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Financing Investment Under Asymmetric Information[†]

The North-Holland Finance Handbook

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A number of articles have analyzed the firm's decision to issue equity when manager's have better information than investors. This chapter reviews these articles within the framework of a model that allows us to compare the efficiency of the various signals introduced in the articles. We show that a number of the proposed signals are uniform cost or *money burning* signals, and show that there is a crucial distinction among these signals, depending on whether the signalling cost is incurred before or after the realization of cash flows. Other proposed signals are shown to be either more or less efficient than burning money, depending on the nature of the information asymmetry. In some cases a combination of signals is optimal. We also analyze the literature that examines how these issues contribute to the firm's debt/equity choice.

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Introduction

When a firm has an opportunity to accept a positive net present value project that requires equity financing, it faces a dilemma. If its managers believe the firm's stock is underpriced, then taking the project forces the firm to issue underpriced stock and thereby dilutes the value of its existing stock. As Myers and Majluf (1984) point out, firms with underpriced stock may forego attractive investment projects for this reason. Given this, analysts and investors will believe that when firms do issue equity, their shares are more likely to be overpriced. Hence, announcements of equity offers are likely to be accompanied by share price declines.

The Myers and Majluf theory has been subject to extensive empirical testing. Chapter * reviews a number of empirical papers that confirm the Myers and Majluf prediction that stock prices will generally decline when firms announce equity issues. The basic theoretical framework provided by Myers and Majluf has also been extended in a variety of ways. Most of these extensions consider ways in which a firm that intends to issue equity can signal its value and thereby reduce adverse selection problem.

This chapter provides a simple framework for understanding the basic Myers and Majluf underinvestment model as well as the related literature that examines how financial decisions provide information to market participants. The financial decisions we analyze include dividend policy (John and Williams (1985) and Ambarish, John and Williams (1987)), the scale of the investment project (Krasker (1986)), the timing of the project's initiation (Choe, Masulis and Nanda (1993)), the underpricing of the issue (Allen and Faulhaber (1989), Grinblatt and Huang (1989) and Welch (1989)) and the overpricing of the issue (Giammarino and Lewis (1988)).

The information content of these financial decisions have typically been examined in isolation. For example, no one has examined whether or not it makes sense to underprice an equity issue if one could also provide information to the market by paying a dividend. To examine issues of this type, we analyze how efficiently the various financial decisions convey information to market participants. Our analysis suggests that only those decisions (or combinations of decisions) which provide the most efficient signals, (i.e., minimum cost), will be used in equilibrium.

A signal can generally be viewed as a certain action or decision that imposes greater costs on low valued firms (or individuals) than on high valued firms. For example, in the classic Spence (1974) model, more talented individuals can successfully signal their talents by acquiring more education, since doing so is less costly for them than for less talented individuals. In general, signals that impose the highest costs on the lower valued firms or individuals, relative to the costs imposed on the higher valued firms or individuals, are the most efficient since they prevent mimicry by the low valued firms at a minimum cost to the high valued firm. In the education example, talented individuals will want to study subjects which are both easiest for them and most difficult for those with less talent.

Our analysis of the signalling efficiency of the various financial decisions includes two forms of "money burning" signals. A signal is typically referred to as a "money burning" signal if it satisfies two conditions: (1) the signalling action must be purely dissipative, meaning that it provides no direct benefits to the signalling firm, and (2) the signalling action must be equally costly for all types. For example Milgrom and Roberts (1986) present a model where advertising is a money burning signal. In their model (1) potential customers learn nothing from a firm's advertising other than that it is spending money, and (2) advertising is equally costly for all firm types, which is in contrast to the education signal suggested by Spence which imposed a higher cost on the less talented individuals. The effect of a money burning signal can always be duplicated by actually burning currency, which is why the signal is so designated.

Given that burning money imposes the same costs on all firms regardless of their types it provides a relatively inefficient signal. As we will show, the efficiency of a money burning signal depends on its timing. We analyze two types of these money burning signals; the first, which is much less efficient, requires burning money prior to the equity offering and the second requires a commitment to burn money subsequent to the equity offering when the project's cash flows are realized. Since we expect that something equivalent to both types of money burning signals should always be available to equity issuers,[1] our analysis of money burning signals provides a useful benchmark for judging the signalling efficiency of the financial decisions suggested in the literature.

[1] For example, the firm can spend an excessive amount promoting the equity issue or alternatively commit to overpay their investment banker.

When there is asymmetric information about the value of the firm's assets-in-place, but not about the value of the project, the overpricing and project scaling signals are shown to be the most efficient signals. However, when there is asymmetric information about the project's value and projects of high valued firms are worth considerably more than those implemented by lower valued firms, the overpricing and project scaling signals are much less efficient and are in fact dominated by signals that commit the firm to burn money subsequent to the offering. The John and Williams dividend signal as well as the Choe, Masulis and Nanda project delay signal turn out to be equivalent to signalling with a commitment to burn money subsequent to the offering. Under some conditions, the most efficient signal will be a combination of committing to burn money, scaling the project, and overpricing the issue. Underpricing the issue, which is equivalent to burning money out of the proceeds of the issue, is the least efficient signal and is always dominated by commitments to burn money in the future.

