March 31, 1997 Comments Welcome

The Equity Premium Puzzle and the Risk-Free Rate Puzzle at Long Horizons †

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Abstract

The failure of consumption based asset pricing models to match the stochastic properties of the equity premium and the risk-free rate has been attributed by some authors to frictions, transaction costs or durability. However, such frictions would primarily affect the higher frequency data components: consumption-based pricing models that concentrate on long-horizon returns should be more successful.

We consider two consumption-based models: time-separable utility, and the habit model of Constantinides (1990). We estimate a vector ARCH model that includes the pricing kernel and the equity return, and use the fitted model to assess the model's implications for the equity premium and for the risk-free rate. Neither model performs well at a quarterly horizon, but at longer horizons the Constantinides model can match the mean and the variance of the observed equity premium, captures time-variation of the equity premium, and can better match the observed risk-free rate. We conclude that the equity premium and risk-free rate puzzles are primarily problems for shorter-horizon returns.

†We thank Steve Cecchetti, Larry Christiano, John Cochrane, George Constantinides, Larry Epstein, Lars Hansen, and an anonymous referee for numerous useful suggestions, and we acknowledge Glenn McAfee for superlative research assistance. Daniel gratefully acknowledges research support from the the Center for Research in Security Prices (CRSP) at the University of Chicago. Some results in this paper were previously circulated in a working paper entitled, "Consumption-Based Modeling of Long-Horizon Returns."

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1 Introduction

Most research on consumption-based asset pricing focuses on short-horizon returns. The starting point is the familiar intertemporal Euler equation

$$1 = E_t\left[m^{\tau}_{t+\tau} r^{\tau}_{t+\tau}\right] \tag{1}$$

where $r^{\tau}_{t+\tau}$ denotes the gross real cumulative equity return from date t to date $t+\tau$ and $m^{\tau}_{t+\tau}$ denotes the intertemporal marginal rate of substitution (IMRS) between wealth at date t and wealth at date $t+\tau$. A model of preferences is then posited that delivers m^{τ}_{t} as a function of aggregate consumption. Implications of equation (1) are tested for return horizons τ equal to one month or one quarter. Typically, the tests reject the model, often decisively.

Of particular interest are the implications of (1) for the equity premium and the risk-free rate. Let rf^{τ}_{t} denote the gross real risk-free return between dates t and $t+\tau$. Equation (1) implies the following characterizations of the equity premium and the risk-free rate:

$$\frac{E_t r^{\tau}_{t+\tau} - rf^{\tau}_{t}}{rf^{\tau}_{t}} = -cov_t\left(m^{\tau}_{t+\tau}, r^{\tau}_{t+\tau}\right) \tag{2}$$

$$rf^{\tau}_{t} = \frac{1}{E_t m^{\tau}_{t+\tau}} \tag{3}$$

The *equity premium puzzle*, as defined by Mehra & Prescott (1985), is the claim that the mean of $\frac{E_t r^{\tau}_{t+\tau} - rf^{\tau}_{t}}{rf^{\tau}_{t}}$ is much bigger than $-E\left[cov_t\left(m^{\tau}_{t+\tau}, r^{\tau}_{t+\tau}\right)\right]$, for plausible consumption-based models of $m^{\tau}_{t+\tau}$. In the context of equation (2) we can also define the *predictability puzzle* as the claim that, for plausible consumption-based models of $m^{\tau}_{t+\tau}$, there is insufficient time-variation in $cov_t\left(m^{\tau}_{t+\tau}, r^{\tau}_{t+\tau}\right)$ to explain the observed variability of $\frac{E_t r^{\tau}_{t+\tau} - rf^{\tau}_{t}}{rf^{\tau}_{t}}$.[1] Finally, the *risk-free rate puzzle* is the claim that, for consumption-based models of m^{τ}_{t}

[1] The predictability of excess returns to equities and to other financial assets is discussed in Fama and French (1988a, 1988b, 1989), Bekaert & Hodrick (1992), Cutler, Poterba, & Summers (1991), and Froot (1990), among others. These observations are anomalous according to the traditional random-walk characterization of market efficiency. They could be explained if, in equilibrium, risk premiums required by investors vary through time. Equation (2) is a precise characterization of the needed variation in risk premiums, in terms of the time-series properties of m^{τ}_{t}.

that come within striking distance of resolving the equity premium puzzle, both the mean[2] and the variance[3] of $\frac{1}{E_t m_{t+\tau}^{\tau}}$ are too high to match the corresponding moments of rf_t^{τ}. Cecchetti, Lam, & Mark (1993) estimate a representative-agent, time-separable model where consumption and dividends are governed by a bivariate model Markov switching model, and find that while they can match the first moments of the equity and risk-free return data, they cannot match both the first and second moments.

The equity premium puzzle and the predictability puzzle are both reflections of a more fundamental problem: the low correlation between consumption-growth and equity returns at short horizons. Consider Figure 1, which displays correlations between real quarterly stock returns and quarterly consumption growth at various leads and lags. (In this figure, consumption is measured as purchases of nondurables plus services, excluding the implicit rental value of owner occupied housing.) The contemporaneous correlation between quarterly returns and quarterly consumption growth is small (approximately 0.15), and the largest correlation at any lead/lag (when returns lead consumption growth by one quarter) is approximately 0.22. Cochrane & Hansen (1992) call this low correlation between the return on market proxies and consumption growth the "correlation puzzle." A number of factors have been proposed to account for the low correlations between stock returns and aggregate consumption growth at short-horizons, including uninsurable cross-sectional heterogeneity,[4] fixed costs of adjusting consumption,[5] costs of portfolio adjustment,[6] and even small deviations from perfect rationality.[7]

While these factors could substantially affect the co-movements of asset returns and aggregate consumption at high frequencies, they should be less disruptive to the theory at longer horizons. Figure 2 suggests that there may be merit in this argument. Figure 2 is analogous to Figure 1, except that the correlations in Figure 2 are between cumulative stock returns over a *one year* horizon and *one year* consumption growth (i.e.,

[2]See Weil (1989) and Cochrane & Hansen (1992).
[3]See the discussion in Cochrane & Hansen (1992, p. 137).
[4]See, e.g., Constantinides & Duffie (1996).
[5]Grossman & Laroque (1990), Marshall (1994), Marshall & Parekh (1996).
[6]Luttmer (1995), He & Modest (1995).
[7]Cochrane (1989).

c_{t+4}/c_t, where the timing interval is one quarter). Figure 2 suggests that the Cochrane & Hansen (1992) correlation puzzle is less pronounced for one-year cumulative returns. The contemporaneous correlation between consumption growth and returns at the one-year horizon is over 0.2, and the maximal correlation over all leads and lags is almost 0.4.[8]

These results suggest that consumption-based models of the equity premium may have more success if they focus on longer horizon returns. Specifically, if the higher *unconditional correlations* displayed in Figure 2 imply higher *conditional covariances* between returns and consumption-growth, then the equation (2) should provide a better fit to observed data as the horizon increases. However, it is unclear how the longer horizon horizon will affect the risk-free rate. In fact, Cochrane & Hansen (1992) find that lengthening the horizon actually exacerbates the risk-free rate puzzle.[9]

In this paper, we explore these questions directly. We consider both time-separable preferences and the Constantinides (1990) time-nonseparable habit-formation preferences. For each of these specifications, we estimate a model of the vector process $\left(r^{\tau}_{t+\tau}, m^{\tau}_{t+\tau}\right)$ that allows for time-varying conditional second moments. We use this model to generate estimates of the conditional moments $E_t r^{\tau}_{t+\tau}$, $E_t m^{\tau}_{t+\tau}$, and $cov_t\left(m^{\tau}_{t+\tau}, r^{\tau}_{t+\tau}\right)$, and we then use these estimates to evaluate (2) and (3) for investment horizons τ ranging from three months through three years.

We use two measures of consumption: consumer expenditures on nondurables plus services, and expenditures on nondurable goods only. The measure of consumer services we use excludes the service flow from the stock of owner-occupied housing, a series that is included in the data on consumer services reported by the Bureau of Economic Analysis. We exclude the service flow from owner-occupied housing for three reasons: (1) the extreme durability of the housing stock, (2) the extremely high transaction costs (pecuniary and non-pecuniary) involved in adjusting one's consumption of owner-occupied housing, and (3) the probable inaccuracies in measuring this flow. Even though we

[8]The frequency-domain analysis in Daniel & Marshall (1995) delivers a similar result.

[9]However, Cecchetti, Lam, & Mark (1994) use annual equity and bond returns from 1890 to 1987, and find that, taking account of sampling variability, volatility bounds are satisfied.

show in Section 2.1 that testing models at long-horizons will mitigate the effects of un-modeled durability and adjustment costs on our tests, the extreme durability and adjustment costs associated with housing are likely to make the housing service data problematic even for the two- and three-year horizons considered in this paper. Furthermore, the data on housing services do not directly measure household expenditures. Rather, they are constructed data that seek to approximate the rental expenditures that a household would spend if they rented, rather than owned, their house. For these reasons, we exclude these data from our measure of consumer expenditures on services.

According to our empirical results, neither of the consumption-based models fit the equity premium or the risk-free rate at the quarterly horizon, regardless of the way consumption is measured. Furthermore, time-separable preferences do poorly at all horizons. However, the Constantinides (1990) model performs remarkably well at the two-year horizon for both consumption specifications. In particular, versions of this model replicate both the mean and the standard deviation of the observed equity premium, and the theoretical equity premium series generated by these models shows some ability to track the observed equity premium through time. We also find that the Constantinides (1990) model provides a better fit to the mean of the risk-free rate as the horizon is lengthened. Interestingly, the excessive variability of the risk-free rate implied by this model at the quarterly horizon completely disappears when the horizon is set between two and three years. Similar results are obtained when we use an alternative specification of time-nonseparable preferences, that of the Abel (1990) "catching-up-with-the-Joneses" model.[10] We conclude that both the equity premium and the risk-free rate puzzles largely disappear when the horizon is lengthened to two years, provided that some form of time-nonseparability is incorporated into consumer preferences.

The remainder of the paper is organized as follows: Section 2 provides the motivation for our paper by showing how frictions or certain types of measurement errors can disrupt link between returns and consumption growth at high frequencies, but leave the relation intact at low frequencies/long horizons. Section 3 describes the models of preferences

[10] To conserve space, we do not report our results using the Abel (1990) model. They are available upon request from the authors.

we use for the remainder of the paper. Section 4 describes the time-series model we use to investigate (2) and (3). Section 5 presents our empirical results, and section 6 summarizes.

2 Why might consumption-based pricing models perform better at longer horizons? Two examples

In this section we give two examples illustrating how small frictions and measurement errors can disrupt the link between consumption and asset returns at high frequencies/short horizons, and lead to an apparent rejection of the model. In these examples, however, the low-frequency or longer-horizon implications of the model are robust to these problems. The first example introduces real frictions into a simple model of consumption- and portfolio-choice. In the second example, we assume that consumption data is contaminated with measurement error.

2.1 The effect of real frictions on consumption-based pricing at short horizons

The example of this subsection is a variant of the model of Grossman & Laroque (1990). These authors introduce fixed costs of consumption adjustment into the Merton (1971) model of consumption and portfolio choice. In particular, a consumer invests in a portfolio composed of a risk-free and a risky asset. The risk-free asset pays a constant return, while the return to the risky asset is a simple Brownian motion with drift. The consumer chooses a path for the flow rate of consumption c_t to maximize

$$E\int_0^\infty e^{-\delta t}\frac{c_t^\gamma}{\gamma}dt \tag{4}$$

Unlike Merton (1971), however, changing the flow rate of consumption requires payment of a fixed cost. Specifically, if the consumption rate immediately before a consumption change at date t is denoted c_{t-}, the cost of consumption adjustment at date t is λc_{t-}, where $0 \leq \lambda \leq 1$. If consumption is thought of as a service flow from a stock of durable goods, this adjustment cost would correspond to the cost of buying or selling durables. If (as is more common in empirical studies of consumption-based asset pricing), consumption is interpreted as the flow rate of nondurable consumption purchases, adjustment costs might include the time spent implementing a new consumption/savings plan, search costs in finding vendors for the new, better (or worse) quality goods to be purchased, or even the psychological effort of consumption re-optimization. (This last category of adjustment costs is related to what Cochrane (1989) calls "near-rational behavior".)

When $\lambda = 0$ (the Merton (1971) model), the consumption/wealth ratio is constant. However, with positive adjustment costs ($\lambda > 0$), the individual's wealth/consumption ratio follows an (S, s) pattern. As shown by Caballero (1993), this reduces both the variance of aggregate consumption growth and the covariance between aggregate consumption growth and the risky return. To see the implications for the equity premium, consider the unconditional version of equation (2):

$$E(r_{t+\tau}^{\tau} - rf_t^{\tau})E\left(\frac{1}{rf_t^{\tau}}\right) = -cov(m_{t+\tau}^{\tau}, r_{t+\tau}^{\tau}) \tag{5}$$

When $\lambda = 0$, the IMRS $m_{t+\tau}^{\tau}$ is given by:

$$m_{t+\tau}^{\tau} = e^{\delta\tau/12}\left(\frac{c_{t+\tau}}{c_t}\right)^{\gamma-1} \tag{6}$$

(In equations (5) and (6), the timing interval is one month, and δ is an annual rate.) It can be shown that equation (5) holds identically when $\lambda = 0$, $m_{t+\tau}^{\tau}$ is defined by (6), and consumption data is *point-in-time sampled*. Suppose, however, that an economist erroneously uses equation (5) to compute $m_{t+\tau}^{\tau}$ in an economy where $\lambda > 0$. In this case, the covariance on the right-hand side of equation (5) is too low to fit the equity premium (given on the left-hand side of (5)). Furthermore, observed monthly consumption data is not point-in-time sampled, but represents an average over the month. This time-averaging further reduces the covariance between $m_{t+\tau}^{\tau}$ and $r_{t+\tau}^{\tau}$.

