (2003) present evidence on how disagreement about inflation expectations, both among professional forecasters and the general public, evolves over time.

3. *Disagreement is persistent.* (Giglio et al., 2021) analyze a bi-monthly survey of wealthy Vanguard clients. Elicited expectations targeted stock returns, GDP, and bond returns. They observe large and persistent individual heterogeneity in beliefs. Optimists tend to stay optimists, and pessimists tend to stay pessimists. Boutros et al. (2020) find strong evidence of CFOs updating beliefs to an insufficient degree, leading to persistent individual miscalibration and disagreement.

4. *Some beliefs are biased.* (Ben-David et al., 2013) report evidence of miscalibration among corporate managers in a 10-year panel of the Duke CFO survey. Interestingly, managers who are miscalibrated regarding the overall stock market are also miscalibrated regarding their own firm.

   Beliefs about future market returns show evidence of return extrapolation (Greenwood & Shleifer, 2014). Investor expectations of future stock market returns are positively correlated with past performance, while many popular representative agent models predict a negative correlation.

All of these facts matter for Miller's argument. It is a necessary condition that beliefs transform into actions. The fact that disagreement fluctuates over time suggests that mispricing of constrained stocks changes over time. If disagreement is persistent, then mispricing should be persistent in the presence of a continuously binding short-sale constraint. If the beliefs of some market participants are biased, then insights from the behavioral sciences about how biases evolve over time may be helpful for predictions about mispricing persistence.

More broadly, the facts above suggest that disagreement matters for financial markets. As of today, somewhat ironically, academics largely agree that belief heterogeneity plays a role in determining financial market prices (Brunnermeier et al., 2021).

3 Why does disagreement arise?

The most obvious reason for disagreement is differences in the information sets around investors. However, there is good reason to believe that different people can also interpret the same piece of information differently. For example, NVIDIA's market capitalization has more than tripled over the previous year at the time of this writing. It seems natural that momentum and value traders think differently about NVIDIA's expected return over the next year. Consistent with this idea, Cookson & Niessner (2020) reports that about half of the disagreement in their social media data set comes from disagreement across investment approaches.

In Section 2, we argue that information shocks matter for the dispersion of beliefs. Especially if disagreement is low initially, new information likely increases disagreement. Investors have different information, differ in their ability to process that information, disagree on the consequences of a piece of information, or may just have their own biases. To make an extreme example, if all investors agree on the value of a stock, then new information almost certainly causes disagreement. Daniel et al. (2023b) show that the dispersion of analysts' beliefs increases after an earnings announcement *if and only if* disagreement is low initially.

One could argue that disagreement might not matter for market outcomes. Biased agents would lose money on average and either be driven out of the market or learn to behave in a Bayesian way. Careful theoretical analysis, however, shows that biased investors can survive in a market environment (Long et al., 1991) and that biased beliefs can even arise as a consequence of successful trading (Gervais & Odean, 2001). A further argument is that strategies run by arbitrageurs with large amounts of money under their control will correct any mispricing. A large literature shows that there are plenty of limits to arbitrage (see Gromb & Vayanos, 2010, for a survey). Short-sale constraints are one important limit to arbitrage, and we will argue that these constraints can cause large overpricing.

4 A static view on disagreement and financial markets

The previous sections established that high disagreement is a prevalent feature of financial markets. This section presents a model that illustrates how a combination of disagreement and constraints on short-selling affects security market prices. We will use the model as guidance in our literature review on the nature of the frictions in the lending market, empirical strategies, and unsettled research questions. Our endeavor here will be static; we will only briefly touch on how disagreement evolves over time. Dynamic issues will be mainly delegated to subsequent sections.

4.1 A static model of disagreement in financial markets

The following highly stylized two-period model illustrates how a combination of disagreement and borrowing constraints can affect security prices. In Section 5, we expand this to a multi-period model to investigate how information arrival and the resolution of uncertainty changes these conclusions. Throughout, we build on the dynamic model in Daniel et al. (2023b), but formulate it in this section in terms of a static model, similar to Blocher et al. (2013).

The model has two types of agents: *passive investors* and *speculators*. In aggregate, passive investors hold all Q of the outstanding shares of the risky security. In addition we specify that the passive investors lend out a fraction λ of these shares even in the limit as the fee they receive for doing so approaches zero. They will lend out more shares if the fee they receive for doing so is large, as we discuss below.

The speculators are further divided into two groups: *optmists* and *pessimists*. In general, we assume that the optimists and pessimists are unit measure. They act as price takers and have CARA utility with a common Arrow-Pratt measure of risk aversion γ. The speculators do not lend out the shares they purchase, and consequently do not receive any loan fees. However, if the speculators choose to take a short position in the risky asset, they are required to borrow it from the passive investors.

There are two periods, a riskfree asset with a return of zero, and a single risky asset. At time 2, the risky asset will pay a liquidating divided $\tilde{D} \sim \mathcal{N}(\mu, \sigma^2)$, where σ^2 is common knowledge. However, at time 1 the speculators disagree about μ, the mean of this distribution: the optimists believe that $\mu = \mathbb{E}^O\{\tilde{D}\}$, while the pessimists belive that $\mu = \mathbb{E}^P\{\tilde{D}\} < \mathbb{E}^O\{\tilde{D}\}$. We are agnostic about why they disagree (but see previous section).

We noted above that the passive investors in our model hold quantity Q of the risky asset, and lend out a fraction λ even if the lending fee they receive for doing so is zero. We also assume that the quantity they will lend increases with the lending fee they earn. Specifically we assume the quantity lent/supplied is

$$X = \lambda Q + \frac{1}{\tau} c, \tag{1}$$

for non-negative borrowing costs $c \geq 0$.

In this setup, notice that: (1) the speculators set the price of the risky asset; (2) the holdings of the optimists and pessimists must sum to zero (because the passive investors hold all outstanding shares); (3) the optimists always take a long position and the pessimists always take a short position in the asset; and (4) if the quantity borrowed and shorted by the pessimists exceeds λQ, then the borrowing cost c will be positive.

With this framework, we can solve for the equilibrium share price P at time 1. Given the CARA-normal setup and the other model assumptions, the demand of the optimist will be postive, and equal to:

$$d^O = \frac{\mathbb{E}^O\{\tilde{D}\} - P}{\gamma \sigma^2} \tag{2}$$

The demand of the pessimists will be negative—i.e., they will go short. Since the pessimists pay a per-share borrowing cost of c, their demand is:

$$d^P = -\frac{P - \mathbb{E}^P\{\tilde{D}\} - c}{\gamma \sigma^2} \quad \text{for} \quad c \geq 0 \tag{3}$$

Market clearing requires that the demand of the optimists and pessimists sum to zero:

$$\frac{\mathbb{E}^O\{\tilde{D}\} - P}{\gamma\sigma^2} - \frac{P - \mathbb{E}^P\{\tilde{D}\} - c}{\gamma\sigma^2} = 0 \tag{4}$$

and that the quantity of shares borrowed (and shorted) equal the quantity supplied by the passive investors:

$$\frac{P - \mathbb{E}^P\{\tilde{D}\} - c}{\gamma\sigma^2} \leq \lambda Q + \frac{1}{\tau}c, \tag{5}$$

In other words, security and lending markets must clear simultaneously (Blocher et al., 2013, Daniel et al., 2023b, Atmaz et al., 2023, Gârleanu et al., 2023, Sikorskaya, 2023). The previous inequality holds if and only if the demand of pessimists at $c = 0$ is smaller than the free lending supply $\left(\frac{P-\mathbb{E}^P\{\tilde{D}\}}{\gamma\sigma^2} \leq \lambda Q \Rightarrow P - \mathbb{E}^P\{\tilde{D}\} - \lambda Q\gamma\sigma^2 \leq 0\right)$. If pessimists want to short more, "new" shares must be found, and a positive lending fee c arises in equilibrium.

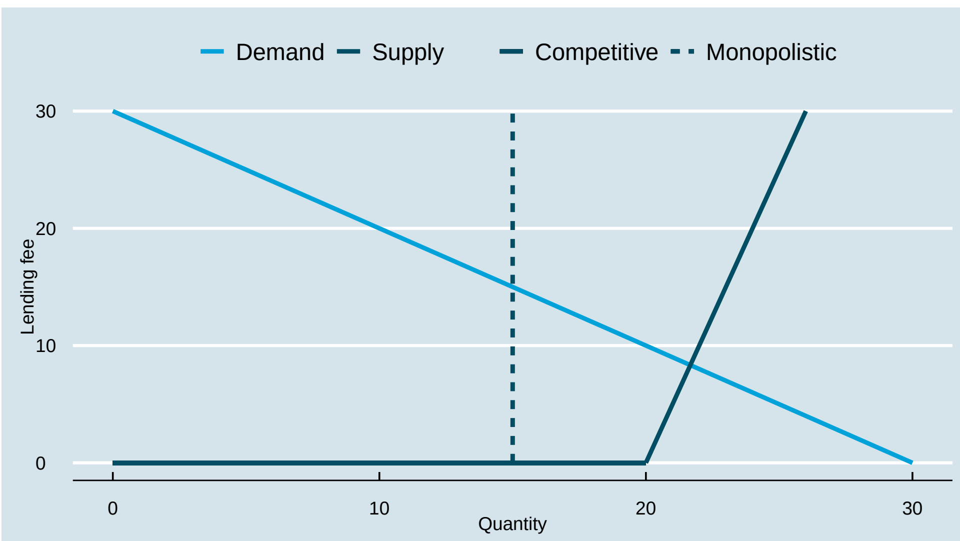

Figure 1: The figure illustrates demand and supply curves in the lending market. Parameters are τ=5, λ=.2, σ=γ=1, Q=100, $\mathbb{E}^P\{\tilde{D}\}$=70, and $\mathbb{E}^O\{\tilde{D}\}$=130.

The solid lines in Figure 1 illustrate the demand and supply curves in this setting. Note, for smaller disagreement, the demand curve were to shift downwards and the equilibrium borrowing cost would equal 0 as soon as demand can be met by the free lending supply. Similarly, if free lending supply were to increase, the kink in the supply curve would move to the right and, eventually, also lead to equilibrium borrowing cost of 0. Lower search costs (τ) make the part of the supply function after the kink less steep, and therefore lower equilibrium borrowing cost.

Rearranging (4) gives the market clearing price of the stock as a function of the (equilibrium) borrowing cost:

$$P = \frac{1}{2}\left(\mathbb{E}^O\{\tilde{D}\} + \mathbb{E}^P\{\tilde{D}\}\right) + \frac{1}{2}c \tag{6}$$

If a "wisdom-of-crowds" effect holds here, that is if $\frac{1}{2}\left(\mathbb{E}^O\{\tilde{D}\} + \mathbb{E}^P\{\tilde{D}\}\right)$ is the rationally expected dividend, then (6) implies P is "rational" if $c = 0$, but that overpricing occurs if $c > 0$. Equation (5) shows that frictions in the lending market are crucial. In a perfectly functioning lending market, where new shares are easy to find ($\tau \to 0$), competition drives fees down to zero and the equilibrium price in (6) would be equal to the rationally expected dividend.

Solving (5) for c yields

$$c = max\left\{0; \frac{\tau}{\gamma\sigma^2 + \tau}\left(P - \mathbb{E}^P\{\tilde{D}\} - \lambda Q\gamma\sigma^2\right)\right\} \tag{7}$$

and for $c \geq 0$, the equilibrium price is

$$P = \frac{\gamma\sigma^2 + \tau}{2\gamma\sigma^2 + \tau}\left(\mathbb{E}^O\{\tilde{D}\} + \mathbb{E}^P\{\tilde{D}\}\right) - \frac{\tau}{2\gamma\sigma^2 + \tau}\left(\mathbb{E}^P\{\tilde{D}\} + \lambda Q\gamma\sigma^2\right) \tag{8}$$

and the equilibrium per-share lending fee is

$$c = \frac{\tau}{2\gamma\sigma^2 + \tau}\left(\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\} - 2\lambda Q\gamma\sigma^2\right) \tag{9}$$

The revenues of the lender are $c\left(\lambda Q + \frac{1}{\tau}c\right)$. If $c > 0$, then we speak of a "constrained" stock.

Figure 2 (solid lines) illustrates the effect on borrow costs and equilibrium prices. Beyond a certain threshold (governed by the kink in the supply curve in the lendinging market, λ), borrow costs become non-zero and equilibrium prices start increasing with disagreement.

The static model nests two interesting special cases. First, note that if shares for lending are easy to find ($\tau \to 0$, no "kink" in the supply curve), then the lending fee will be zero ($\lim_{\tau\to 0} c = 0$) and the asset price will be equal to the wisdom of the crowds (or the rationally expected dividend, $\lim_{\tau\to 0} P = \frac{1}{2}\left(\mathbb{E}^O\{\tilde{D}\} + \mathbb{E}^P\{\tilde{D}\}\right)$. Intuitively, if there is no friction in the lending market, prices are right.

Second, think of a stock with extreme disagreement and infinite search costs. Here, short sellers will compete for the few available shares. The lending fee will rise to a level, where pessimists are indifferent between staying out of the market and shorting the stock at the expense of the fee. The optimists are setting the price ($\lim_{\tau\to\infty} P = \mathbb{E}^O\{\tilde{D}\} - \lambda Q\gamma\sigma^2$), and pessimists are completely sidelined. The lending fee is proportional to the degree of disagreement ($\lim_{\tau\to\infty} c = \mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\} - 2\lambda Q\gamma\sigma^2$). If either risk aversion

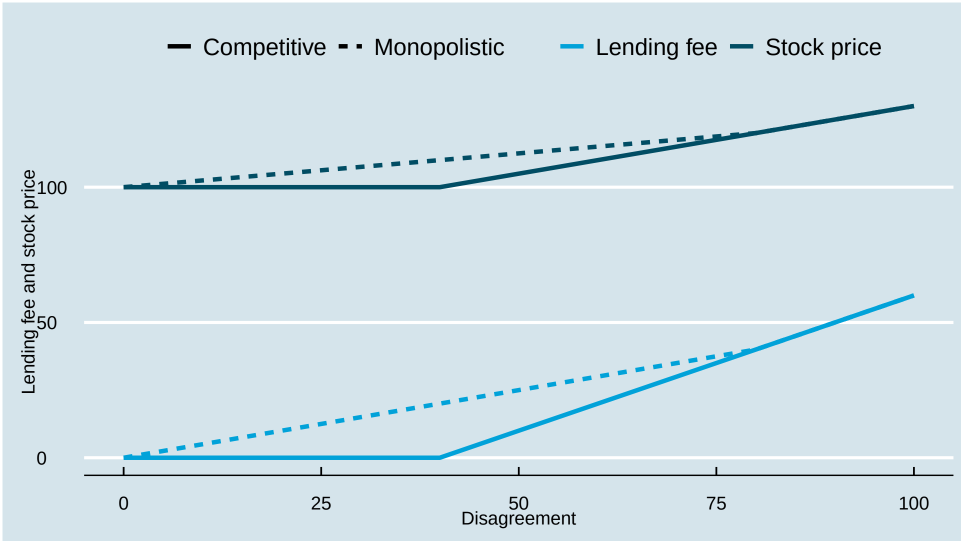

Figure 2: The figure plots equilibrium price and lending fee as a function of disagreement. Disagreement is defined as $\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\}$, and the average expected payout next period is always 100. Further parameters are τ=∞, λ=.2, σ=γ=1, and Q=100.