A typical example of this effect is displayed in Figure 3.[11] This figure illustrates, for a particular parameterization, the behavior of both the actual equity premium (the left-hand side of (5)) and what might be called the "theoretical equity premium"(the

[11] The simulations of the Grossman & Laroque (1990) model displayed in Figure 3 use the procedure described in Marshall & Parekh (1996).

right-hand side of (5), with $m + \tau$ defined by (6)) as the horizon τ increases. In this figure, the adjustment cost parameter λ is set to 0.002. The risk-free rate and the drift and diffusion parameters of the risky return process are calibrated to post-war US data,[12] $\gamma = -4$ (which would correspond to a coefficient of relative risk aversion of five when $\lambda = 0$), and $\delta = .04$. To handle the increasing horizon in a simple fashion, we display continuously-compounded equity premiums, in percent per annum. That is, the true (continuously-compounded) mean annual equity premium is computed as

$$\frac{12}{\tau} log \left(\frac{E(r^{\tau}_{t+\tau} - rf^{\tau})}{rf^{\tau}} + 1 \right). \tag{7}$$

The expression in (7) is invariant to τ, since the equity return is a simple Brownian motion and the risk-free rate is constant.[13] When calibrated to post-war US data, the expression in (7) takes on a value of 6.2%. It is plotted as a dotted line in Figure 3. For horizons τ ranging from 1 month to 36 months, the dashed line in Figure 3 plots $(12/\tau)$ times the values of the (continuously-compounded) theoretical equity premium,

$$log(-cov(e^{-\delta\tau/12}[c_{t+\tau}/c_t]^{\gamma-1}, r^{\tau}_{t+\tau}) + 1),$$

implied by the model when consumption c_t is sampled point-in-time at a monthly frequency. Notice that the mean theoretical equity premium is over four times lower than the true equity premium for the monthly horizon ($\tau = 1$ month). However, this discrepancy virtually disappears as the horizon increases to four years. The solid line plots the theoretical equity premium when consumption is measured as a monthly average. Notice that this averaging process further reduces the theoretical equity premium. Again, the theoretical equity premium computed in this way moves close to the true equity premium as the horizon increases to four years.

What is striking about Figure 3 is how small an adjustment cost is needed to induce substantial discrepancies between the theoretical equity premium computed from

[12] The risky return is calibrated to match the mean and variance of the monthly real return to the CRSP value-weighted stock portfolio from 1959:2 through 1993:12. The risk-free rate r is calibrated to the mean of the monthly real return to one-month treasury bills over this same period.

[13] In deriving (7) from (5), we use the constancy of the risk-free rate.

consumption data and the actual equity premium. To give an idea of the magnitude of this adjustment cost, per capita consumption of nondurables and services in the United States in 1993 was approximately \$14,000. A value of λ equal to 0.002 would correspond to a fixed cost of consumption adjustment of \$28. Costs this small are virtually undetectable. Moreover, since agents modify their behavior to avoid paying this cost, the equilibrium reduction in per capita consumption due to the adjustment cost is even smaller. For the parameterization used in Figure 3, the average fraction of economy-wide consumption paid out in adjustment costs is only 18 parts per million. An alternative measure of the size of the adjustment cost is its welfare cost, measured as the maximum amount that a consumer would pay to permanently reduce λ to zero. For the parameterization in Figure 3, the average welfare cost is 0.0049 (about 1/2%), less than a typical mutual fund's yearly management fee.[14]

This example suggests that extremely small frictions (modelled here as costs of consumption adjustment) can substantially reduce the covariance between consumption growth and equity returns at short horizons, and therefore reduce the theoretical equity premium computed from consumption data. However, these frictions ought to have little effect on this covariance at longer horizons.

2.2 The effect of measurement error on consumption-based pricing at short horizons

In the last subsection we presented an example that illustrates how the presence of real frictions, such as consumption adjustment costs or durability, can de-link consumption and asset returns at high frequencies without affecting their co-movements at lower frequencies. In this subsection, we present an example that illustrates how certain types of measurement errors in consumption can have a similar effect.

We start by assuming, as in Hansen & Singleton (1983), that the logarithms of both

[14]See Marshall & Parekh (1996) for a more complete exploration of this model, and a comparison of its implications with observed data.

consumption and equity prices can be represented as Gaussian random walks with drift:

$$log(c_t) = log(c_{t-1}) + \mu_c + \epsilon_t \tag{8}$$

and

$$log(p_t) = log(p_{t-1}) + \mu_S + u_t \tag{9}$$

where p_t denotes the price per share of equity at date t, and ϵ_t and u_t are i.i.d. Gaussian processes with zero means and standard deviations σ_ϵ and σ_u, respectively. The covariance between these two processes is denoted $\sigma_{\epsilon,u}$. We assume for simplicity that $cov(\epsilon_t, u_{t+\tau}) = 0$ for $\tau \neq 0$. We further assume that there are no dividends, so the logarithm of the τ-period equity return is:

$$log(r^\tau_{t+\tau}) \equiv log(p_{t+\tau}) - log(p_t) = \tau\mu_S + \sum_{i=1}^{\tau} u_{t+i}. \tag{10}$$

Consumer preferences are described by the discrete-time analogue to the time-separable power utility specification in (4). In particular, agents solve the following maximization problem:

$$\max_{\{c_{t+j}\}_{j=0}^{\infty}} U \equiv E_t \sum_{j=0}^{\infty} \beta^j \frac{c_{t+j}^{1-\gamma}}{1-\gamma} \tag{11}$$

subject to the usual budget constraint. The τ-period IMRS is:

$$m^\tau_{t+\tau} = \beta^\tau \left(\frac{c_{t+\tau}}{c_t}\right)^{-\gamma}, \tag{12}$$

so, using (8) and (12),

$$log(m^\tau_t) = \tau log(\beta) + \tau\gamma\mu_c + \gamma \sum_{i=1}^{\tau} \epsilon_{t+i}. \tag{13}$$

We can take logarithms of both sides of equation (1) and use (10) and (13), along with the properties of the log normal distribution, to obtain

$$\tau log(\beta) - \tau\gamma\mu_c + \tau\mu_S + \frac{1}{2}\tau(\gamma^2\sigma^2_\epsilon + \sigma^2_u) - \tau\gamma\sigma_{\epsilon,u} = 0.$$

Rearranging, and using the fact that the log of the expected equity return $log\left(E\left[r^\tau_{t+\tau}\right]\right) = \tau\mu_S + \frac{1}{2}\tau\sigma^2_u$, we obtain:

$$log\left(E\left[r^\tau_{t+\tau}\right]\right) = \tau\left(\gamma\mu_c - log(\beta) - \frac{1}{2}\gamma^2\sigma^2_\epsilon + \gamma\sigma_{\epsilon,u}\right) \tag{14}$$

In the case of a (real) risk-free asset, we have:

$$log(rf_t^\tau) = \tau\left(\gamma\mu_c - log(\beta) - \frac{1}{2}\gamma^2\sigma_\epsilon^2\right) \tag{15}$$

It is convenient in this example to express the equity premium in this model as the log expected equity return minus the risk-free rate. Equations (14) and (15) imply that this annualized equity premium is given by:

$$\frac{1}{\tau}\left[log\left(E[r_{t+\tau}^\tau]\right) - log(rf_t^\tau)\right] = \gamma\sigma_{\epsilon,u} \tag{16}$$

Equation (16) is the analogue in this model to equation (2). It clearly illustrates how underestimation of $\sigma_{\epsilon,u}$, the correlation between consumption growth and equity returns, can induce an "equity premium puzzle". Notice that, in this simple example, $\sigma_{\epsilon,u}$ could be estimated either by computing $cov\left(\Delta log(c_t), log(r_t^1)\right)$ or by computing $\frac{1}{\tau}cov\left(log(c_{t+\tau}) - log(c_t), log(r_t^\tau)\right)$, for $\tau > 1$. If consumption is accurately measured, the estimates at all horizons τ will be the same.

Now consider the case where observed log consumption growth (denoted $\widehat{\Delta c_t}$) is measured with error:

$$\widehat{\Delta c_t} = \Delta log(c_t) + \nu_t,$$

where ν_t is some mean zero measurement-error process. If the errors in measured consumption are uncorrelated with true consumption growth and returns, they will not affect the estimates of $\sigma_{\epsilon,u}$. However, there are plausible models of measurement error that imply negative correlation between the measurement error and the variable being measured. For example, suppose consumption innovations are not fully incorporated into measured consumption data at the time they occur, but are incorporated into the reported data series only over time. We can formulate a tractable model of this sort of measurement error by specifying that current observed consumption is a weighted average of current true consumption and lagged reported consumption:

$$\widehat{\Delta c_t} = \rho\widehat{\Delta c_{t-1}} + (1-\rho)\Delta log(c_t) = \rho\widehat{\Delta c_{t-1}} + (1-\rho)(\mu_c + \epsilon_t) \tag{17}$$

where ρ is between zero and one. A process similar to equation (17) could arise because the data collection agency does not trust reports of large deviations from the consumption level reported the previous period. They may round downward these large reported changes, inducing a smoother path for reported Δc_t. Alternatively, if all sectors of the economy are not observed every period, it is possible that a sector with a large innovation at date t might not actually be observed at t. This large innovation will not be incorporated into reported consumption until some date after t. Process (17) is a tractable way to approximate such an effect.

Under (17), the error in consumption-growth is given by

$$\nu_t = \widehat{\Delta c_t} - \Delta log(c_t) = -\rho\epsilon_t + (1-\rho)\cdot[\rho\epsilon_{t-1} + \rho^2\epsilon_{t-2} + \cdots],$$

and the autocovariogram of observed log consumption growth is given by:

$$cov(\widehat{\Delta c_t}, \widehat{\Delta c}_{t-\tau}) = \frac{(1-\rho)^2}{1-\rho^2}\rho^\tau \sigma_\epsilon^2.$$

Furthermore, it is straightforward to show that the cross-covariogram between *observed* log consumption growth and returns is given by:

$$cov(\widehat{\Delta c_t}, u_{t-\tau}) = (1-\rho)\sigma_{\epsilon,u}\rho^\tau$$

for $\tau \geq 0$, and zero otherwise. Given this, the observed long-horizon covariance between log consumption growth and log returns is:

$$\frac{1}{\tau}\cdot cov(\widehat{\Delta c}^\tau_{t+\tau}, r^\tau_{t+\tau}) = \sigma_{\epsilon,u}(1-\rho)\left[1 + \left(\frac{\tau-1}{\tau}\right)\rho + \left(\frac{\tau-2}{\tau}\right)\rho^2 + \cdots + \left(\frac{1}{\tau}\right)\rho^{\tau-1}\right] \quad (18)$$

where $\widehat{\Delta c}^\tau_{t+\tau}$ denotes the growth rate in observed log consumption from date t through date $t+\tau$. Unlike the case where consumption is measured without error, it now matters which horizon τ is used when estimating $\sigma_{\epsilon,u}$. If τ is small, the right-hand side of (18) will be smaller than $\sigma_{\epsilon,u}$. If this estimate is used in equation (16), an erroneously high value of γ would be needed to make this equation hold, giving the appearance of an "equity premium puzzle". When this value of γ is then used in equation (15), the right-hand side of that equation will be too high, giving the appearance of a "risk-free rate puzzle." Note, however, that the right-hand side of (18) increases with the horizon, and that in the limit, as τ goes to infinity,

$$\lim_{\tau\to\infty}\frac{1}{\tau}\cdot cov(\widehat{\Delta c}^\tau_{t+\tau}, r^\tau_{t+\tau}) = \sigma_{\epsilon,u}$$

This means that, as the horizon τ grows, the apparent equity premium puzzle induced by the measurement error process (17) vanishes.

3 Models of Preferences

In the preceding section, we gave two examples where an equity premium puzzle arises at short horizons, but not at longer horizons. For the remainder of this paper we examine this conjecture in detail. We ask whether the equity premium puzzle and the risk-free rate puzzle are primarily short-horizon phenomena. While the simple examples of section 2 use only time-separable preferences, we will examine consumption-based pricing using both time-separable power utility and Constantinides (1990) habit-formation preferences.

3.1 Time Separable Power Utility

The most widely-studied (and widely-rejected) preference specification in the consumption-based pricing literature is time-separable power utility, described in equations (11) and (12). Cochrane & Hansen (1992) find that the performance of time-separable utility actually *deteriorates* as the horizon lengthens. The problem is that aggregate consumption is a (stochastically) growing series. In the time-separable model, agents seek to transfer some of the high future consumption to the present by borrowing. A counterfactually high risk-free rate is needed to discourage this borrowing. (Recall that net borrowing must equal zero in equilibrium.) In principle, this effect could be countered by a strong precautionary motive for saving: agents may wish to insure against the possibility of consumption downturns. However, the probability of a consumption downturn gets smaller as the horizon lengthens: Cochrane & Hansen (1992) note that there is no five-year period in post-war US data over which aggregate consumption declines. As a result, the time-separable model predicts a lower precautionary demand for savings, and a higher equilibrium risk-free rate, as the horizon lengthens.

What is needed, then, is a reason why the precautionary motive for saving remains strong at longer horizons. One possible reason is that agent's within-period utility-of-consumption changes through time. In particular, suppose agents seek protection, not against an absolute decline in consumption, but against a decline in consumption *relative to some reference point*, where the reference point itself grows at the same rate that consumption grows. In such a model, the precautionary motive for saving would not become attenuated as the horizon grows. Preference specifications with this property include the Constantinides (1990) habit-formation preferences and the Abel (1990) "catching-up-with-the-Joneses" preferences. In the following section, we describe the version of the Constantinides (1990) preference specification that we will use in our empirical analysis.

3.2 Constantinides (1990) Habit-Formation Preferences

In the Constantinides (1990) model, the agent solves

$$\max_{\{c_{t+j}\}_{j=0}^{\infty}} U \equiv E_t \sum_{j=0}^{\infty} \beta^j \frac{(c_{t+j} - h_{t+j})^{1-\gamma}}{1-\gamma} \tag{19}$$

subject to the usual budget constraint, where the stochastic subsistence point h_t is a function of lagged consumption:

$$h_t \equiv \frac{\eta(1-\delta)}{\delta - \delta^{m+1}} \sum_{i=1}^{m} \delta^i c_{t-i}, \ \eta > 0, \ 0 < \delta < 1. \tag{20}$$

The marginal rate of substitution is

$$m_{t+\tau}^{\tau} = \beta^{\tau} \frac{MU_{t+\tau}}{MU_t} \tag{21}$$

where the marginal utility of consumption MU_t is defined by

$$MU_t \equiv (c_t - h_t)^{-\gamma} - \frac{\eta(1-\delta)}{\delta - \delta^{m+1}} \sum_{i=1}^{m} (\beta\delta)^i E_t \left[c_{t+i} - h_{t+i})\right]^{-\gamma} \tag{22}$$

Terms involving conditional expectations appear in equation (22) because agents consider the effect of their current consumption on future values of h_t. These conditional expectations must be computed when we construct m_t^{τ}. We do this as follows. First, define the variable D_t by:

$$D_t \equiv 1 - \frac{\eta(1-\delta)}{\delta - \delta^{m+1}} \frac{\sum_{i=1}^{m} (\beta\delta)^i \left[c_{t-m+i} - h_{t-m+i})\right]^{1-\gamma}}{(c_{t-m} - h_{t-m})^{-\gamma}}. \tag{23}$$

The variable D_t behaves as a stationary stochastic process. Equations (21) and (23) imply that, in the Constantinides model,

$$m^{\tau}_{t+\tau} = \beta^{\tau} \frac{(c_{t+\tau} - h_{t+\tau})\, E_{t+\tau} D_{t+\tau+m}}{(c_t - h_t)\, E_t D_{t+m}} \tag{24}$$

Since D_t is stationary, we can fit an autoregressive time-series model for this variable: we use the fitted values as our estimate of $E_t D_{t+m}$. For most models, the likelihood ratio statistics testing n lags against $n-1$ lags in the autoregression for D_t (for n between 1 and 5) favor four lags. We estimate a fourth-order autoregression in D_t, and project the fitted regression m periods into the future. We estimated $\{m^{\tau}_t\}$ only for values of γ that do not imply negative marginal utilities (as defined by equation (22)) for any observations.[15] The model parameters we use are $\eta = 0.8$, $\delta = 0.7$, $n = 8$, $\beta = 1$.

4 A Vector ARCH Model of Conditional Covariances

4.1 The Basic Set-Up

In this section, we describe the time-series model we use to evaluate (2) and (3). Since there is no observable asset with a risk-free real payoff over a multi-year horizon, we examine the implications of (1) for nominal returns. Let P_t denotes the price level at date t, and let $R^{\tau}_{t+\tau}$ denote the nominal cumulative equity return from date t to date $t+\tau$ (so $R^{\tau}_{t+\tau} \equiv r^{\tau}_{t+\tau}\left[\frac{P_{t+\tau}}{P_t}\right]$). Equation (1) then implies:

$$1 = E_t\left[M^{\tau}_{t+\tau} R^{\tau}_{t+\tau}\right] \tag{25}$$

where $M^{\tau}_{t+\tau}$ is the marginal-rate-of-substitution in nominal wealth between t and $t+\tau$:

$$M^{\tau}_{t+\tau} \equiv m^{\tau}_{t+\tau}\left[\frac{P_t}{P_{t+\tau}}\right]. \tag{26}$$

Let RF^{τ}_t denote the risk-free nominal return from t to $t+\tau$. The observable analogue to RF^{τ}_t is the return on a τ-period zero-coupon dollar bond. Equation (25) implies the

[15] When consumption is measured as nondurables-plus-services, the maximum usable value of γ is 12. With nondurable consumption the maximum value of γ is 9.

analogues to (2) and (3):

$$\frac{E_t R^{\tau}_{t+\tau} - RF^{\tau}_t}{RF^{\tau}_t} = -cov_t\left(M^{\tau}_{t+\tau},\ R^{\tau}_{t+\tau}\right) \tag{27}$$

$$RF^{\tau}_t = \frac{1}{E_t M^{\tau}_{t+\tau}} \tag{28}$$

To test (27), we need a model of the conditional first moment of $R^{\tau}_{t+\tau}$ and of the conditional second moments of the joint $\left\{M^{\tau}_{t+\tau},\ R^{\tau}_{t+\tau}\right\}$ process. We use the following vector ARCH model. Let X_t denote an $(N-2)\times 1$ vector of variables that is useful in predicting $\left\{M^{\tau}_{t+\tau},\ R^{\tau}_{t+\tau}\right\}$, and let $Y'_t \equiv (\ R^{\tau}_t, M^{\tau}_t, X'_t)$. We assume that the $(N \times 1)$-dimensional process Y_t follows a vector autoregression:

$$Y_{t+1} = A_0 + A_1 Y_t + A_2 Y_{t-1} +\ ... + A_p Y_{t-p+1} + u_{t+1} \tag{29}$$

where A_0 is an $N \times 1$ vector of constants, $A_i,\ i = 1,\ ...,\ p$ are $N \times N$ matrices, and

$$u_{t+1} = L_{t+1} v_{t+1}\ ,\ v_{t+1} \sim i.i.d. \mathcal{N}(0, I),$$

and L_{t+1} is a lower triangular matrix such that

$$L_{t+1} L'_{t+1} \equiv H_{t+1} = E_t\left(u_{t+1} u'_{t+1}\right). \tag{30}$$

We now must specify the law-of-motion for H_{t+1}. We use the following notation: For any $N \times N$ symmetric matrix Σ let $vec(\Sigma)$ stack the distinct elements of Σ into a $\frac{N(N+1)}{2} \times 1$ vector. Following the ARCH approach of Engle (1982), we assume that $vec(H_{t+1})$ can be approximated by a linear function of squared residuals dated t and earlier (i.e., elements of the matrices $u_t u'_t,\ u_{t-1} u'_{t-1},\ .\ .\ .,\ u_{t-q} u'_{t-q}$). That is,

$$\begin{aligned} vec(H_{t+1}) = \quad & B_0 + \quad B_1 vec(u_t u'_t) + B_2 vec(u_{t-1} u'_{t-1}) + \dots \\ & + B_q vec(u_{t-q+1} u'_{t-q+1}) \end{aligned} \tag{31}$$

where B_0 is an $N(N+1)/2 \times 1$ vector of constants, and B_i, $i = 1, \dots, q$, are $N(N+1)/2 \times N(N+1)/2$ matrices. The parameters in (31) can be estimated by fitting the regression

$$\begin{aligned} vec(u_{t+1} u'_{t+1}) = \quad & B_0 + \quad B_1 vec(u_t u'_t) + B_2 vec(u_{t-1} u'_{t-1}) +\ ... \\ & + B_q vec(u_{t-q+1} u'_{t-q+1}) + w_{t+1} \end{aligned} \tag{32}$$

where w_{t+1} is an $i.i.d$ $N(N+1)/2 \times 1$ vector process. The linear model (29) - (31) allows for easy computation of the τ-step-ahead conditional first and second moments of Y_t: Let us write (29) in first-order "companion" form by defining $\mathcal{Y}_t' \equiv \left(Y_t', Y_{t-1}', ..., Y_{t-p+1}'\right)$, $\mathcal{U}_t' \equiv \left(u_t, 0_{(N\times N)}, ..., 0_{(N\times N)}\right)$, and $\mathcal{A}_0' \equiv \left(A_0', 0_{(N\times N)}, ..., 0_{(N\times N)}\right)$. We define the $Np\times Np$ coefficient matrix $\mathcal{A}$ by:

$$\mathcal{A} \equiv \begin{bmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}.$$

Equation (29) can now be written

$$\mathcal{Y}_{t+1} = \mathcal{A}_0 + \mathcal{A}\mathcal{Y}_t + \mathcal{U}_{t+1}$$

so

$$E_t\mathcal{Y}_{t+\tau} = (I - \mathcal{A}^\tau)(I - \mathcal{A})^{-1}\mathcal{A}_0 + \mathcal{A}^\tau\mathcal{Y}_t \tag{33}$$

and

$$var_t\mathcal{Y}_{t+\tau} = \sum_{i=0}^{\tau-1} \mathcal{A}^i E_t\left[\mathcal{U}_{t+\tau-i}\mathcal{U}_{t+\tau-i}'\right]\left[\mathcal{A}^i\right]' \tag{34}$$

In (34), $E_t\left[\mathcal{U}_{t+\tau-i}\mathcal{U}_{t+\tau-i}'\right]$ is computed from (32): If we use the notation that $B(L) \equiv \sum_{k=1}^{q} B_k L^k$, (32) implies

$$E_t vec\left[u_{t+j}u_{t+j}'\right] = B_0 \sum_{k=0}^{j-1}\left[B(L)\right]^k + \left[B(L)\right]^j vec\left[u_t u_t'\right]. \tag{35}$$

Equations (33) and (34) are used to evaluate the conditional moments in (27) and (28): $E_t R_{t+\tau}^\tau$ and $E_t M_{t+\tau}^\tau$ are the first and second elements of the vector $E_t\mathcal{Y}_{t+\tau}$ in (33), and $cov_t\left(M_{t+\tau}^\tau, R_{t+\tau}^\tau\right)$ is the $(2,1)^{th}$ element of the matrix $var_t\mathcal{Y}_{t+\tau}$ in (34). Notice that, in principle, $cov_t\left(M_{t+\tau}^\tau, R_{t+\tau}^\tau\right)$ depends on *all* elements of the matrices $E_t\left[u_{t+\tau-i}u_{t+\tau-i}'\right], i = 1, ..., \tau - 1$.

We use the linear model (29) - (32) because it provides the straightforward analytic expressions (33) and (34) for $E_t R_{t+\tau}^\tau$, $E_t M_{t+\tau}^\tau$, and $cov_t\left(M_{t+\tau}^\tau, R_{t+\tau}^\tau\right)$. However, the linear model is not without drawbacks. First, it tends to generate a large number of free parameters. For example, if no ex-ante restrictions were placed on the matrices $B_1, ..., B_q$, (that is, if equation (32) were treated as an unrestricted VAR in the $\frac{N(N+1)}{2}$ elements of $u_{t+1}u'_{t+1}$), there would be a total of $\frac{3N+N^2}{2}+pN^2+q\left[\frac{N(N+1)}{2}\right]^2$ free parameters to be estimated This number grows at rate N^4. It is easy to see that modest values of N, p, and q can give severe degrees-of-freedom problems.

Given this problem of parameter proliferation, we experimented with rather ruthless zero-restrictions on the coefficient matrices $B_1, ..., B_q$. We arrived at the following specification: First, we exclude all cross-terms (of the form $u_{i,t-k}u_{j,t-k}, k = 0, 1, ..., q-1, i \neq j$) from the right-hand side of (32). Second, in those equations of (32) where the dependent variable is a squared residual of the form $u^2_{i,t+1}$, only own lagged dependent variables (i.e., $u^2_{i,t},\ u^2_{i,t-1},\ ...,\ u^2_{i,t-q+1}$) are used as regressors. Third, where the dependent variable is a cross-term of the form $u_{i,t+1}u_{j,t+1}, i \neq j$, only lagged squared residuals $u^2_{i,t}, u^2_{j,t}$ $u^2_{i,t-1}, u^2_{j,t-1}\ ...,\ \ u^2_{i,t-q+1}, u^2_{j,t-q+1}$ are used as regressors. (That is, $u^2_{k,t-n}, k \neq i, j$ is never used as an explanatory variable for $u_{i,t+1}u_{j,t+1}$.) These restrictions were loosely patterned after the constant-correlation model, which also excludes cross-terms as explanatory variables and only uses lagged dependent variables as explanatory variables in the squared-residual equations. These restrictions reduce the number of free parameters to $\frac{3N+N^2}{2} + (p+q)\,N^2$, a number which grows at rate N^2. In our empirical work, we set $N = 4$, $p = 1$, and $q = 8$, so the total number of free parameters is 158.

A second drawback of our linear model is that it does not guarantee positive-definiteness of the $var_t Y_{t+\tau}$ matrix. Positive-definiteness is a nonlinear restriction, so multivariate models with time-varying second moments that impose positive-definiteness necessarily must introduce nonlinearities either into the model structure or into the estimation procedure. These sorts of nonlinearities substantially increase the computational burden in estimating and solving the model. For example, a widely-used multivariate model that guarantees positive-definite conditional covariance matrices is the constant-correlation model of Bollerslev (1990). This model is not suitable for our purposes, since, for $i > 1$, the elements of the matrix $\mathcal{U}_{t+i}\mathcal{U}'_{t+i}$ are *nonlinear* functions of the innovations $w_{t+1}, w_{t+2}, ..., w_{t+i}$. As a result, the matrices $E_t\left[\mathcal{U}_{t+\tau-i}\mathcal{U}'_{t+\tau-i}\right]$ in equation (34) cannot be computed as a linear projection, as in equation (35). For $\tau > 1$, computing these conditional covariance matrices would require integrating out the innovations $w_{t+1}, w_{t+2}, ..., w_{t+\tau-i}$, a computationally burdensome task. An alternative way of imposing positive-definiteness is the diagonal GARCH model of Bollerslev, Engle, & Wooldridge (1988). This model delivers a linear model of the general form (32), but guarantees positive-definiteness by imposing a nonlinear restriction on the coefficient matrices $B_1, B_2, ..., B_q$. As such, this model must be estimated using nonlinear techniques, such as maximum likelihood. Due to the large number of parameters in the models we use, nonlinear estimation would be extremely burdensome. Furthermore, it is not clear how much our inference is distorted by our failure to impose positive-definiteness as a restriction. For these reasons, we estimate (29) and (32) by OLS. We report the number of violations of positive-definiteness for each model studied, and we use the number of such violations as a check for model mis-specification.

We include two predictor variables in X_t: the term spread and the default spread.[16] The inflation rate $\frac{P_{t+\tau}}{P_t}$ is constructed from the deflator associated with the consumption series being used. For each model of m_t^τ, vector process $\{Y_t\}$ is constructed, the first vector autoregression (29) is estimated by OLS, vector process $\{vec\,(u_t u_t')\}$ is constructed from the residuals of (29), and the second regression (32) is estimated by OLS.