(γ), free lending supply (λ) or asset risk (σ) is small enough for the last term in Equation 9 to be neglected, the lending fee fully reveals the degree of disagreement.

4.2 Frictions in the lending market

Short sellers must borrow the share before selling it. In Equation (1), we model the lending supply as $\lambda Q + \frac{1}{\tau}c$. This simplified framework captures the basics empirical facts of the share lending market. D'Avolio (2002) shows that the majority of stocks can be borrowed at a small fee. We capture this fact by allowing for free lending up to a fraction λ of the available shares Q. Free lending supply could come from index funds and other large institutional investors with large lending programs. Here, intense competition and very small marginal costs should lead to a willingness to lend out shares for a tiny fee that captures administrative costs. In the model, we assume that passive institutional investors with large holdings are lending out for free. Institutional ownership is a useful proxy for free lending supply when taking the model to the data.

Once, the free lending supply is exhausted, brokers must search for additional shares (Duffie et al., 2002). Finding these shares to borrow involves leg work: Identifying potential lenders among smaller or even individual investors, and convincing them to convert their accounts to margin accounts. The smaller and/or otherwise obscure the stock, the more difficult such an endeavor would be.

The model captures search costs by assuming that the search cost of finding a new share is constant and equal to τ. The equilibrium fee in Equation (1) is $c = \tau(X - \lambda Q)$ and reflects that these search cost must be paid by the short sellers for every share that exceeds free lending supply.

An interesting additional friction discussed in Chen et al. (2023) and Prado et al. (2016) can arise out of a lack of competition. We consider an extreme example within our model to illustrate the argument. Say the lender is a monopolist and able to hold back shares. Assume for simplicity, that it is impossible to find additional shares ($\tau \to \infty$), and that disagreement is large enough such that the shorting demand at zero costs exceeds the free lending supply $\left(\frac{P-\mathbb{E}^P\{\tilde{D}\}}{\gamma\sigma^2} > \lambda Q\right)$. If the monopolistic lender chooses to lent out S shares instead of λQ, then her revenues are

$$cS = \left(\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\} - 2S\gamma\sigma^2\right) S \tag{10}$$

Maximizing (10) yields

$$S^* = \frac{\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\}}{4\gamma\sigma^2} \tag{11}$$

A monopolistic lender has an incentive to restrict supply whenever $S^* < \lambda Q$. If disagreement is low initially and the equilibrium lending fee would be zero in a competitive market $\left(\frac{P-\mathbb{E}^P\{\tilde{D}\}}{\gamma\sigma^2} \leq \lambda Q\right)$, then a single lender would generate revenues by withholding shares, and artificially creating a constrained stock.

The dashed vertical line in Figure 1 illustrates the behavior of a monopolistic lender in a numerical example. Instead of lending out the amount of shares implied by the competitive equilibrium represented by the solid lines, she holds back shares and only lends out S^* shares (15 in this example). Thereby, she maximizes her revenues, but higher borrow cost and, consequently, overpricing in the stock market ensue.

The comparative statics of this can be seen in Figure 2, focusing on the dashed lines. Borrow cost (and the equilibrium stock price) increase with disagreement from zero onward. This means that stocks that would be unconstrained and fairly priced with a competitive lending market (for any level of disagreement left of the "kink") are now short-sale constrained and exhibit overpricing. Beyond a certain high level of disagreement ($S^* \geq \lambda Q$), the maximal fee can be achieved without holding back shares, and the monopolistic market generates the same outcomes as the competitive market.

There is an additional friction in the lending market that is rarely discussed in the literature. Lenders do not directly lend to borrowers, but do so through an intermediary: their broker. Chen et al. (2023) discuss how this may inhibit competition, potentially (and paradoxically) even benefiting everyone (if short-sellers value secrecy).

Anecdote 1: Short sellers against lenders in a legal case

In 2001, some of the major investment banks founded a platform called EquiLend to improve the securities lending workflow. In 2017 they were sued by pension funds and other investors, who accused them of "relegat[ing] the stock lending market to the stone age" by using their board positions on EquiLend to boycott startup platforms in order to keep monopoly control over the market and charge excessive lending fees since 2009. Up until August 2023, a subset of the banks has settled for a combined $580m in damages.

Case 1:17-cv-06221-KPF-SLC

4.3 How to test Miller?

The model developed in Section 4.1 allows us to think about empirical strategies in a structured way. Equation (6) suggests that asset prices should be proportional to lending fees, and more recent papers do indeed use lending fee data (Boehme et al., 2006, Cohen et al., 2007, Blocher & Ringgenberg, 2018, Cookson et al., 2022). Panel data of lending fees with broad cross-sectional and time-series coverage has become available in this century but was unavailable previously (see Jones & Lamont, 2002, Jones, 2012, for studies useing lending market data from 1926 to 1933). This observation begs the question of what data to use if researchers need to study longer time periods.

With a fixed number of outstanding shares, Miller's empirical prediction is that overpricing for a single stock is an increasing function of the dispersion in beliefs. Short interest is an observable equilibrium outcome that is undoubtedly strongly related to disagreement.

In our model, short interest is equal to the shorting demand of the pessimist. However, Equation (7) shows that short interest for a constrained stock does not only depend on the amount of disagreement ($\mathbb{E}^O\{\tilde{D}\} - \mathbb{E}^P\{\tilde{D}\}$) but also on the free lending supply λQ, the riskness of the asset σ^2, the risk aversion of speculators γ, and the search costs τ to find new shares to borrow. All of these quantities are probably stock-specific and make cross-sectional analyses harder to interpret. In other words, stocks with low short interest may be constrained stocks with no disagreement or stocks with strongly binding short-sale constraints. Stocks with high short interest may be stocks with high disagreement or stocks with moderate disagreement and no short-sale constraints. Examining the relationship between disagreement proxies and stock returns is certainly interesting in and by itself. Still, it is not a direct test of Miller (1977) without an appropriate control for binding short-sale constraints on the lending market.

A similar argument applies to looking at institutional holdings and future stock returns. The institutional ownership ratio, defined as the market value of the holdings scaled by the stock's market capitalization, is a reasonable proxy for the free lending supply λQ. However, Equation (5) shows that cross-sectional variation in institutional ownership should only matter for constrained stocks with high search costs. Here, the missing shorting demand and the unobserved search costs confound results.

The model suggests that the most convincing proxy for shorting costs is a suitable combination of short interest and institutional ownership. Recent studies use short interest divided by institutional ownership (SIRIO, Drechsler & Drechsler, 2016), or independent sorts on these two quantities (Asquith et al., 2005, Daniel et al., 2023a). Historically, however, short interest and institutional ownership have been used as individual proxies for short-sale constraints on their own. We include these studies in our review of qualitative results in the next section. From a quantitative perspective, results reported in papers that use only short interest or only institutional ownership should be viewed as lower bounds for the real economic importance of short-sale constraints. Readers who want to dig deeper into this literature should keep that in mind.