We used the multivariate Schwartz and Akaike Information Criteria to determine the appropriate order p of the first VAR, equation (29). In most models, these criteria favored a single lag, so we set $p = 1$ for all models. It is unclear how relevant these information-based criteria are for the second regression (32), since the elements of $vec\,(u_t u_t')$are generated from a smaller number of distinct information sources. Instead, we use a more informal procedure to choose the order q of regression (32). We seek to maximize the variability of $cov_t\left(M_{t+\tau}^\tau,\ R_{t+\tau}^\tau\right)$ while keeping the number of non-positive-definite

[16]We initially included the dividend yield on the CRSP value-weighted portfolio as a third predictor variable in X_t. We found, however, that for most preference specifications the dividend yield was insignificant in the equations for M_{t+1}^τ and R_{t+1}^τ, according to standard F-tests. This result is consistent with Fama & French (1989). In the interest of parsimony, we therefore exclude the dividend yield from X_t. (Recall that the number of free parameters grows at rate N^2.)

estimates for $var_t Y_{t+\tau}$ low. For most models, we found that $q = 8$ worked well according to this standard. For each model, Table 1 reports the number of times that our proxy for $var_t Y_{t+\tau}$ failed to be a positive-definite matrix. Failures of positive-definiteness are distressingly frequent at the quarterly horizon ($\tau = 1$), but are infrequent or nonexistent at longer horizons. These results could be interpreted as evidence of misspecification at when $\tau = 1$: our linear time-series model (29) - (32) may simply be inappropriate for modeling conditional second moments of the $\{r_t^\tau, m_t^\tau\}$ process for very small τ's. Alternatively, the problem may be that the true eigenvalues of $var_t Y_{t+1}$ are very close to zero at the quarterly horizon. When taking a linear approximation to $var_t Y_{t+1}$, it would not be surprising that the smallest eigenvalue of the approximate covariance matrices frequently falls below zero.[17]

Using the estimated values for parameters$\{A_1, B_j, j = 1, ..., 8\}$, we construct the equity-premium series EP_t^τ

$$EP_t^\tau \equiv \frac{E_t R_{t+\tau}^\tau - RF_t^\tau}{RF_t^\tau}, \tag{36}$$

the "theoretical equity-premium" series implied by the particular model, which we denote $\widetilde{EP}_t$:

$$\widetilde{EP}_t^\tau \equiv -cov_t \left(M_{t+\tau}^\tau, R_{t+\tau}^\tau \right),$$

and the "theoretical risk-free rate" series, which we denote $\widetilde{RF}_t$:

$$\frac{1}{E_t M_{t+\tau}^\tau} \equiv \widetilde{RF}_t^\tau \tag{37}$$

5 Results

5.1 Implications for the Equity Premium

In this section we examine the implications of the models for equation (27). In order to make our results comparable across different time horizons, we compute annualized

[17] In support of this interpretation, we note that when positive-definiteness fails, there is usually only one negative eigenvalue, and its absolute value is usually several orders of magnitude smaller than the other three eigenvalues of the conditional covariance matrix estimate.

continuously-compounded equity premiums, denoted $EP_t^{\tau A}$ and $\widetilde{EP}_t^{\tau A}$ as follows:[18]

$$EP_t^{\tau A} \equiv \frac{4}{\tau} \log \left[EP_t^{\tau} + 1 \right] \tag{38}$$

$$\widetilde{EP}_t^{\tau A} \equiv \frac{4}{\tau} \log \left[\widetilde{EP}_t^{\tau} + 1 \right] \tag{39}$$

If a model of m_t^τ and the time-series model (29) - (32) together described the data perfectly, we would find $EP_t^{\tau A} = \widetilde{EP}_t^{\tau A}$ for every date t. No one would expect such an outcome even for a successful model. Rather, we wish to see whether, for any of the pricing models, $\widetilde{EP}_t^{\tau A}$ approximates some of the key properties of $EP_t^{\tau A}$. In particular, we ask whether the following hold:

$$mean \left[EP_t^{\tau A} \right] \approx mean \left[\widetilde{EP}_t^{\tau A} \right] \tag{40}$$

$$var \left[EP_t^{\tau A} \right] \approx var \left[\widetilde{EP}_t^{\tau A} \right] \tag{41}$$

$$corr \left(EP_t^{\tau A}, \widetilde{EP}_t^{\tau A} \right) >> 0 \tag{42}$$

5.1.1 Consumption Measured by Nondurables Plus Services

Table 2 summarizes our results for (40) and (41) when we measure consumption by purchases of nondurables plus services. The first line of the table gives our estimate of the mean and the variance of the equity premium $EP_t^{\tau A}$ at horizons equal to one quarter, one year, two years, and three years. The remainder of the table gives the corresponding moments of the theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the two preference models. The numbers in parentheses are asymptotic p-values testing whether $(mean[EP_t^{\tau A}] - mean[\widetilde{EP}_t^{A}])$ and $(var[EP_t^{\tau A}] - var[\widetilde{EP}_t^{A}])$ are significantly different from zero. In the case of the means, we use the test statistic

$$Z_{mean} \equiv \frac{\frac{1}{T}\sum_{t=1}^{T} \left(EP_t^{\tau A} - \widetilde{EP}_t^{\tau A} \right)}{std \left[\frac{1}{T}\sum_{t=1}^{T} \left(EP_t^{\tau A} - \widetilde{EP}_t^{\tau A} \right) \right]} \tag{43}$$

[18]The annualization in (38) is appropriate, since $EP_t + 1 = E_t \frac{R_{t+\tau}^\tau}{RF_t^\tau}$, where both R_t^τ and $RF_{t+\tau}^\tau$ are gross rates of return. Also, recall that the horizon τ is in units of quarter-years.

In the case of the variances, we use the test statistic

$$Z_{var} \equiv \frac{\frac{1}{T}\sum_{t=1}^{T}\left(\left[EP_t^{\tau A}-\mu\right]^2-\left[\widetilde{EP}_t^{\tau A}-\widetilde{\mu}\right]^2\right)}{std\left[\frac{1}{T}\sum_{t=1}^{T}\left(\left[EP_t^{\tau A}-\mu\right]^2-\left[\widetilde{EP}_t^{\tau A}-\widetilde{\mu}\right]^2\right)\right]} \tag{44}$$

where μ and $\widetilde{\mu}$ are constants set equal to the sample means of $EP_t^{\tau A}$ and $\widetilde{EP}_t^{\tau A}$, respectively. Under the hypothesis that $E\left[EP_t^{\tau A}\right]=E\left[\widetilde{EP}_t^{\tau A}\right]$, statistic Z_{mean} is asymptotically distributed as a standard normal variate; similarly, if $var\left[EP_t^{\tau A}\right]=var\left[\widetilde{EP}_t^{\tau A}\right]$, statistic Z_{var} is asymptotically standard normal. We compute the standard deviations in the denominators of (43) and (44) using 12 Newey-West lags for the quarterly, yearly, and two-year horizons, and 16 lags for the three-year horizon.[19] Note that we treat $EP_t^{\tau A}$ and $\widetilde{EP}_t^{A}$ as known data series, not generated series, so the uncertainty in estimating the VAR parameters in (29) and (32) is not taken into consideration. As a result the standard deviations used in constructing Z_{mean} and Z_{var} are understated.

The time-separable model exhibits both the equity-premium puzzle and the predictability puzzle at all horizons. The observed annualized equity premiums have means between 4.5% and 6.5%. The time-separable model has difficulty generating a mean equity premium in excess of one percent for any horizon. Furthermore, the equity-premium variance generated by the model is an order of magnitude too small at all horizons.

Let us now turn to the Constantinides model. Figure 4 plots the results reported in Table 2 for this model. The upper panel plots the mean of $EP_t^{\tau A}$ (solid lines) at the four horizons, along with the mean of $\widetilde{EP}_t^{\tau A}$ for four different values of γ. The lower panel in each figure displays the analogous plots for the standard deviations of EP_t^{A} and $\widetilde{EP}_t^{A}$, scaled by $\left(\frac{\tau}{4}\right)^{\frac{1}{2}}$.[20] The Constantinides model does not fare much better

[19] We want the number of Newey-West lags to equal the maximum of the appropriate lag-lengths for $EP_t^{\tau A}$ and for $\widetilde{EP}_t^{A}$. According to equation (32), $\widetilde{EP}_t^{A}$ is a function of eight lagged regressors, each of which is serially correlated, so the appropriate lag-length for this variable is at least 9. The τ-horizon equity premium involves τ overlapping observations, so the appropriate equity-premium lag-length is at least $\tau+1$. We then experimented lag-lengths above $max\,(9,\tau+1)$ until there were no large changes in the standard deviations.

[20] We multiply by $(\frac{\tau}{4})^{\frac{1}{2}}$ to counteract the effect of annualization on the standard deviation

than the time-separable model at the shortest and longest horizons: the means of $\widetilde{EP}_t^A$ are less than 2% when $\tau = 1$ and $\tau = 12$, even with extremely high risk-aversion. According to Table 2, these point estimates are significantly below the mean equity premium. However, the model performs surprisingly well at the two-year horizon: when the Constantinides model is implemented using two-year returns with $\gamma = 12$, the mean of $\widetilde{EP}_t^{8A}$ is about 5.5%, which is extremely close to the mean value of EP_t^{8A}. According to Table 2, equality of the means of $\widetilde{EP}_t^A$ and EP_t^A is not rejected at any conventional significance level. The point estimate for the variance of $\widetilde{EP}_t^{8A}$, 0.0003, is somewhat below the estimate of 0.0015 for EP_t^{8A}. However, equality of these variances cannot be rejected at any conventional significance level.

A general pattern that emerges from Table 2 is that the mean of $\widetilde{EP}_t^{\tau A}$ increases as τ is increased from one quarter to one or two years. In other words, the mean conditional covariance between the intertemporal marginal rate of substitution and the equity return is higher (in absolute value) for the one- or two-year horizon than for the quarterly horizon. It is of interest to see whether this increase in the conditional covariance is due to an increase in the conditional correlation coefficient between these two variables, or if it is due to increased variability of one or both of these variables. In the left-hand panel of Table 3, we address this question. For each value of τ and γ in the Constantinides model, we display the mean of $\left(\frac{4}{\tau}\right) cov_t \left(M_{t+\tau}^\tau, R_{t+\tau}^\tau\right)$, as well as the mean values of $corr_t \left(M_{t+\tau}^\tau, R_{t+\tau}^\tau\right)$, $\sqrt{\frac{4}{\tau}}\sigma_{R_{t+\tau}^\tau}$, and $\sqrt{\frac{4}{\tau}}\sigma_{M_{t+\tau}^\tau}$. As can be seen, the increase in the conditional covariance as the horizon τ is lengthened is primarily due to an increase in the conditional correlation between $M_{t+\tau}^\tau$ and $R_{t+\tau}^\tau$. For example, in the Constantinides model with $\gamma = 7$, the doubling of the conditional covariance (from -0.0107 to -0.0204) as τ increases from one quarter to eight quarters is due exclusively to the increase in the conditional correlation from -0.1681 to -0.3988. (Both conditional standard deviations actually decrease with the increased horizon.) At higher levels of

of $EP_t^{\tau A}$. If $\log\left[EP_t^\tau + 1\right]$were the sum of τ i.i.d. random processes, then $std(EP_t^{\tau A})$ would decline at rate $\tau^{\frac{1}{2}}$ as τ increases, but $(\frac{\tau}{4})^{\frac{1}{2}}\, std(EP_t^{\tau A})$ would be constant. While definition (36) of EP_t^τ does not imply that $\log\left[EP_t^\tau + 1\right]$ is determined in precisely this way, we find that, in practice, $(\frac{\tau}{4})^{\frac{1}{2}}\, std(EP_t^{\tau A})$ is approximately constant in τ.

risk-aversion, however, this increase in the conditional correlation is accompanied by a moderate increase in the conditional standard deviation of $M^{\tau}_{t+\tau}$. Notice that when γ is increased to 12 in the Constantinides model, $\sqrt{\frac{4}{\tau}}\sigma_{M^{\tau}_{t+\tau}}$ increases from 0.8068 to 1.0623 as τ increases from one to eight quarters. However, the principal reason for the three-fold increase in $\left(\frac{4}{\tau}\right) cov_t\left(M^{\tau}_{t+\tau}, R^{\tau}_{t+\tau}\right)$ (from -0.0185 to -0.0586) is that the conditional correlation more than doubles.

While the results displayed in Table 2 and Figure 4 suggest that the performance of the Constantinides model improves when the horizon is lengthened to two years, the real test is whether $\widetilde{EP}^{8A}_t$ actually tracks EP^{8A}_t through time. We examine this question in Figure 5, which plots the time-series for EP^{8A}_t (dotted lines) and $\widetilde{EP}^{8A}_t$ (solid lines) generated by the Constantinides model with $\gamma = 12$. If (27) held exactly, the two series would be identical. In fact, Figure 5 shows clear (albeit imperfect) co-movement between the theoretical and observed equity premium series. The main discrepancy is that the theoretical equity premium $\widetilde{EP}^{\tau A}_t$ does not capture the secular decline in the observed equity premium from 1954 through 1980: the theoretical series is too low in the 1950's and too high in the early 1980's. However, our construct for the theoretical premium does appear to capture some of the cyclical fluctuation in the equity premium: note equity-premium peaks in 1956, 1965, and 1976-77, as well as the sharp fall-offs in 1977-78 and 1988. The overall correlation between EP^{8A}_t and $\widetilde{EP}^{8A}_t$ implied by this model is 0.11. On the whole, Figure 5 provides some evidence that consumption-based models can generate time-varying risk premiums appropriate to the observed data. As a more formal test, we regress EP^{8A}_t on the $\widetilde{EP}^{8A}_t$ series implied by the Constantinides model, along with the linear and quadratic trends.[21] In order to account for the high serial persistence in these series, we compute standard errors using 12 Newey-West lags. The coefficient on $\widetilde{EP}^{\tau A}_t$ is 0.186 with a standard error of 0.131 (significant at the 16%

[21] We include the trend terms to accommodate the slight "U" shape in EP^{8A}_t. This pattern is due almost entirely to the secular rise in the nominal two-year risk free rate over this period. (Nominal equity returns do not display any pronounced trend in post-war data.) Both the linear and quadratic trend terms enter highly significantly. As in Table 2, we do not take into consideration the fact that we are using generated regressors, so the standard errors are understated.

marginal significance level). This regression evidence confirms the visual impression of Figure 5.

The evidence of this section suggests that the equity premium puzzle is less puzzling at a two-year horizon than at a one-quarter horizon. However, most studies of consumption-based asset pricing [22] use consumption data that include the implicit service flow from owner-occupied housing. In an earlier version of this paper, [23] we performed the analysis of this section using consumption data that includes this housing service flow. The qualitative results are similar, in that the moments of the theoretical equity premium $\widetilde{EP}_t^{\tau A}$ fail to match those of the true equity premium $EP_t^{\tau A}$ at the quarterly horizon ($\tau = 1$), but come much closer at the two-year horizon ($\tau = 8$). However, both the mean and the variance of $\widetilde{EP}_t^{8A}$ are still somewhat below the corresponding moments of EP_t^{8A} when the data on the service flow from owner-occupied housing are included. In particular, the version of the Constantinides model that comes closes to matching the data sets $\gamma = 12$ and $\tau = 8$. For this specification, the mean of $\widetilde{EP}_t^{8A}$ is 0.0282, as compared to the value of 0.0551 for the mean of EP_t^{8A}. One can reject equality of these means at the 5% marginal significance level, although not at the 1% marginal significance levels. The variance of $\widetilde{EP}_t^{8A}$ is insignificantly different from the variance of EP_t^{8A} at any conventional significance level (p-value = .305).

5.1.2 Consumption Measured by Purchases of Nondurables

In this section, we replicate our analysis with consumption measured by nondurables only. Table 4 is analogous to Table 2, except that consumption is measured by purchases of nondurable consumption goods. (That is, the services component is omitted from the consumption data.) The results for the time-separable model improve somewhat at the highest levels of risk aversion for the one- and two year horizons, but they still fail to come close to matching the data. The Constantinides model, however, does quite well. Figure 6 plots the point estimates reported in Table 4 for the Constantinides model.

[22] Hansen and Singleton (1982, 1983), Grossman, Melino, & Shiller (1987), Cochrane & Hansen (1992)

[23] Daniel & Marshall (1995)

Note that the theoretical equity premium matches the mean equity premium observed in the data at the two-year horizon with $\gamma = 7$. According to the p-values in Table 4, the means of the theoretical equity premiums for these models are insignificantly different from the means of the observed equity premiums at any conventional significance level The variances are insignificantly different at the two-year horizon. The second panel of Table 3 shows that, as with nondurables plus services, the improved performance of the models at the two-year horizon is primarily due to increased correlation between the marginal rate of substitution and the equity return.

Figure 7 plots the EP_t^{8A} against the $\widetilde{EP}_t^{8A}$ series implied by the Constantinides model with $\gamma = 7$. The results are similar to Figure 5. Again, the model cannot replicate the "U-shaped" secular movement in EP_t^{8A} (the correlation between EP_t^{8A} and $\widetilde{EP}_t^{8A}$ is only 0.08). However, when EP_t^{8A} is regressed on this $\widetilde{EP}_t^{8A}$ process, along with a linear and quadratic time-trend, the coefficient on $\widetilde{EP}_t^{\tau A}$ is 0.296, with a standard error of 0.126 (significant at the 2% marginal significance level).

5.2 Implications for the Risk-Free Rate

We now test equation (28) for the models studied in the previous section. We ask whether the means and standard deviations of $\widetilde{RF}_t^{\tau}$ match those of RF_t^{τ}, and whether these two series have substantial positive correlation. As with the equity premium, we annualize by setting

$$RF_t^{\tau A} \equiv \frac{4}{\tau} \log\left(RF_t^{\tau}\right)$$

$$\widetilde{RF}_t^{\tau A} \equiv \frac{4}{\tau} \log\left(\widetilde{RF}_t^{\tau}\right)$$

Table 5 displays our results for the risk-free rate when consumption is measured by expenditures on nondurables plus services. The first line of the table gives our estimate of the mean and the variance of the observed nominal risk- free rate at the four horizons. The remainder of the table gives the corresponding moments of the theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ implied by our two preference models. The numbers in parentheses are asymptotic p-values testing whether $(mean[RF_t^{\tau A}] - mean[\widetilde{RF}_t^{A}])$ and

$(var[RF_t^{\tau A}] - var[\widetilde{RF}_t^A])$ are significantly different from zero.[24]

Our results for the time-separable model clearly illustrate the risk-free rate puzzles: For all γ's up to 50, all horizons, and both measures of consumption, the means and variances of the theoretical risk-free rates implied by the models vastly exceed the values observed in the data. Furthermore, the mean of the theoretical risk-free rate rises dramatically as risk-aversion increases.

However, the risk-free rate puzzle is far less of a problem with the Constantinides model. In Figure 8, we plot the point estimates from Table 5 for the Constantinides model. The top panels in Figure 8 plots the mean of $RF_t^{\tau A}$ (heavy lines) at the four horizons, along with the means of $\widetilde{RF}_t^{\tau A}$ for the Constantinides model using four values of γ. The bottom panels plot $\left(\frac{\tau}{4}\right)^{\frac{1}{2}}$ times the standard deviation of $RF_t^{\tau A}$ against the corresponding statistic for $\widetilde{RF}_t^{\tau A}$. Notice, first, that the means of the theoretical risk-free rate implied by the Constantinides model are all much closer to the observed mean risk-free rate than with time-separable preferences. In particular, when γ equals nine, the mean theoretical risk-free rate closely approximates the value observed in the data for all horizons up to two years. (The formal hypothesis tests reported in Table 5 reveals virtually no evidence against the hypothesis that these means are identical.) Unlike the time-separable model, the mean of the theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ in the Constantinides model is *decreasing* in the level of risk-aversion. Intuitively, the precautionary demand is much more important in these models than in the time-separable model. When γ increases from 9 to 11, the mean of $\widetilde{RF}_t^{8A}$ falls from 7.77% to 3.74%. This latter number is somewhat below the value of 6.3% estimated from the data. As indicated in Table 5, it is significantly below RF_t^{8A} at the 5% marginal significance level, although not at the 1% marginal significance level.

Let us now turn to the implications of longer horizons for the variance of $\widetilde{RF}_t^{\tau A}$. The Constantinides model has been criticized for implying counterfactually high interest-rate

[24] As in Table 2, we construct the p-values using 12 Newey-West lags for the quarterly, yearly, and two-year horizons, and 16 lags for the three-year horizon. The fact that $\widetilde{RF}_t^A$ is a generated series is not taken into consideration in computing the p-values.

variability at short horizons. This problem can be seen clearly in the bottom panel of Figure 8. In particular, at the quarterly horizon all (scaled) standard deviations of $\widetilde{RF}_t^{1A}$ are over an order of magnitude greater than the corresponding standard deviation for RF_t^{1A}. However, this excessive volatility of the theoretical risk-free rate becomes less of a problem at longer horizons with the Constantinides model. For all values of γ studied, the variance of $\widetilde{RF}_t^{\tau A}$ actually approximates the variance of the observed $RF_t^{\tau A}$ for a horizon τ somewhere between two and three years. According to Table 5, the variance of $\widetilde{RF}_t^{12A}$ implied by the Constantinides model is insignificantly different from the variance of RF_t^{12A} for all γ's studied. At the 8-quarter horizon, the variance of the theoretical risk-free rate is significantly different from the variance of the observed risk-free rate only for the highest γ's.

It would appear, then, that the problem of excessive variability in the risk-free rate implied by time-nonseparable models is primarily a short-horizon problem. The intuition behind this result is that the short-term interest rate RF_t^1 is an extremely persistent series, while M_t^1 displays little persistence. Let us assume, for purposes of exposition, that long interest rates satisfy the following version of the expectations hypothesis:

$$\log RF_{t+\tau}^{\tau A} = \frac{4}{\tau}\sum_{i=1}^{\tau} E_t\left[\log(RF_{t+i}^1)\right]. \tag{45}$$

In addition,

$$\widetilde{RF}_t^{\tau A} \equiv \frac{4}{\tau}\log\left(\frac{1}{E_t M_{t+\tau}^{\tau}}\right) \approx \frac{4}{\tau}E_t\left[\log(M_{t+\tau}^{\tau})^{-1}\right] = \frac{-4}{\tau}\sum_{i=1}^{\tau} E_t\left[\log(M_{t+i}^1)\right], \tag{46}$$

where, in (46), we ignore Jensen's inequality in making the approximation. Since $log(RF_{t+i}^1)$ is a near random walk, the variability of $E_t\left[\log(RF_{t+\tau}^1)\right]$ does not decline substantially as τ gets big. According to (45), this implies that the variance of $\log RF_{t+\tau}^{\tau A}$ is relatively insensitive to τ. On the other hand, M_{t+i}^1 displays rapid mean-reversion, so $E_t\left[\log(M_{t+i}^1)\right] \cong E\left[\log(M_{t+i}^1)\right]$ for moderately large values of i. As a result, most of the terms in the right-hand side of (46) are approximately non stochastic, so the variance of $\frac{1}{\tau}E_t\left[\log(M_{t+\tau}^{\tau})\right]$ drops off rapidly as τ increases.

In Table 6 and in Figure 9, we display implications of the models for the risk-free rate when consumption is measured as expenditures on nondurables. The qualitative behavior of both the mean and the variance of $\widetilde{RF}_t^{\tau A}$ is similar to the case of nondurables plus services. For all models, the variance is decreasing in τ, but this effect is far more pronounced in the time-nonseparable models than with time-separable preferences. With Constantinides preferences, $mean\left(\widetilde{RF}_t^{\tau A}\right)$ is somewhat lower with nondurable consumption than when the standard nondurables-plus-services measure is used. At the quarterly horizon, this mean takes on a large *negative* value for the higher γ's. As with the estimates using nondurables plus services, the mean of $\widetilde{RF}_t^{\tau A}$ is increasing in τ for these models, so these means do turn positive at a sufficiently long horizon. For example, in the Constantinides model with nondurable consumption, the mean of $\widetilde{RF}_t^{12A}$ is insignificantly different from the mean of RF_t^{12A} at the 1% marginal significance level.

While both measures of consumption allow the models to match the mean and standard deviation of the risk-free rate for a horizons between two and three years, the models do not capture any of the time-series variation in the observed risk-free rate. As in section 5.1, we regress the observed risk- free rate $RF_t^{\tau A}$ on the theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ along with a constant and linear and quadratic time-trends. Our point estimates for the coefficient on $\widetilde{RF}_t^{\tau A}$ are in all cases negative; the estimates are insignificant (at the 10% marginal significance level) for the two- year horizon, but are significantly negative (at the 1% marginal significance level) for the three year horizon. We conclude that these models do not succeed in replicating the time-series movement of the nominal risk-free rate.

6 Conclusions

In this paper, we ask whether consumption-based models are better able to match observed equity premiums and risk-free rates as the horizon lengthens. We find that time separable utility fails at all horizons with all measures of consumption, and the Constantinides model does not fare particularly well at the quarterly horizon. However, this model displays a substantial improvement in fit when the horizon is lengthened to two years. In particular, when consumption is measured as nondurables plus services (net of the implied service flow from owner- occupied housing), the Constantinides model with curvature parameter γ set to 11 comes quite close to matching the moments of the two- year equity premium, and replicates significant features of the way the two- year equity-premium varies through time. Furthermore, these models do surprisingly well at matching the moments of the risk-free rate at horizons between two and three years.

These results are intriguing. They suggest that the equity premium and risk-free rate puzzles can be substantially resolved for two-year returns. There are a number of possible reasons why the consumption-based asset pricing paradigm may fail at short horizons. In this paper, we review two candidate explanations: consumption adjustment costs and consumption measurement error. An important issue for future research is to distinguish among these possible explanations. To this end, a major puzzle is why all models dramatically fail to match the equity-premium at the three-year horizon. Market frictions could disrupt the linkage between asset returns and the consumption-based pricing kernel at short horizons. It is not clear what economic model would similarly disrupt this linkage at the very long horizons.

Finally, an alternative way of measuring consumption would be to use total consumption expenditures, including purchases of nondurables, services, and durable consumption goods. Durables are usually omitted when testing consumption-based asset pricing models because durables purchases do not directly enter agents' utility functions. Rather, agents derive utility from the service flow derived from the stock of consumer durables. However, one can argue that the omission of consumer durables may be inappropriate when studying longer-horizon returns. If the durable good depreciates quickly enough, the distinction between durables purchases and the durables stock becomes less important as the horizon lengthens. We have performed the analysis of this paper with consumption measured as total consumption expenditures. The results are broadly similar to those for nondurables plus services, so we do not report them here. These results are available from the authors upon request.

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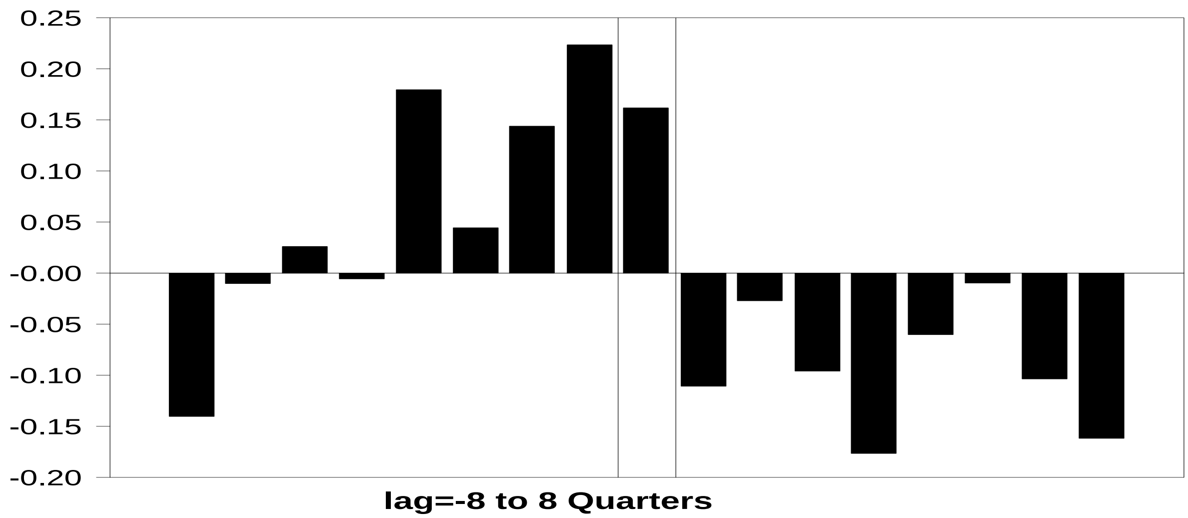
Appendix

A Construction of the Data

The quarterly real non-durable and services consumption series, the deflators for each of these series, and the population series (GPOP) were extracted from CITIBASE for the 1947:1-1994:1 period.

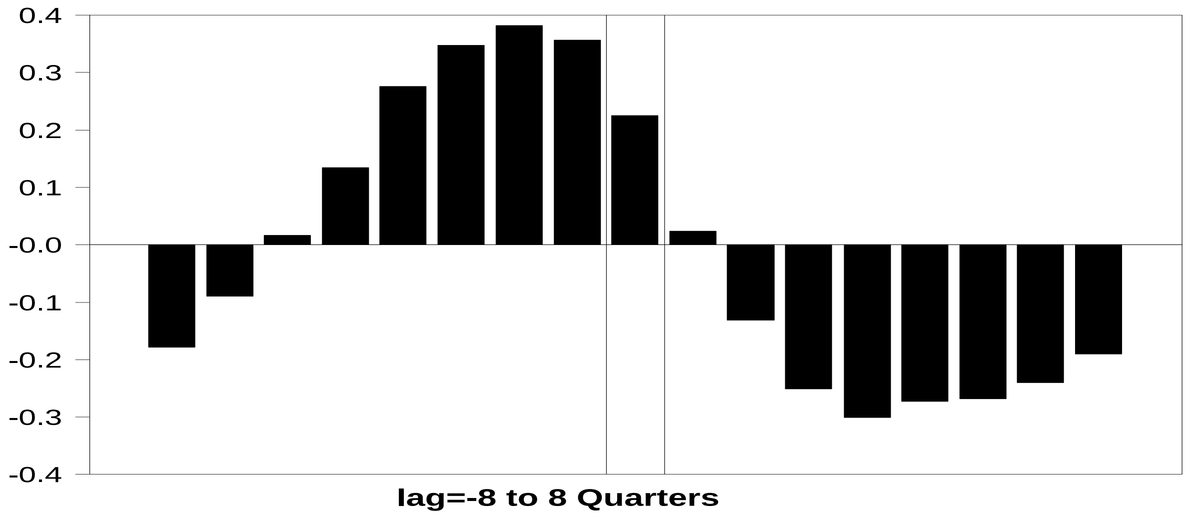
Monthly VW index returns were obtained from CRSP, and were cumulated to obtain quarterly returns. One month T-Bill returns were taken from the CRSP RISKFREE file. One, two, and three year nominally risk-free rates were computed as the returns to one-, two-, and three-year zero-coupon bonds, computed from the Fama-Bliss data in CRSP.