4.4 Empirical evidence on Miller

Miller's hypothesis is overwhelmingly supported by empirical analyses. Stocks with high short interest perform poorly subsequently (Figlewski, 1978, Asquith & Meulbroek, 1996, Desai et al., 2002). High disagreement predicts negative abnormal performance going forward (Diether et al., 2002, Goetzmann & Massa, 2005). Stocks experiencing a reduction in the number of mutual fund owners face presumably tighter short-sale constraints and underperform subsequently (Chen et al., 2002). Disagreement can predict increases in short-interest and low returns around options introductions, consistent with the idea that the possibility to short through options mitigates short-sale constraints (Boehme et al., 2006). Stock returns are negative around earnings announcements (Berkman et al., 2009) and EDGAR inclusions (Chang et al., 2022), suggesting resolution of disagreement. Studies using proprietary data from the lending market or studies that simultaneously proxy for supply and demand in the lending market allow a better identification of constrained stocks and their findings further support Miller's hypothesis (Jones & Lamont, 2002, Asquith et al., 2005, Cohen et al., 2007).

Using daily disagreement measures and daily lending fees, Cookson et al. (2022) find high returns on days with high disagreement, the more so for constrained stocks with a high ratio of short interest to lendable value. They also show that overpricing caused by disagreement attracts trading activities of activists and short sellers. Further shorts make sense in a model with multiple heterogeneous agents (Daniel et al., 2023a). Some pessimists may want to short the stock, but refrain from doing so because of the high lending fee. If disagreement causes additional overpricing among these stocks, shorting may become attractive for previously sidelined pessimists.

Miller's overpricing hypothesis is a leading explanation for the long-run underperformance of IPOs (Ritter & Welch, 2002). It is difficult to short a new firm, and optimists set the price at the end of the first trading day. As the firm ages, more shares become available in the lending market, and short-sale constraints become less binding. In experimental asset markets in the spirit of Smith et al. (1988), relaxing short-sale constraints, ceteris paribus, leads to lower prices (see, for example, Haruvy & Noussair, 2006). Last, short-sale constraints are positively related to the profitability of quantitative strategies designed to exploit mispricing (Nagel, 2005, Hirshleifer et al., 2011, Stambaugh et al., 2012, Drechsler & Drechsler, 2016, Engelberg et al., 2022, Muravyev et al., 2023).

4.5 Some directions for further research

4.5.1 Why do short-sale constraints matter that much?

A natural question to ask is how severe short-sale constraints are in reality. One of the earliest papers examining this question empirically is D'Avolio (2002). Using proprietary data on loan fees, he finds the vast majority of stocks are easy to borrow. 91% of stocks in his sample (April 2000 – September 2001) exhibit fees $< 1\%$.

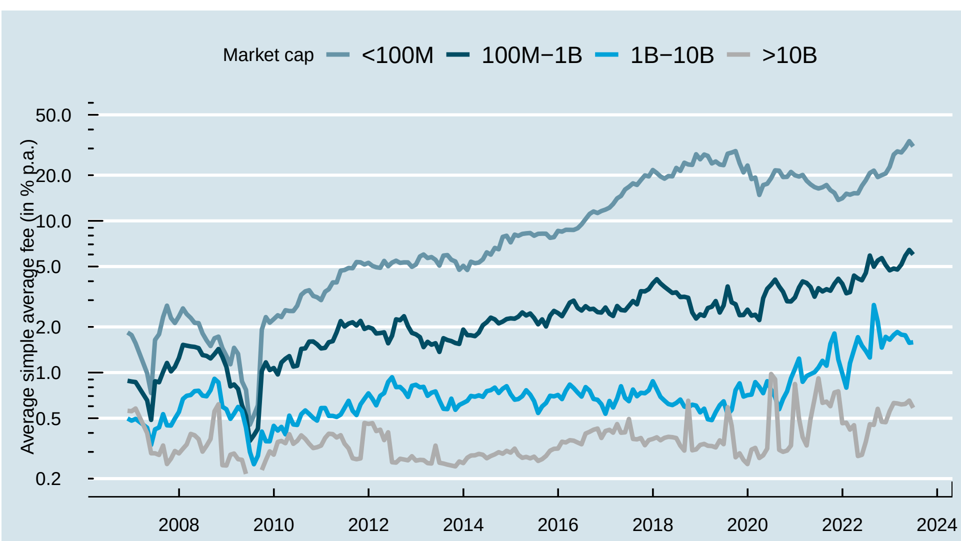

Figure 3: The figure shows the time-series of daily fees by different size group over time.

This has changed. While financial markets tend to have become more efficient in many dimensions, borrowing stock has, at least for the majority of stocks, become *more* expensive over the last two decades. Daniel et al. (2023c) report an increase in average fees from around 2.5% in 2010 to over 30% in 2023 for the smallest stocks (<100M USD). Even for larger stocks, the fee has more than quintupled, from roughly 1% in 2010 to over 5% in 2023 for stocks between 100M and 1B USD market capitalization, and more than tripled for stocks between 1B and 10B USD. While D'Avolio (2002) finds around 9% of stocks were "hard-to-borrow" by his definition (fee >1% p.a., which is also often cited by practitioners as the threshold for "specialness") in 2010, that share was already up to 12%, and it has increased to almost 50% of all stocks in 2023.

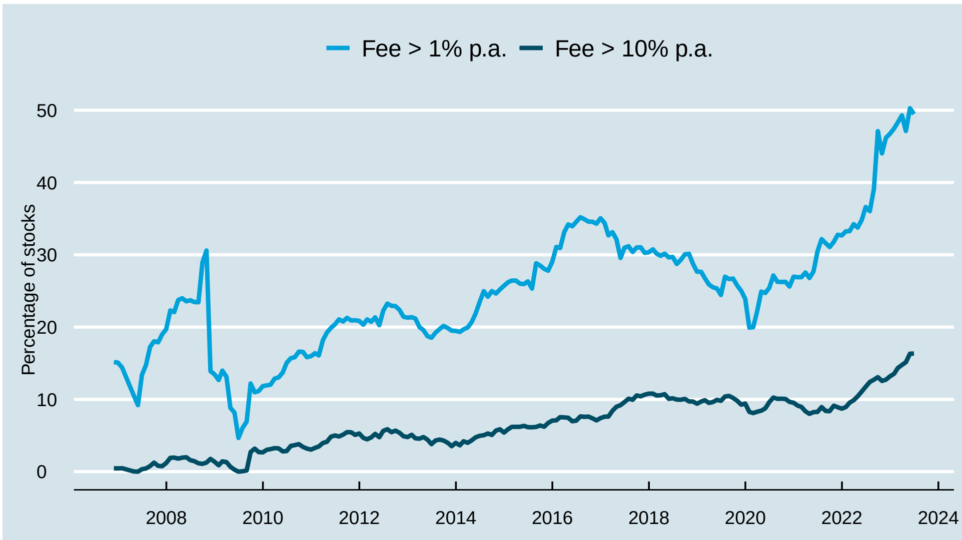

Figure 4: The figure shows the time-series of the daily calculated percentage of stocks with a simple average fee larger than 1% or 10% per annum.

This means that almost half of American stocks are "hard-to-borrow" and thereby subject to the Miller mechanism illustrated earlier. The picture does not look much better internationally (Japan being the exception), where similar trends in lending fees can be observed (Daniel et al., 2023c). The trend implies a potentially less efficient price discovery process (Bris et al., 2007, Saffi & Sigurdsson, 2010, Boehmer & Wu, 2012, Beber & Pagano, 2013). It could also be related the recent revival of quantitative strategies (Jensen et al., 2023) and the findings of Muravyev et al. (2022) that these strategies are unprofitable after accounting for short-selling costs.

This trend raises questions. What could have driven the increase in fees for small firms? Is the lending market competitive or could there be monopolistic tendencies (see Chen et al., 2023, as well as our box on a recent legal case) and have such tendencies worsened over time? Why does supply not react to such large fees? Any long-investor should be happy to receive fees of 10% or more in additional income for simply lending out stock. What keeps them from making their shares available to the market? Is there a connection to recent trends in concentration of market capitalization in fewer much larger firms (Gao et al., 2013, Doidge et al., 2017, Autor et al., 2020, Schlingemann & Stulz, 2022) and/or frictions related to passive indexing or institutional lending regulation (e.g. Sikorskaya, 2023)?