The default spread, term spread, and dividend yield are calculated following Fama & French (1989), using data supplied by Roger Ibbotson.



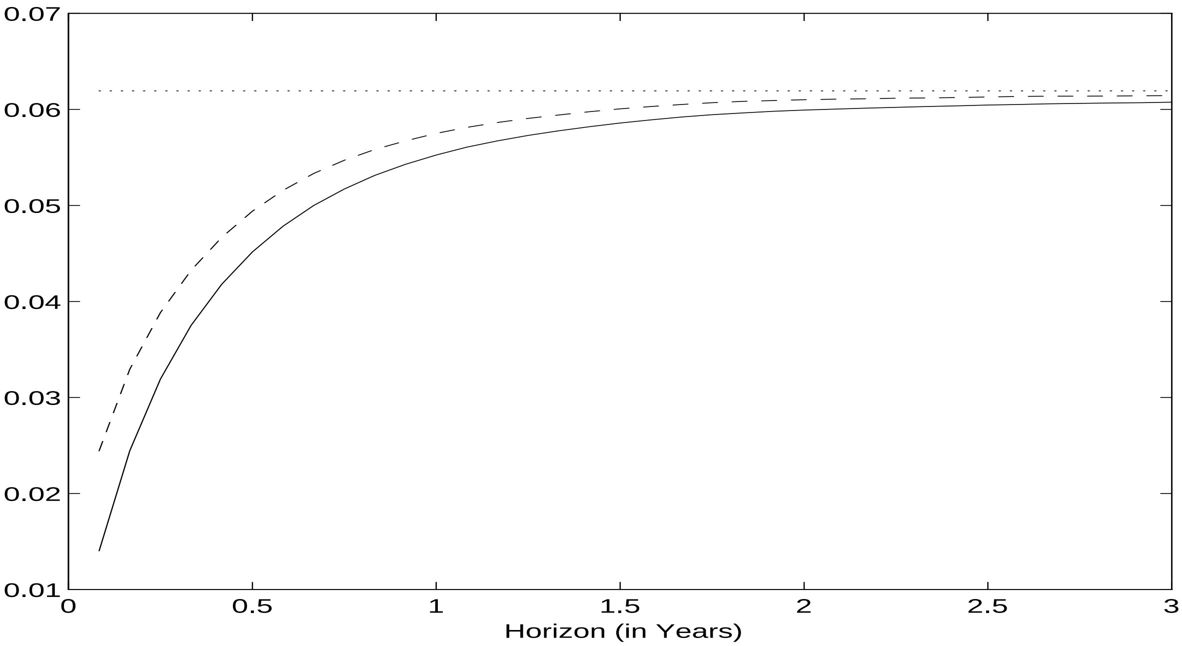
This plot presents the correlation between quarterly real VW index returns at t and the quarterly growth rate of real non-durable and services consumption at $(t - lag)$ for lag = -8 through +8 quarters. The vertical lines in the center of the graph indicate the contemporaneous correlation.

Figure 1: **Correlation between Real Quarterly VW Index Returns at t and Real Quarterly Non-Durable and Services Consumption Growth**



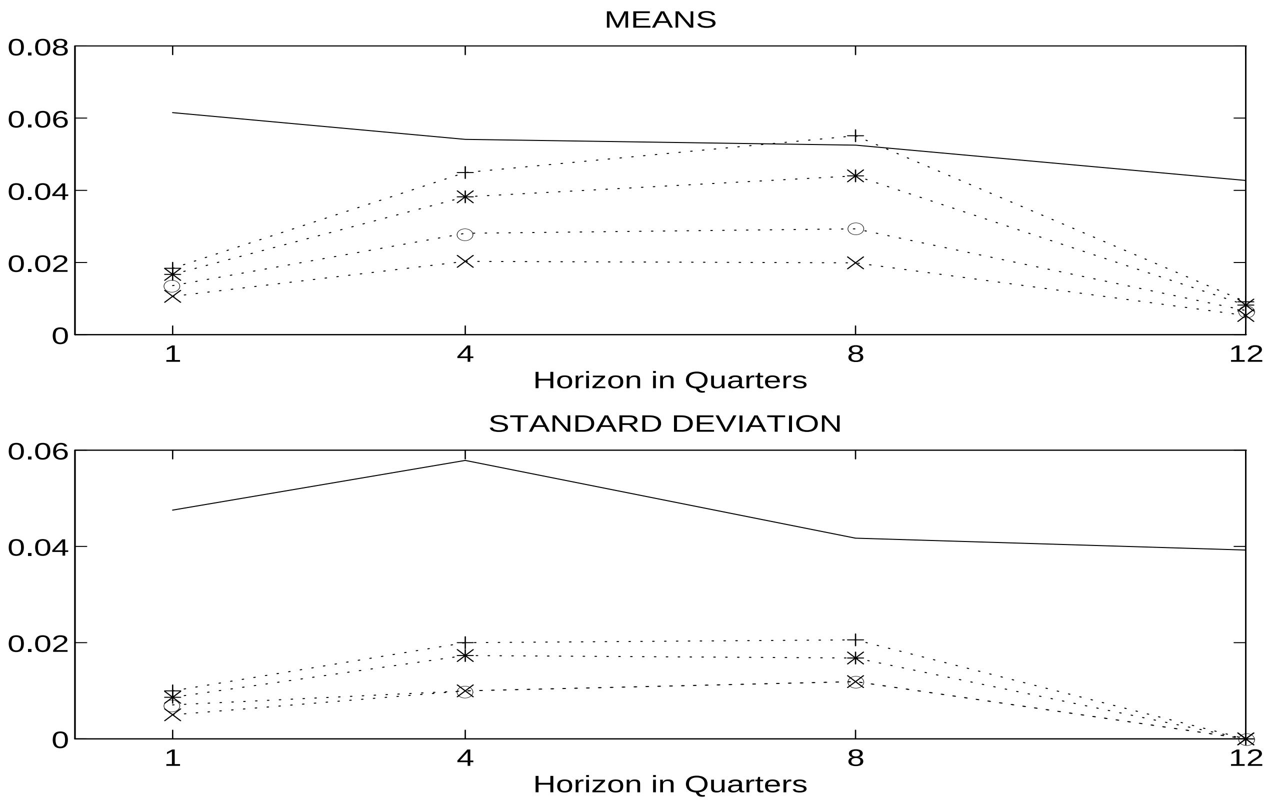
This plot presents the correlation between real cumulative one-year VW index returns at t and the one-year growth rate of real non-durable and services consumption at $(t - lag)$ for $lag = -8$ through $+8$ quarters. The vertical lines in the center of the graph indicate the contemporaneous correlation

Figure 2: **Correlation between Real One-Year VW Index Returns at t and Real One-Year Non-Durable and Services Consumption Growth**



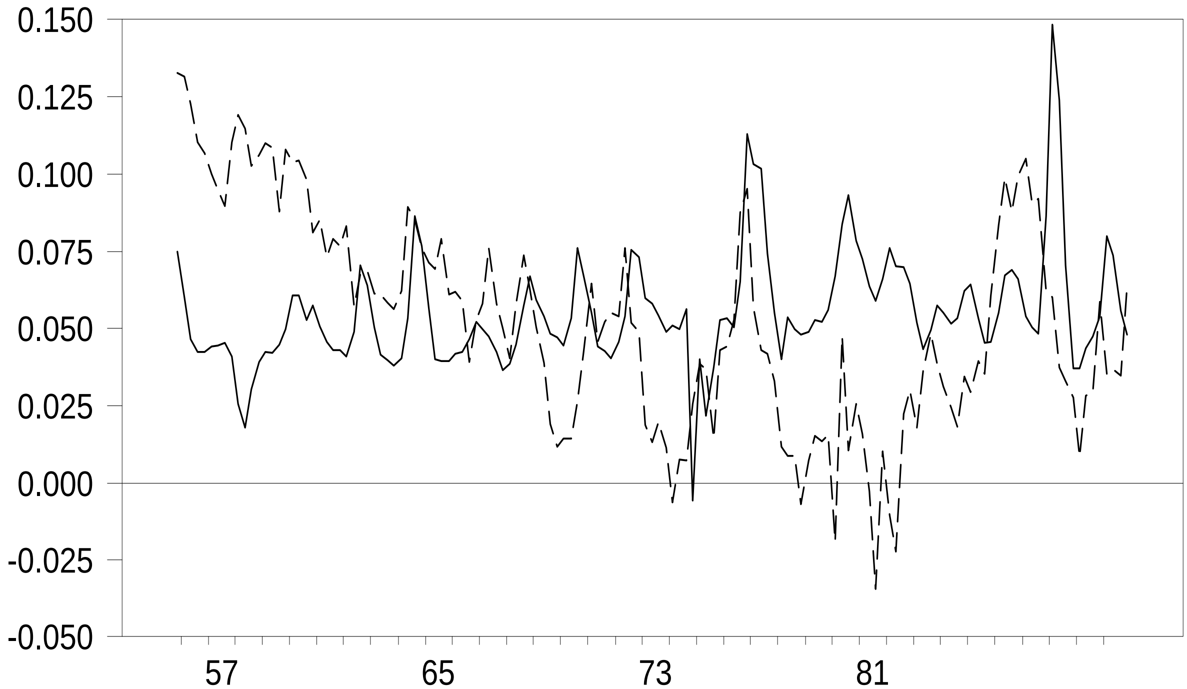
Notes: For horizons τ ranging from 1 month to 36 months, this figure plots the true (continuously-compounded) mean equity premium (dotted line), as well as $(12/\tau)$ times the values of the (continuously-compounded) theoretical equity premium, $log(-cov(e^{-\delta\tau/12}[c_{t+\tau}/c_t]^{a-1}, r^{\tau}_{t+\tau}) + 1)$, implied by the model. The dashed line plots the theoretical equity premium with consumption point-in-time sampled, while the solid line plots the theoretical equity premium with consumption measured as a monthly average. The model parameters are set as follows: $\lambda = 0.002$; $\gamma = -4$; $\delta = .04$; instantaneous risk-free rate (annualized) = .00926; instantaneous excess return to risky asset (annualized) = .06227; diffusion parameter for the risky asset (= instantaneous standard deviation, annualized) = 0.1505. The aggregate consumption process c_t was computed by simulating the optimal decisions of 100 individuals for 2,500 years and summing cross-sectionally.

Figure 3: **Theoretical Equity Premium (with and without time aggregation) verses actual Equity Premium (%/year)**



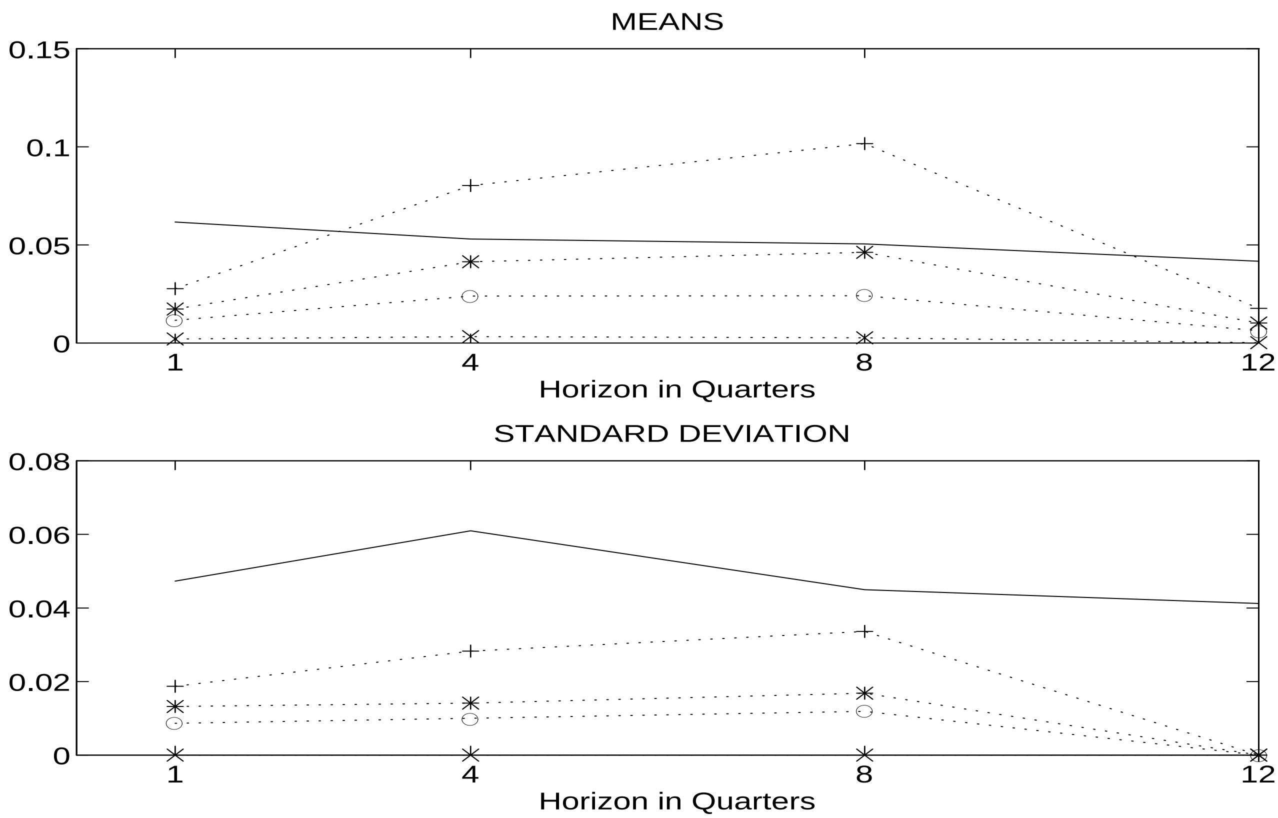
This figure displays means (top panel) and standard deviations (bottom panel) of the annualized equity premium $EP_t^{\tau A}$ (solid line) and the annualized theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the Constantinides model with $\gamma = 7$ (indicated by "$\times$"), 9 (indicated by "$\circ$") , 11 (indicated by "*"), and 12 (indicated by "+"), and with consumption measured by expenditures on nondurables plus services. The standard deviations are all scaled by $(\frac{\tau}{4})^{12}$ as in (41), to facilitate comparison across horizons. The horizons are $\tau = 1$, 4, 8, and 12 quarters.