There is one caveat about the lending market data. The figures here are based on the lending fee data from Markit, which is now widely used in academic papers. Daniel et al. (2023c) observe the trends reported in Figures 3 and 4 for the simple average fee (reported here) and the indicative fee (not reported here). However, to the extent that there is price discrimination in the equity lending market (as, for example, shown in Brazil by Barbosa et al., 2020), the simple average fees reported may not reflect the lending income of the "typical" lender or the borrowing fee of the "typical" borrower.

4.5.2 Understanding supply and demand in the lending market

Estimating demand and supply curves is hard and a classic topic in economics (Wright, 1928). Estimating demand and supply curves in the lending market is arguably harder than in most other markets. Supply and demand of different stocks are probably different, and supply and demand of the same stock likely changes quickly over time.

A standard response is to find a set of instruments that influences demand but not supply. Kolasinski et al. (2013) use short-term technical trading indicators as instruments for short-term demand and fit a quadratic supply curve. Beneish et al. (2015) examine the empirical relationship between common indicators of overvaluation and lending supply. Chen et al. (2023) present evidence consistent with big lenders holding back inventory and analyze the economic consequences.

However, financial economists have not net fully understood the drivers of supply and demand in the lending market (just as for supply and demand in equity markets, see, for example Koijen & Yogo, 2019). For example, what causes large disagreement shocks? Answering this and related questions is a promising field for further research.

4.5.3 Are short-sellers always right?

In Section 4.1, when defining $\frac{1}{2}\left(\mathbb{E}^O\{\tilde{D}\} + \mathbb{E}^P\{\tilde{D}\}\right)$ to be the rational expectation, we assumed symmetric disagreement. It implies both optimists and pessimists are equally wrong in their assessment of firm value (and both lose money) but their beliefs' aggregation leads to a "wisdom-of-crowds" effect (see Da & Huang, 2020, for empirical evidence of this phenomenon). This need not always be the case. Another potentially reasonable assumption could be that pessimists are better-informed market participants than the average investor. There is some research indeed arguing short-sellers tend to possess superior information and make money, on average (Boehmer et al., 2008, Engelberg et al., 2012, Jones et al., 2016, Boehmer et al., 2020).

Note that the two assumptions are conceptionally incompatible. If short-sellers tend to be right, then the crowd cannot be wise.

The dynamic model in Daniel et al. (2023a) suggests pessimists suffer from the same biases as optimists. Our empirical evidence is consistent with this, in that short-sellers seem to lose money when looking at longer horizons than previous studies have. Thus, it seems fair to say that short-sellers are not always smarter than everybody else and that incorrect beliefs of firm value can persist on both the positive and negative side of the spectrum.

Consistent with the idea that not all institutions are fully rational, there is an emerging literature documenting suboptimal investment behavior among institutional investors (Edelen et al., 2016, von Beschwitz et al., 2021, Akepanidtaworn et al., 2023). One may argue that short sellers are a particular educated subgroup of institutional investors, but at least in the data set of von Beschwitz & Massa (2020) short sellers exhibit a disposition effect.

5 The dynamics of disagreement

In the previous sections we have mainly taken a static perspective. In this section we investigate the influence of disagreement on stock prices in a dynamic perspective where agents can learn and disagreement can evolve. If a high-disagreement stock is overpriced, how will mispricing evolve over time?

An important question is how biased beliefs can persist in the first place. A lot of information is available to investors, and there are plenty of opportunities to learn.

In economics, a large literature has evolved on motivated beliefs over the previous twenty years (see Bénabou & Tirole, 2016, for a review). In short, there are things that people want to believe. If they want to believe that they are skilled or that their initially formed beliefs are correct, they will be incentivized to downplay information contradicting this view. Instead, they will focus on information supporting their priors. This idea reminds of confirmation or confirmatory bias (Wason, 1960, Klayman & Ha, 1987) studied in the psychology literature. Rabin & Schrag (1999) define confirmatory bias as follows (page 38): "A person suffers from confirmatory bias if he tends to misinterpret ambiguous evidence as confirming his current hypotheses about the world." It will take time to convince such a person that he got it wrong in the first place. These ideas are consistent with the persistence of retail investors' beliefs reported in Giglio et al. (2021) and the persistence of analyst disagreement documented in Daniel et al. (2023a). These ideas are further consistent with the self-attribution bias that has been applied previously in the behavioral finance literature (Daniel et al., 1998).

Direct evidence on motivated beliefs over time comes from the lab and the field. Zimmermann (2020) shows in laboratory experiments that positive feedback has a persistent, while negative feedback only has a transitory effect on beliefs about one's own relative performance in an IQ task. After a month, negative feedback is less accurately recalled than positive feedback. Huffman et al. (2022) study food store managers that are partly paid based on past performance. These managers are overconfident about their own performance. Further, they have overly positive memories of their past performance. Biased recollection of positive performance and overconfidence are positively correlated.

Perhaps surprisingly, having more information than others may even lead to higher degrees of overconfidence. In Oskamp (1965), respondents received information for a prediction task in chunks. Participants felt more competent with more information, although their prediction accuracy remained unchanged.

5.1 Evidence from Equity Markets

In Daniel et al. (2023a), we explore the predictable returns of constrained stocks following shocks to their market values. The motivation of this examination is to study the evolution of disagreement. Consistent with the simple model in Section 4.1, we argue that market prices for the constrained stocks we examine serve as proxies for the optimistic investors' estimation of the stocks' values (*i.e., the optimists' expectation of the discounted cashflows*), and that the borrow-costs and other proxies for disagreement serve as proxies for the difference between the valuations of the optimistic and pessimistic investors. We explore the predictable returns over a long-horizon as a way of determining how the resolution of disagreement occurs.

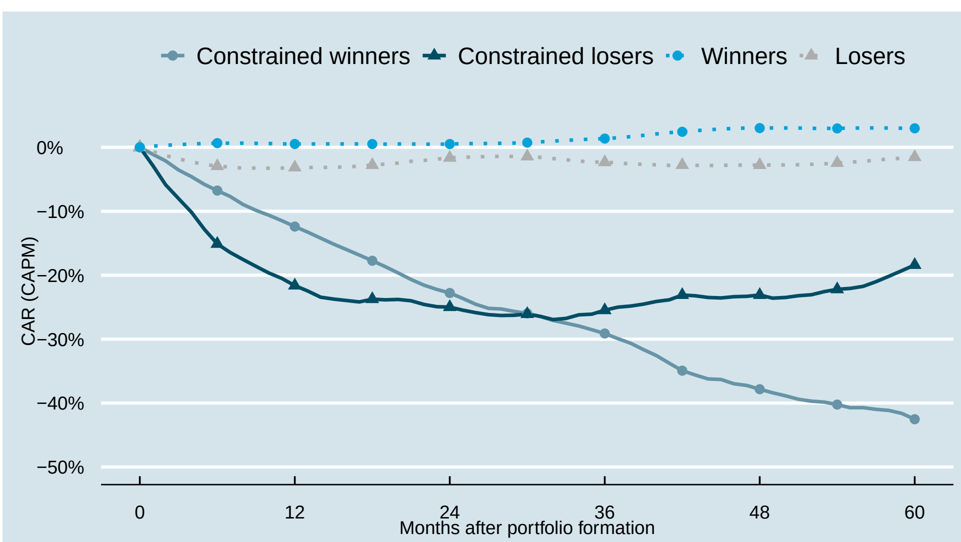

Figure 5: The solid/dotted lines present CARs for value-weighted portfolios constrained/unconstrained firms that, over the period from $t-12$ to $t-1$ month relative to the formation date t, earned a cumulative return that put them in in the top ("winner") or bottom ("loser") 30% of firms.