Figure 4: **Equity Premium vs. Theoretical Equity Premium: Constantinides Model, Nondurables Plus Services**



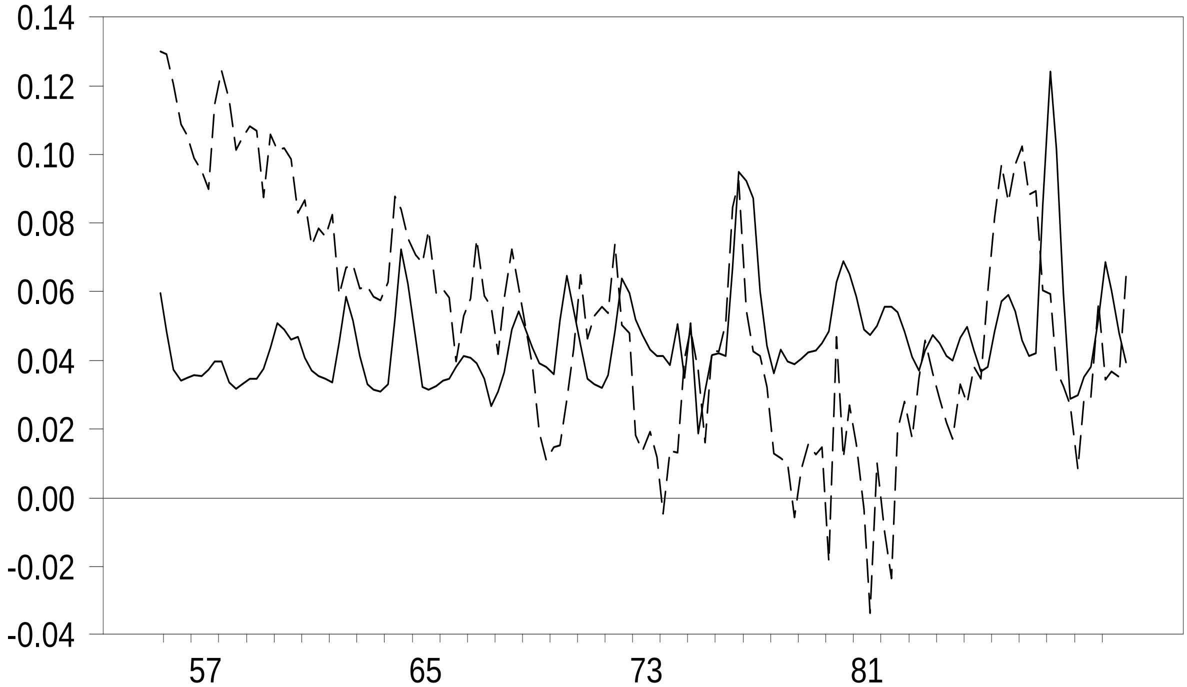
This figure displays time-series plots of the annualized equity premium $EP_t^{\tau A}$ (dashed lines) and the annualized theoretical equity premium $\widetilde{EP}_t^{\tau A}$ (solid lines) implied by the Constantinides model with $\gamma = 12$. Consumption is measured by expenditures on nondurables plus services, and the horizon $\tau = 8$ quarters.

Figure 5: **Equity Premium vs. Theoretical Equity Premium (Constantinides Model) Using Nondurables Plus Services: Time-Series Plots**



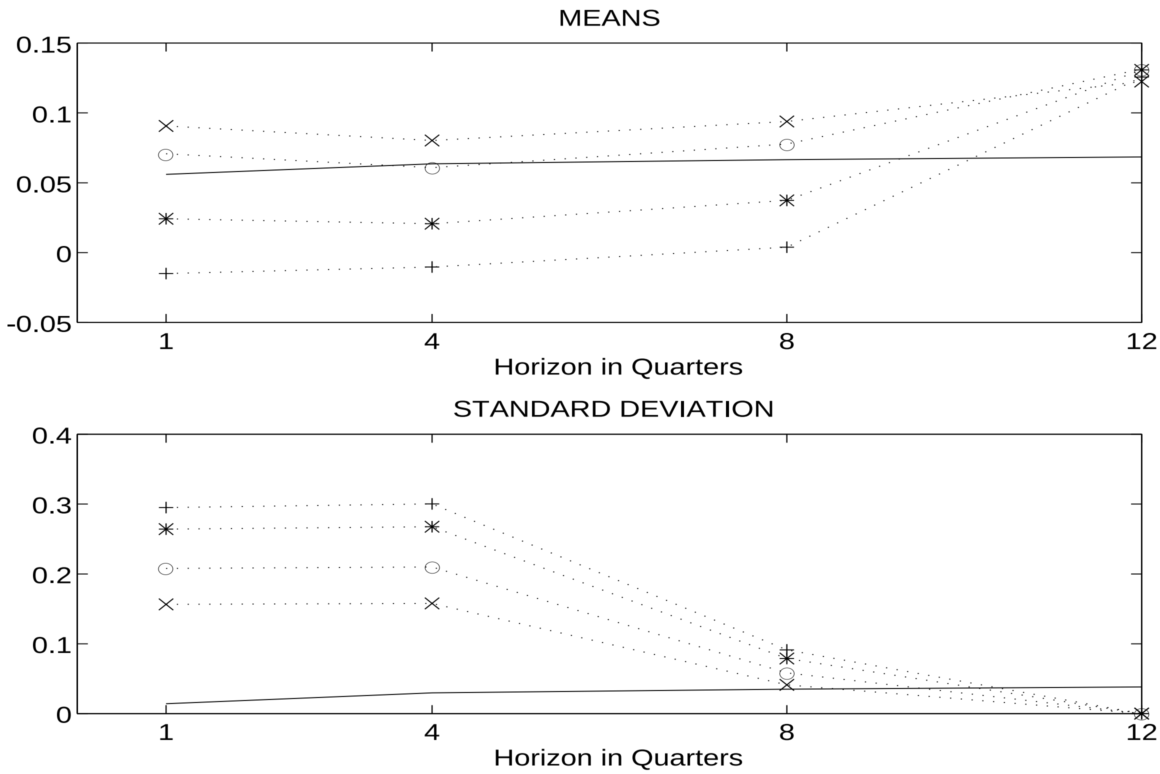
This figure displays means (top panel) and standard deviations (bottom panel) of the annualized equity premium $EP_t^{\tau A}$ (solid line) and the annualized theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the Constantinides model with $\gamma = 1$ (indicated by "$\times$"), 5 (indicated by "$\circ$") , 7 (indicated by "*"), and 9 (indicated by "+"), with consumption measured by purchases of consumer nondurables. The standard deviations are all scaled by $(\frac{\tau}{4})^{12}$ as in (41), to facilitate comparison across horizons. The horizons are $\tau = 1$, 4, 8, and 12 quarters.

Figure 6: **Equity Premium vs. Theoretical Equity Premium: Constantinides Model, Nondurables**



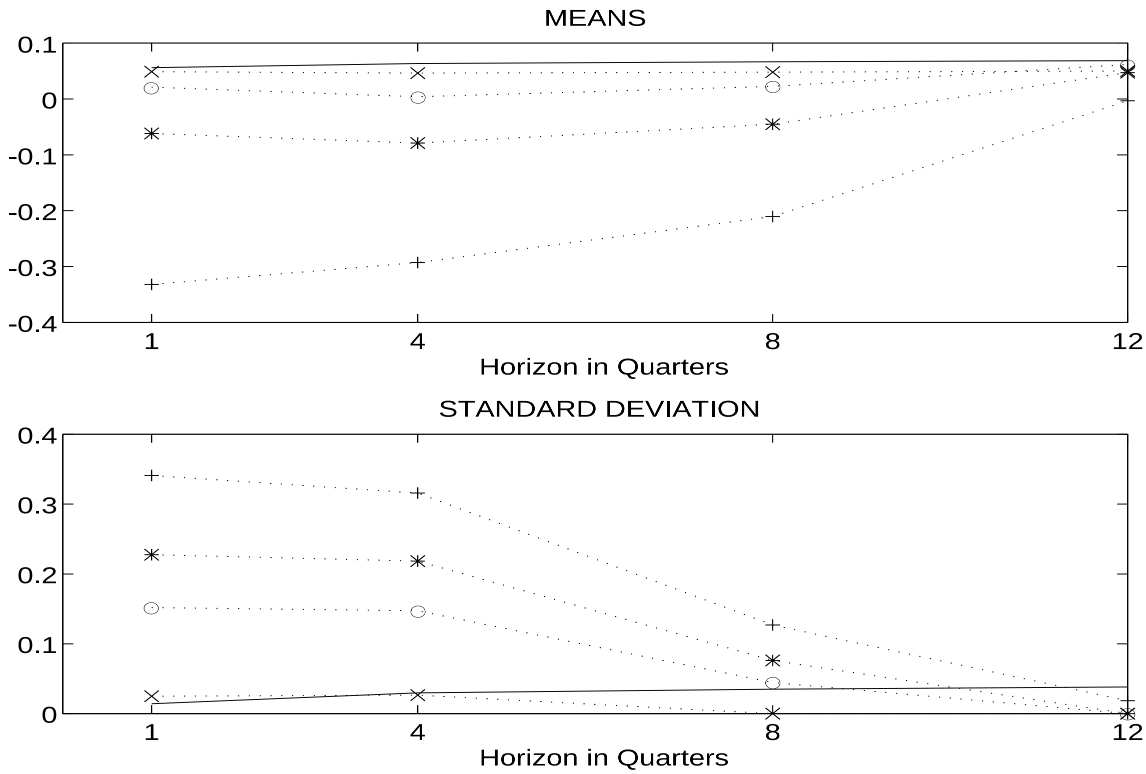
This figure displays time-series plots of the annualized equity premium $EP_t^{\tau A}$ (dashed lines) and the annualized theoretical equity premium $\widetilde{EP}_t^{\tau A}$ (solid lines) implied by the Constantinides model with $\gamma = 7$. Consumption is measured by expenditures on consumer nondurables, and the horizon $\tau = 8$ quarters.

Figure 7: **Equity Premium vs. Theoretical Equity Premium: Time-Series Plots, Nondurable Consumption**



The top panel plots the mean of the annualized risk-free rate $RF_t^{\tau A}$ at the four horizons $\tau = 1$, 4, 8, and 12 quarters (solid line), along with the means of the annualized theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ for the Constantinides model with $\gamma = 7$ (indicated by "$\times$"), 9 (indicated by "$\circ$") , 11 (indicated by "*"), and 12 (indicated by "+"). The bottom panel plot $(\frac{\tau}{4})^{\frac{1}{2}}$ times the standard deviation of $RF_t^{\tau A}$ at the four horizons (solid line) against the corresponding statistic for $\widetilde{RF}_t^{\tau A}$ for the four values of γ. Consumption is measured by expenditures on nondurables plus services.

Figure 8: **Risk-Free Rate vs. Theoretical Risk-Free Rate: Constantinides Model, Nondurables Plus Services Consumption**



These figures display means and standard deviations of the annualized risk-free rate $RF_t^{\tau A}$ and the annualized theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ implied by the Constantinides model with $\gamma = 1$ (indicated by "$\times$"), 5 (indicated by "$\circ$") , 7 (indicated by "*"), and 9 (indicated by "+"), and with consumption measured by consumption expenditures on nondurables. The standard deviations are all scaled by $(\frac{\tau}{4})^{\frac{1}{2}}$ as in (41), to facilitate comparison across horizons. The horizons are $\tau = 1$, 4, 8, and 12 quarters.

Figure 9: **Risk-Free Rate vs. Theoretical Risk-Free Rate: Constantinides Model, Nondurable Consumption**

Table 1: **Number of non-positive-definite approximate covariance matrices generated by the linear model**

For each model in equations (29) and (32), this gives the number of observations for which we obtained non-positive-definite estimates for the matrix H_{t+1}, defined in (30). The columns labeled "Nondur + Serv." are for the models where consumption is measured as expenditures on consumer nondurables plus services; the columns labeled "Nondurables" are for the models where consumption is measured as nondurable consumption expenditures. In the Constantinides model, the maximum values of γ for which all marginal utilities were positive equaled 12 for nondurables plus services and 9 for nondurable consumption.

	Time-Separable Preferences							
	Nondur. + Serv.				Nondurables			
	Horizon (Yrs.)				Horizon (Yrs)			
γ	0.25	1	2	3	0.25	1	2	3
20	43	1	0	0	46	1	1	0
30	44	1	0	0	47	3	2	0
40	48	2	0	0	47	5	1	0
50	52	2	0	0	44	5	1	0
	Constantinides Preferences							
	Nondur. + Serv.				Nondurables			
	Horizon (Yrs.)				Horizon (Yrs)			
γ	0.25	1	2	3	0.25	1	2	3
1	49	2	0	0	53	1	0	0
5	50	1	0	0	57	1	0	0
7	53	0	0	0	55	1	0	0
9	54	0	0	0	52	2	0	0
11	56	0	0	0				
12	56	1	2	0				

Table 2: **Tests of Equity-Premium Model: Nondurables + Services**

This table displays means and variances of $EP_t^{\tau A}$, along with the corresponding moments of the theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the time-separable model and the Constantinides (1990) model. Each model is evaluated at four values for the curvature parameter γ, with consumption measured as expenditures on nondurables plus services. The numbers in parentheses are asymptotic p-values testing whether the means and variance of $EP_t^{\tau A}$ equal the corresponding moments of $\widetilde{EP}_t^{\tau A}$. Specifically, the numbers in parentheses the two-sided p-values for the standard normal distribution evaluated at Z_{mean} (for the columns labeled "mean") and Z_{var} (for the columns labeled "var"), as defined in equations (43) and (44)

	HORIZON							
	quarterly		1-year		2-year		3-year	
γ	mean	var	mean	var	mean	var	mean	var
	Estimated Equity Premium							
	0.0621	0.0075	0.0556	0.0036	0.0551	0.0015	0.0478	0.0011
	Time-Separable Model, Nondurables + Services							
20	0.0041	0.0000	0.0080	0.0000	0.0057	0.0000	0.0014	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0410)	(0.0000)	(0.2633)	(0.0000)	(0 .3524)
30	0.0057	0.0001	0.0112	0.0000	0.0071	0.0000	0.0013	0.0000
	(0.0001)	(0.0001)	(0.0004)	(0.0424)	(0.0000)	(0.2647)	(0.0000)	(0 .3524)
40	0.0070	0.0002	0.0140	0.0000	0.0080	0.0000	0.0011	0.0000
	(0.0001)	(0.0002)	(0.0010)	(0.0437)	(0.0000)	(0.2658)	(0.0000)	(0 .3524)
50	0.0078	0.0003	0.0166	0.0001	0.0089	0.0000	0.0008	0.0000
	(0.0002)	(0.0002)	(0.0021)	(0.0450)	(0.0000)	(0.2669)	(0.0000)	(0 .3523)
	Constantinides Model, Nondurables + Services							
7	0.0106	0.0001	0.0203	0.0001	0.0199	0.0001	0.0053	0.0000
	(0.0027)	(0.0002)	(0.0028)	(0.0198)	(0.0004)	(0.2538)	(0.0000)	(0 .2790)
9	0.0136	0.0002	0.0281	0.0001	0.0293	0.0001	0.0067	0.0000
	(0.0048)	(0.0002)	(0.0248)	(0.0240)	(0.0141)	(0.2920)	(0.0001)	(0 .2796)
11	0.0167	0.0003	0.0382	0.0003	0.0440	0.0002	0.0082	0.0000
	(0.0084)	(0.0002)	(0.1874)	(0.0368)	(0.3853)	(0.3777)	(0.0001)	(0 .2805)
12	0.0184	0.0004	0.0449	0.0004	0.0551	0.0003	0.0091	0.0000
	(0.0115)	(0.0003)	(0.4596)	(0.0529)	(0.7992)	(0.4636)	(0.0002)	(0 .2811)

Table 3: **Average Conditional Covariances, Standard Deviations, and Correlations**

This table gives the (average) decomposition of the conditional covariance between the τ-period ahead marginal rate of substitution, $M^{\tau}_{t+\tau}$, and the τ-period cumulative equity return $R^{\tau}_{t+\tau}$, into the conditional correlation coefficient, the conditional standard deviation of $M^{\tau}_{t+\tau}$, and the conditional standard deviation of $R^{\tau}_{t+\tau}$. In particular, for each value of τ, $\overline{cov_t}$ denotes the mean of $\left(\frac{\tau}{4}\right) cov_t\left(M^{\tau}_{t+\tau}, r^{\tau}_{t+\tau}\right)$, $\overline{\rho_t}$ denotes the mean of $corr_t\left(M^{\tau}_{t+\tau}, R^{\tau}_{t+\tau}\right)$, and $\overline{\sigma_{M,t}}$ denotes $sqrt\frac{4}{\tau}$ times the mean of the conditional standard deviation of $M^{\tau}_{t+\tau}$. The final component of this decomposition is $\sqrt{\frac{4}{\tau}}E\left(\sigma_{R^{\tau}_{t+\tau},t}\right)$, the mean of the conditional standard deviation of $R^{\tau}_{t+\tau}$. This moment is invariant to the preference specification or consumption measure. For the four values of τ tabulated, this moment is given by: $\sqrt{\frac{4}{\tau}}E\left(\sigma_{R^{1}_{t+1},t}\right) = 0.1543$; $\sqrt{\frac{4}{\tau}}E\left(\sigma_{R^{4}_{t+4},t}\right) = 0.1617$; $\sqrt{\frac{4}{\tau}}E\left(\sigma_{R^{8}_{t+8},t}\right) = 0.1497$; $\sqrt{\frac{4}{\tau}}E\left(\sigma_{R^{12}_{t+12},t}\right) = 0.1377$.

	Constantinides Model							
	ND & S Consumption				Non-Durable Consumption			
τ	γ	$\overline{cov_t}$	$\overline{\rho_t}$	$\overline{\sigma_{M,t}}$	γ	$\overline{cov_t}$	$\overline{\rho_t}$	$\overline{\sigma_{M,t}}$
1	7	-0.0107	-0.1681	0.4016	1	-0.0021	-0.1606	0.0732
4	7	-0.0206	-0.3491	0.3584	1	-0.0033	-0.2882	0.0687
8	7	-0.0204	-0.3988	0.3332	1	-0.0027	-0.2998	0.0560
12	7	-0.0054	-0.2223	0.1749	1	-0.0002	-0.0440	0.0387
1	9	-0.0136	-0.1652	0.5413	5	-0.0116	-0.1718	0.3895
4	9	-0.0285	-0.3522	0.4928	5	-0.0243	-0.3801	0.3918
8	9	-0.0303	-0.3931	0.5085	5	-0.0248	-0.4284	0.3821
12	9	-0.0068	-0.2143	0.2288	5	-0.0063	-0.2549	0.1796
1	11	-0.0168	-0.1575	0.7062	7	-0.0174	-0.1734	0.6001
4	11	-0.0391	-0.3555	0.6717	7	-0.0423	-0.4328	0.6723
8	11	-0.0462	-0.3743	0.8149	7	-0.0486	-0.4028	0.8002
12	11	-0.0083	-0.2046	0.2938	7	-0.0104	-0.2624	0.2875
1	12	-0.0185	-0.1530	0.8068	9	-0.0279	-0.1647	1.0555
4	12	-0.0461	-0.3553	0.7960	9	-0.0840	-0.3527	1.4821
8	12	-0.0586	-0.3644	1.0623	9	-0.1137	-0.3671	2.0543
12	12	-0.0092	-0.1992	0.3345	9	-0.0181	-0.2532	0.5202

Table 4: **Tests of Equity-Premium Model: Nondurable Consumption**

This table displays means and variances of $EP_t^{\tau A}$, along with the corresponding moments of the theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the time-separable model, and the Constantinides (1990) model. Each model is evaluated at four values for the curvature parameter γ, with consumption measured as expenditures on nondurables services.
The numbers in parentheses are asymptotic p-values testing whether the means and variance of $EP_t^{\tau A}$ equal the corresponding moments of $\widetilde{EP}_t^{\tau A}$: Specifically, the numbers in parentheses the two-sided p-values for the standard normal distribution evaluated at Z_{mean} (for the columns labeled "mean") and Z_{var} (for the columns labeled "var"), as defined in equations (43) and (44)

	HORIZON							
	quarterly		1-year		2-year		3-year	
γ	mean	var	mean	var	mean	var	mean	var
	Estimated Equity Premium							
	0.0621	0.0075	0.0556	0.0036	0.0551	0.0015	0.0478	0.0011
	Time-Separable Model, Nondurable Consumption							
20	0.0045	0.00007	0.0107	0.00002	0.0105	0.00002	0.0030	0.00000
	(0.0001)	(0.0000)	(0.0000)	(0.0150)	(0.0000)	(0.2737)	(0.0000)	(0.3456)
30	0.0062	0.00015	0.0173	0.00004	0.0161	0.00004	0.0040	0.00000
	(0.0001)	(0.0000)	(0.0004)	(0.0157)	(0.0000)	(0.2848)	(0.0000)	(0.3458)
40	0.0076	0.00026	0.0250	0.00009	0.0236	0.00008	0.0049	0.00000
	(0.0001)	(0.0000)	(0.0044)	(0.0172)	(0.0009)	(0.3056)	(0.0000)	(0.3459)
50	0.0084	0.00039	0.0348	0.00017	0.0351	0.00017	0.0058	0.00000
	(0.0002)	(0.0000)	(0.0490)	(0.0200)	(0.0353)	(0.3525)	(0.0000)	(0.3460)
	Constantinides Model, Nondurable Consumption							
1	0.0021	0.00001	0.0033	0.00000	0.0027	0.00000	0.0002	0.00000
	(0.0006)	(0.0002)	(0.0000)	(0.0110)	(0.0000)	(0.1971)	(0.0000)	(0.2693)
5	0.0116	0.00031	0.0239	0.00008	0.0241	0.00008	0.0063	0.00000
	(0.0035)	(0.0002)	(0.0165)	(0.0118)	(0.0092)	(0.2342)	(0.0002)	(0.2698)
7	0.0173	0.00067	0.0414	0.00016	0.0462	0.00022	0.0102	0.00000
	(0.0094)	(0.0004)	(0.3481)	(0.0149)	(0.6804)	(0.3215)	(0.0009)	(0.2706)
9	0.0277	0.00143	0.0803	0.00082	0.1016	0.00081	0.0177	0.00000
	(0.0472)	(0.0015)	(0.0322)	(0.0603)	(0.0000)	(0.7086)	(0.0120)	(0.2723)

Table 5: **Tests of Riskfree Rate Model: Nondurables + Services**

This table displays means and variances of the nominal risk-free rate $RF_t^{\tau A}$, along with the corresponding moments of the theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ implied by the time-separable model and the Constantinides (1990) model. Each model is evaluated at four values for the curvature parameter γ, with consumption measured as expenditures on nondurables plus services. The numbers in parentheses are asymptotic p-values testing whether the means and variance of $RF_t^{\tau A}$ equal the corresponding moments of $\widetilde{RF}_t^{\tau A}$: Specifically, the numbers in parentheses the two-sided p-values for the standard normal distribution evaluated analogously to Z_{mean} (for the columns labeled "mean") and Z_{var} (for the columns labeled "var"), as defined in equations (43) and (44)

	HORIZON							
	quarterly		1-year		2-year		3-year	
γ	mean	var	mean	var	mean	var	mean	var
	Observed Risk-Free Rate							
	0.0541	0.00084	0.0608	0.00096	0.0630	0.00097	0.0664	0.00088
	Time-Separable Model, Nondurables + Services Consumption							
20	0.3735	0.0403	0.3605	0.0231	0.3556	0.0054	0.3628	0.0008
	(0.0000)	(0.0793)	(0.0000)	(0.1384)	(0.0000)	(0.5839)	(0.0000)	(0 .9870)
30	0.5263	0.0999	0.4990	0.0571	0.4807	0.0118	0.4929	0.0008
	(0.0000)	(0.0541)	(0.0000)	(0.0983)	(0.0000)	(0.4962)	(0.0000)	(0 .9899)
40	0.6674	0.1948	0.6215	0.1126	0.5803	0.0204	0.6025	0.0006
	(0.0000)	(0.0408)	(0.0000)	(0.0767)	(0.0000)	(0.4466)	(0.0000)	(0 .9671)
50	0.7971	0.3348	0.7312	0.2055	0.6561	0.0322	0.6945	0.0004
	(0.0000)	(0.0321)	(0.0000)	(0.0655)	(0.0000)	(0.4092)	(0.0000)	(0 .9449)
	Constantinides Model, Nondurables + Services Consumption							
7	0.0907	0.0978	0.0803	0.0249	0.0938	0.0012	0.1224	0.0000
	(0.3308)	(0.0000)	(0.4749)	(0.0002)	(0.0017)	(0.8054)	(0.0000)	(0 .5888)
9	0.0707	0.1727	0.0608	0.0441	0.0777	0.0024	0.1314	0.0000
	(0.7515)	(0.0000)	(0.9315)	(0.0000)	(0.2586)	(0.3633)	(0.0000)	(0 .5862)
11	0.0242	0.2791	0.0207	0.0716	0.0374	0.0044	0.1308	0.0000
	(0.5863)	(0.0000)	(0.2961)	(0.0000)	(0.0123)	(0.0338)	(0.0000)	(0 .5774)
12	-0.0150	0.3482	-0.0103	0.0902	0.0039	0.0059	0.1257	0.0000
	(0.2725)	(0.0000)	(0.1109)	(0.0000)	(0.0000)	(0.0021)	(0.0000)	(0 .5717)

Table 6: **Tests of Risk-Free Rate Model: Nondurable Consumption**

This table displays means and variances of the nominal risk-free rate $RF_t^{\tau A}$, along with the corresponding moments of the theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ implied by the time-separable model and the Constantinides (1990) model. Each model is evaluated at four values for the curvature parameter γ, with consumption measured as expenditures on nondurables. The numbers in parentheses are asymptotic p-values testing whether the means and variance of $RF_t^{\tau A}$ equal the corresponding moments of $\widetilde{RF}_t^{\tau A}$: Specifically, the numbers in parentheses the two-sided p-values for the standard normal distribution evaluated analogously to Z_{mean} (for the columns labeled "mean") and Z_{var} (for the columns labeled "var"), as defined in equations (43) and (44)

	HORIZON							
	quarterly		1-year		2-year		3-year	
γ	mean	var	mean	var	mean	var	mean	var
	Observed Risk-Free Rate							
	0.0541	0.00084	0.0608	0.00096	0.0630	0.00097	0.0664	0.00088
	Time-Separable Model, Nondurable Consumption							
20	0.2250	0.0424	0.2062	0.0250	0.2100	0.0054	0.2159	0.0016
	(0.0001)	(0.0003)	(0.0000)	(0.0051)	(0.0000)	(0.2570)	(0.0000)	(0.8393)
30	0.2873	0.1039	0.2500	0.0631	0.2324	0.0128	0.2646	0.0033
	(0.0000)	(0.0001)	(0.0000)	(0.0024)	(0.0000)	(0.1133)	(0.0000)	(0.6770)
40	0.3292	0.2001	0.2675	0.1283	0.2280	0.0234	0.2870	0.0045
	(0.0000)	(0.0000)	(0.0000)	(0.0030)	(0.0000)	(0.0572)	(0.0000)	(0.5999)
50	0.3508	0.3395	0.2407	0.1876	0.1907	0.0400	0.2849	0.0046
	(0.0002)	(0.0000)	(0.0004)	(0.0002)	(0.0000)	(0.0300)	(0.0000)	(0.5692)
	Constantinides Model, Nondurable Consumption							
1	0.0489	0.0025	0.0462	0.0007	0.0480	0.0000	0.0500	0.0000
	(0.3889)	(0.0633)	(0.0210)	(0.8545)	(0.0200)	(0.5030)	(0.0461)	(0.5738)
5	0.0211	0.0921	0.0040	0.0217	0.0226	0.0014	0.0618	0.0000
	(0.1881)	(0.0000)	(0.0016)	(0.0000)	(0.0000)	(0.6653)	(0.4573)	(0.5600)
7	-0.0618	0.2071	-0.0791	0.0477	-0.0452	0.0041	0.0470	0.0000
	(0.0021)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0563)	(0.0141)	(0.5662)
9	-0.3320	0.4651	-0.2929	0.0998	-0.2104	0.0114	-0.0031	0.0002
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1001)	(0.0000)	(0.6362)