The dotted lines in Figure 5 are cumulative abnormal returns (CARs) for *unconstrained* past-winners and past-losers, over the period from 1927:01–2020:06. These value-weighted portfolios, formed each month, are based on rankings on the cumulative return from 12 months before to 1 month before the portfolio formation date. The "winner" portfolio contains the top 30% of firms, and the "loser" portfolio the bottom 30%. Roughly speaking, what is plotted is the average cumulative abormal return that would have been earned over horizons between 1 and 60-months.

Daniel et al. (1998) show that the momentum (Jegadeesh & Titman, 1993) and long-term-reversal (DeBondt & Thaler, 1985, 1987) effects imply a hump-shaped impulse response function: that is, a positive return should be followed by positive returns for about one year, and negative returns for about 3–5 years, and vice-versa for losers. Given the plot's scaling, it is difficult to see here, but both the winners and losers exhibit this hump-shaped CAR.

The solid lines in Figure 5 show the corresponding CAR plots functions for *constrained* past-winners and losers, over the period from 1980:05–2020:06 where we have access to data that allows us to identify constrained firms using a combination of short-interest and institutional-ownership. A comparison with the unconstrained firms shows that the patterns are dramatically different for constrained stocks. Both the constrained-past-winners and the constrained-past-losers earn strongly negative abnormal returns following the portfolio formation date. The striking difference between the winners and losers is the horizon over which these negative abnormal returns persist: for the past-losers, the negative returns persist for one year: starting one year post-formation, the portfolio returns are economically small and are not statistically different from zero. In contrast, for the past-winners the returns are also negative in years 2–5 following portfolio formation.

Note also that the magnitudes of these forecastable returns are large: the constrained-past-losers lose about 20% in the first year post-formation, and the constrained-past-winners lose just over 50% of their value in the 5-years post-formation, relative to a market benchmark.

While statistical inference associated with CARs can be problematic (Barber & Lyon, 1996, Lyon et al., 1999), in Daniel et al. (2023a) we confirm that these results are robust and highly statistically significant using a set of time-series regressions on monthly-rebalanced buy-and-hold portfolio returns. Specifically, we show that in the first year following portfolio formation, the constrained winner and loser portfolios earn monthly abnormal returns of -137 and -95 basis-points ($t = -3.95, -4.07$). However, in years 2-5 post-formation, the monthly abnormal return is -62 bps ($t = -4.93$) for the past-winners, but +26 bps/month ($t = 1.03$) for the past-losers, and that difference in monthly alpha of 88 bps is highly signficant ($t = 4.33$).[4]

[4] These abnormal returns are relative to a Fama & French (1993)-Carhart (1997) four-factor benchmark. The t-statistics are Newey & West (1987), with automatic lag selection (Newey & West, 1994).

In Daniel et al. (2023a) we also show that measures of the disagreement between the optimists and pessimists—including the indicative fee from *Markit* and analyst disagreement—are large and exhibit the same level of persistence for the constrained-past-losers and winners, suggesting that the difference in the persistence of predictable returns does not result from differences in the resolution of disagreement for winners and losers.

What can explain the difference in the persistence for the constrained past winners and losers? We argue that this must result from persistent over-optimism on the part of the agents who take long positions in the constrained past-winners, and strong, but far less persistent optimism on the part of those who hold the constrained past-losers. The following dynamic-disagreement model generates this set of belief patterns, and the belief distortions that also result in momentum and long-horizon reversal effects for unconstrained stocks.

5.2 A dynamic model

The single-period model we presented in Section 4.1 built on the intuition of Miller (1977), and showed that a combination of disagreement and short-sale constraints can lead to overpricing even in a "wisdom of crowds" setting where the average market participant holds correct views. Here, we extend that model to a multi-period setting with the goal of studying the evolution of prices and expected returns and disagreement/borrow costs.

The specific dynamic model we present here delivers implications consistent with the price and disagreement dynamics for constrained and unconstrained common stocks that we presented in Section 5.1. In this Section, we present the model setup, the basic findings, and some intuition for these findings. Our paper Daniel et al. (2023a) and the corresponding online appendix present the full derivation of the model.

In this dynamic model, there are two assets: a risk free asset which earns a return of zero each period and a risky asset which pays an uncertain liquidating dividend $\tilde{D}_T$ at time T. To capture the information dynamics that drive the dynamics of return predictability, we follow Hong & Stein (1999) and specify that the liquidating dividend is a sum of dividend innovations each period.

In contrast to the Hong & Stein (1999) specification, where the innovations are mean zero, in our specification the innovations are drawn from a (time-invariant) *i.i.d.* distribution $\tilde{\epsilon}_t \sim \mathcal{N}(\mu_\epsilon, \sigma^2)$. Importantly, the agents in our model do not directly observe μ_ϵ, but do have a valid, common prior distribution for μ_ϵ at time $t = -1$, $\mu_\epsilon \sim \mathcal{N}\left(0, \zeta^2\right)$, which they update over time as they observe the realized dividend innovations (ϵ_t's). All agents are Bayesian, but do not optimally use all information available to them.

The motivation for this specification is the following: given symmetric information at $t = -1$, all agents agree on the firm value in period $t = -1$. However, because after this point they see different parts of the information set and process this information differently, they will start to disagree about the firm's value over time. Their disagreement will result in different posterior distributions for μ_ϵ. One group will become more optimistic, meaning they think that the firm will generate higher average cashflows going forward, and the second group will be more pessimistic. Our objective in writing the model this way is to develop an understanding of how this disagreement will evolve over time, and how this disagreement will affect price dynamics.

Given our modeling assumptions, each agent's posterior distribution for μ_ϵ will be normal, but the distributions will have different means and variances. Specifically, for an agent from subgroup i, we denote the mean and variance of their posterior distribution over μ_ϵ, after observing the new information at time t, as $\mu_\epsilon \sim \mathcal{N}\left(\hat{\alpha}_{i,t}, \hat{\eta}^2_{i,t}\right)$. What kind of information different agents see and how they update their priors will define the subgroup of an agent, and will be specified later.

5.2.1 Agents

There are four types of agents in our model. For each type, there is a measure of agents of this type who form beliefs in the same way. The first type consists of passive investors. In aggregate, the group of passive investors demands exactly the total outstanding supply of shares, independent of the share price. The set of passive investors is further stratified into institutional and individual investors. Institutions are willing to lend out shares at zero cost, while individuals are not.

The other two types of agents are active. Each forms beliefs, trades, and sets prices so as to maximize individual utility. Since the passive investors demand the total outstanding supply of shares, the active agents must hold zero shares in aggregate. Each period t, all active agents maximize utility over their period $t + 1$ wealth.

There are no trading costs. However all active agents are required to locate and borrow any shares they sell short. As in the static model in Section 4.1, costly search results in shares becoming hard to borrow, and the borrowing cost here is again determined endogenously. For simplicity, we again assume that share lending takes place in a centralized market—so the cost c_t is the same for any agent borrowing the stock. We further assume that any active agent who buys shares does not lend out these shares. In the following, we refer to active agents by using the single word agents (as opposed to passive investors, who do not trade actively).

The first set of active agents overreact to new information. In Daniel et al. (2023b), we model these agents as *overconfident-informed* (OC). Their overconfidence is paired with access to information, in that they immediately observe the innovations $\tilde{\epsilon}_t$ at time t, presumably as a result of effort on their part. Consistent with Daniel et al. (1998, DHS) and Gervais & Odean (2001), they overestimate the precision of their views on the value of the risky security.

The second set of active agents underreact to new information. We model these agents as *newswatchers* (NW), following Hong & Stein (1999, HS). As in HS, the NW do not have access to the innovations $\tilde{\epsilon}_t$ at time t (when the OC access this information). Rather, the information slowly diffuses through the population of NW as in HS. Finally, again following HS, the NW ignore the information content of prices; they deviate from full rationality in that they do not infer the signals of the other agents from prices.

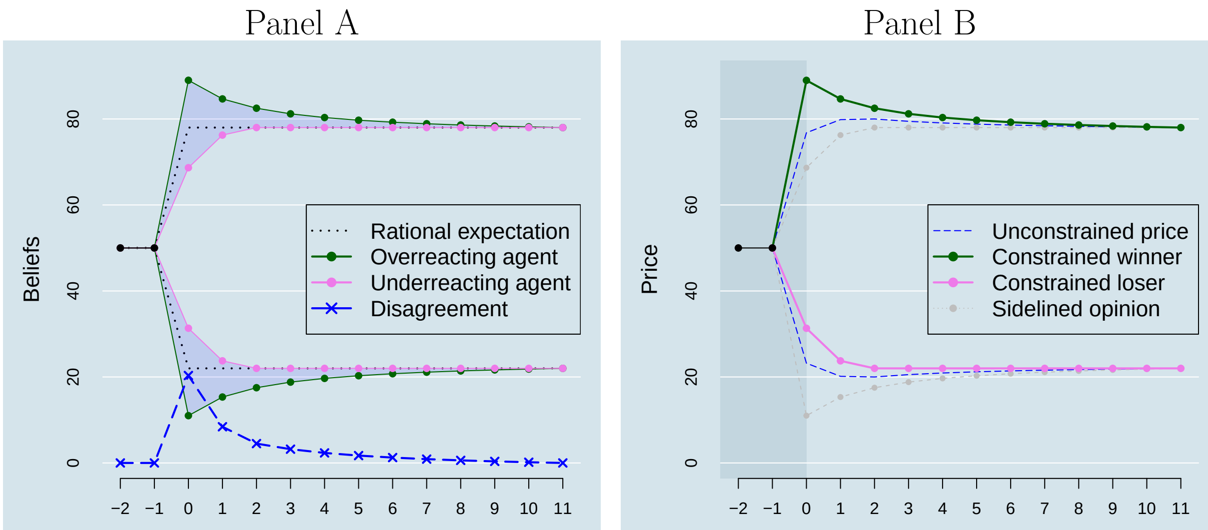

Figure 6: Panel A plots belief (i.e., the expectation of the single liquidating dividend of the risky asset) paths for overreacting (green) and underreacting (pink) agents, for positive and negative information shocks (at time $t = 0$). The dotted lines represent rational expectations beliefs for those same shocks. The dashed blue line (labeled disagreement) plots the difference between the overreacting and underreacting agents' beliefs. Panel B plots the resulting prices in unconstrained (dashed blue) and fully short-sale constrained (solid green and pink) markets. The dashed gray line represents the opinions of the sidelined agents.

5.2.2 Dynamic Model Implication

Consider first the upper three lines in Panel A of Figure 6. These illustrate the evolution of beliefs of the OC and NW (over- and underreacting) agents following a positive shock to firm value at $t = 0$.

At $t = -1$, before the release of the first dividend innovation, all active agents share a common prior on the distribution of $\tilde{D}_T$, which is 50 in the example. Following a positive innovation at $t = 0$, fully observed only by the OC, their view (ie., $\mathbb{E}^{OC}_{t=0}[\tilde{D}_T]$) jumps above the rational expected value. This is because, as a result of their overconfidence, they place too much weight on the positive innovation they observe at $t = 0$. In contrast, the underreacting NW see only part of the innovation, and as a result stay close to their prior.

However, as the positive signal $\tilde{\epsilon}_0$ diffuses through the population of NW, they revise upward their estimates of the final dividend.

Here, two of the model parameters that are required for calibration are the speed of the diffusion of information through the population of NW and the degree of overconfidence in the population of OC. These parameters determine how quickly the OC- and NW-beliefs converge to rationality. We parameterize the model so that NW diffusion takes about one year, and so that the resolution of the OC's biased beliefs takes about 5 years. In the case of an equal-magnitude positive/negative shock, the belief dynamics for the two types of agents will be symmetric: the overreacting agents overreact to the new information, and the underreacting underreact.

The right panel of Figure 6 plots the prices that result from these belief dynamics in constrained and unconstrained markets. First, consider a setting where there are a sufficient number of passive institutional investors so that borrowing costs are zero. In this case, prices will be approximately an average of the beliefs of the two types of active agents. In the model, this leads to the hump-shaped impulse response function labeled "unconstrained price".

However, when the number of passive institutional investors is small, then the price will (approximately) equal beliefs of the optimists; the pessimistic type will be sidelined. Following a positive shock, the over-reacting investors will set the price, and following a negative shock the underreacting investors will set the price.

These implications are consistent with the price patterns documented in Section 5.1: the hump-shaped impulse response function for unconstrained stocks (i.e., momentum followed by long-horizon reversal), and the negative returns for constrained stocks which are highly persistent for past winners and less persistent for past-losers.

Thus, the Daniel et al. (2023a) model can capture the return dynamics of constrained stocks. However as we discuss in Sections 5.3 and 5.4 it cannot explain other notable features of security markets—high trading volume and short squeezes—that possibly result from disagreement.

5.3 Trading volume and disagreement

Hong & Stein (2007) make the compelling case that asset pricing models, rational and behavioral, should be able to generate large amounts of trading volume. French (2008) highlights how trading volume has increased dramatically over his 80 year sample from 1926 to 2007. Extending his sample, Figure 7 shows annual turnover has remained at extremely high levels of around 200% (to the right of the vertical line, which represents the year 2007). The model outlined in Section 5.2 does not generate trading high volumes. Trading occurs if beliefs change quickly, while in Daniel et al. (2023a) biased beliefs are persistent.

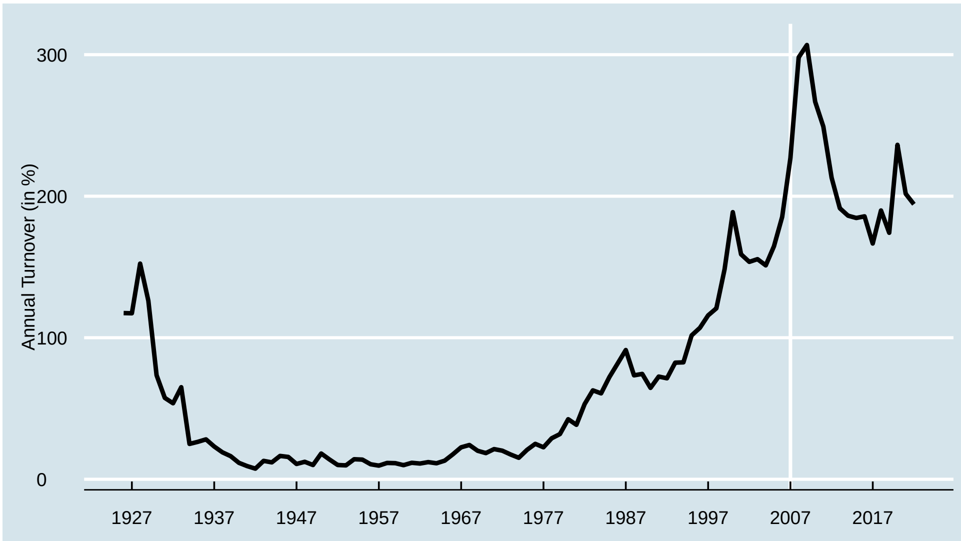

Figure 7: Annual turnover of NYSE/AMEX/NASDAQ stocks from 1926 to 2022. We replicate French (2008)'s Figure 1, extending the original sample by 15 years.

A solution to this modeling problem is "wavering" among biased agents (Barberis et al., 2018). The idea is to model biased beliefs as weighted averages of a Bayesian and a non-Bayesian component. Assume that there are many biased agents, each of them with his own weight. If these weights independently vary over time, that is, if they waver, there will be a lot of trading within the group of biased agents. Barberis et al. (2018) introduce wavering in a return extrapolation model and provide empirical evidence in favor of wavering using data from the dot-com era. The model predicts high trading volume if Bayesian and non-Bayesian beliefs differ substantially, consistent with the data.

The idea of wavering is a plausible and straightforward extension that generates high trading volume in dynamic heterogeneous agent models.

5.4 Short squeezes

The models in Sections 4.1 and 5.2 show that disagreement and limited lending can lead to overpricing. A key question is, of course, why optimists who have a long position in a security would not lend out that security. As discussed elsewhere, when the share lending market is well developed, an atomistic investor who is not behaving strategically will always find it optimal to lend out their shares and collect the lending fees. Of course, there are a lot of caveats here: most importantly, agents have to behave in a non-strategic manner and competitive intermediaries must facilitate stock loans in a frictionless manner.

There are many examples of situations where agents choose not to lend their shares out to potential short sellers. The motivation for this may be strategic, and specifically to facilitate a short squeeze.

Jones & Lamont (2002) examine the cost of borrowing over the period from 1926–1933, when a centralized stock lending desk existed on the floor of the NYSE. They find that shorting stocks that recently entered the borrowing market, presumably as a result of increased disagreement, leads to positive abnormal returns.

Lamont (2012, p. 21) discusses short squeezes, and specifically what he calls "urge events" where firm management urges their shareholders to stop lending their shares to short sellers. He finds that these events do not raise prices of their shares, on average, though he also argues that these attempts at a coordinated share withdrawal may delay price declines. However, he finds that the longer-term abnormal returns following these urge events are "abysmal".[5]

We have noted elsewhere that, generally, only small market capitalization stocks with low institutional ownership become constrained. A counterexample to this is the case of Volkswagen. In 2008, Porsche announced that they would attempt to acquire Volkswagen and moreover announced actions that would result in the cessation of lending to short-sellers, forcing them to buy to cover their short positions. Allen et al. (2021) argue that this announcement led to buying in anticipation of the withdrawal of share lending. As a result, Volkswagen temporarily became the world's largest company by equity market capitalization.

[5]See Schultz (2023) for a more recent discussion of short squeezes.

Anecdote 2: Short squeeze narratives told in the general public

According to the SEC, the short squeeze "refers to the pressure on short sellers to cover their positions as a result of sharp price increases or difficulty in borrowing the security the sellers are short. The rush by short sellers to cover produces additional upward pressure on the price of the stock, which then can cause an even greater squeeze" (see https://www.sec.gov/investor/pubs/regsho.htm, last accessed, October 24, 2023).

Selective historical short squeezes have been entertaining enough to generate interest beyond financial economists and the financial press. The most salient example in recent years is probably the GameStop case, which is not only a major test case for new theories in the academic literature (Pedersen, 2022, Atmaz et al., 2023, Gârleanu et al., 2023), but also has its own Netflix documentary as of 2022.

A widely recognized historical example is Piggly Wiggly in the 1920s. According to the New York Times Bestseller *Business Adventures* (Brooks, 2019), Piggly Wiggly founder Clarence Saunders decided "to beat the Wall Street professionals at their own game" and started a debt-financed buying campaign in response to a group of market participants short-selling Piggly Wiggly shares. As of Monday, March 19, 1923, Saunders acquired 198,872 of the 200,000 outstanding shares. Saunders then recalled the shares he owned, putting pressure on the short sellers who were unable to buy them at the open market. As a response, the Governing Committee of Exchange suspended trading in Piggly Wiggly stocks and extended the delivery deadline. Short sellers and their brokers located shares over the counter, which eventually drove Saunders into bankruptcy.

5.5 Some directions for further research

Dynamic models with disagreeing agents have become increasing popular (see Harrison & Kreps, 1978, Scheinkman & Xiong, 2003, 2004, Hong et al., 2006, Pedersen, 2022, for an incomplete list). In recent years, many authors simultaneously model security and lending markets (Blocher et al., 2013, Daniel et al., 2023b, Chen et al., 2023, Atmaz et al., 2023, Gârleanu et al., 2023, Sikorskaya, 2023). The models have been applied to quite different empirical applications. Chen et al. (2023) examine market power in the lending market, Daniel et al. (2023a) describe long-term returns of constrained stocks, Sikorskaya (2023) studies prices and lending supply around index inclusions, while the GameStop case is examined empirically in Atmaz et al. (2023) and Gârleanu et al. (2023). Each model captures at least one mechanism that is absent in the other models. For each of these models, it would be interesting to see if additional empirical phenomena could be understood over and above the original application.

The dynamic model of Miller (1977) presented in Section 5.2 seems to capture the dynamics of short-constrained common stocks but, as noted above, cannot explain high trading volume or short-squeezes. While there is a satisfying and straight-forward solution to the problem of trading volume (see Section 5.3), it is not clear how wavering could explain the time trend in Figure 7. Are there now more agents present who waver than there were 60 years ago?

Short squeezes are a fascinating interaction between the share-lending and the share-exchange market. An open question is whether beliefs can become completely detached from fundamentals. Perhaps investors purchasing a stock like GameStop do so not because of any view on the cashflows the security will eventually distribute, but rather because they believe that short sellers and others who hold and buy the stock will push prices further away from fundamentals, which will allow them to close out their position at a higher price. In his discussion of the Piggly Wiggly short squeeze, Brooks (2019) states that at one point during the episode, "...the real worth of the company was irrelevant; the point was the game." A model that employs an exaggerated speculative motive (as in Scheinkman & Xiong, 2003) may be a direction to take in modeling such behavior.

6 Conclusion

We survey the literature surrounding Miller's (1977) idea that disagreement paired with short-sale constraints causes overpricing. There is now ample evidence that constrained high-disagreement stocks have low returns going forward. Recent research examines dynamic extension of this idea.

We conclude with three remarks on the broader literature on heterogeneous agents, beliefs, and bubbles. First, there is a debate on whether low returns are predictable (Fama, 2014, Greenwood et al., 2019). The literature surveyed here shows that under a binding short-sale friction high-disagreement stocks underperform. In an extreme case, a portfolio of constrained stocks loses about 50% against the market over a 5-year horizon. These stocks have experienced a price run-up up before the crash and exhibited classic "bubble" features (Daniel et al., 2023a).

Second, Hong & Stein (2007) argue that disagreement models are the natural candidates to think about trading volume and prices simultaneously. The literature we survey here adds another argument in favor of heterogeneous agent models. Constrained stocks underperform and their asset price dynamics differ significantly from unconstrained ones. Disagreement models in which prices of constrained stocks disproportionately reflect the beliefs of optimists provide a more natural framework to think about these empirical facts than representative agent models.

Third, the theories reviewed here suggest that prices of constrained stocks reflect the beliefs of optimists, and lending fees are closely tied to the beliefs of pessimists, the more so, the more binding the short-sale constrained is. Heavily constrained stocks therefore offer a new testing ground for theories of belief formation in asset markets.

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