Although most of this chapter is devoted to an analysis of how financial signals mitigate the adverse selection problem that arises when firms issue equity, we also briefly examine the choice between debt and equity financing in this setting. As Myers and Majluf discuss, when a firm can issue risk-free debt, there is no adverse selection problem and thus no need for costly signals. However, when the firm must issue risky debt, the problem becomes substantially more interesting. While we expect that in general the Myers and Majluf adverse selection problem will exist when firms are unable to issue risk-free debt, there are certain cases described in the literature where firms can costlessly signal their type with their financing choices and as a result invest efficiently.

The rest of this chapter is organized as follows: The first and second sections describe the basic Myers and Majluf model in the case where the more highly valued issuing firm can burn money to signal its type. The third and fourth sections expand the potential signals of the issuing firm to include price setting, dividends, project scaling and project delay. Section 5 analyzes the model when firms can issue debt as well as equity and Section 6 presents our conclusions.

Figure 1: Basic Model Timeline

$t = -1$	$t = 0$	$t = 1$
-Firm type revealed to manager	-Manager commits to burn \$C. -Firm issues equity. -Investors purchase equity at mkt. price.	-Firm receives cash flows from assets-in-place and project. -Firm burns \$C. -Firm liquidated.

1 Model Description

Our basic model includes a group of risk-neutral investors and a firm that has the opportunity to take on a positive NPV project. We will initially assume that the project requires equity capital. Perhaps, the firm has existing debt covenants that prevents it from issuing additional debt. Alternatively, the firm may want to avoid issuing debt because of concerns about costs associated with financial distress and bankruptcy.

The firm's manager is assumed to maximize the intrinsic value of its shares or equivalently the wealth of its shareholders who choose to retain their shares and do not purchase any part of a new issue.[2] This assumption will be discussed in detail at the end of the chapter. Initially, we assume that the firm is one of two types: high or low. The high type is characterized by a larger value of its assets-in-place and in some cases by a more valuable investment opportunity. In this model, the firm's management knows the firm's type, but investors do not.

To simplify notation, we assume that the risk-free interest rate is zero and that the competitive equity market purchases equity issues at a price equal to the expected future cash flows of the shares. If the issuer specifies a price, then the market will not purchase the issue if it is priced above this level, and at any issue price below this level there will be rationing of the issue. In one case we allow the investors to mix if the issue price is equal to the conditional expected value.

[2]The model can easily be generalized to have a manager's objective function that includes the firm's current market price as well as its intrinsic value, though the conclusions will change somewhat. This issue is explored further in Section 5.4.

The timeline for the two type model is illustrated in Figure 1. At $t=-1$ the firm's type is revealed to its management. At $t=0$, the firm's manager decides whether or not to take the project. If his decision is affirmative, he issues just enough equity to fund the project. At this point in time he can also commit to burn money at $t=1$. Examples of mechanisms by which the manager can make this commitment are given later in this section.

We vary the strategy space of the model throughout the paper. In the first part of Section 2 we restrict the firm to either undertaking or foregoing the project; no other actions are possible. The model with this restriction is equivalent to the Myers and Majluf model and the conclusions are identical. In the second half of the section we extend the basic model's strategy space to allow the firm to commit to burn money, in Sections 3.1 and 3.2 we extend the strategy space to allow the firm's manager to specify an issue price (as in Giammarino and Lewis (1988)) and to scale the new project (as in Krasker (1986)).

The following notation is employed: θ denotes the value of the firm's assets-in-place. In our basic two type model when the manager's private information concerns the value of the firm's assets in place $\theta \in \{H, L\}$, where L denotes the value of the assets-in-place of the low value firm and H the value of the assets-in-place of high value firm. I the cost of project and the size of the equity issue and V the revenue from project (so that $V-I$ is equal to the NPV of the project). In this section of the paper V is assumed to be common knowledge. In later sections of the paper, we will consider a setting where the manager has private information about the value of the project in which case V will be a random variable observable only by the manager. C is the amount of money which the firm commits to burn as a signal.

2 Analysis of the Model

In Sections 2.1 and 2.2 a model with two possible firm types is used to illustrate the Myers and Majluf model. We analyze the various equilibria that can arise in this setting and the effect on these equilibria of the addition of committing to burn money to the strategy space. In Section 2.3 these results are extended to a continuum of types.

2.1 The Basic Model

Table 1 illustrates the first example. The model parameters are listed at the top of the table. A high valued firm has assets-in-place worth $H = 100$, compared to $L = 50$ for a low valued firm. Both types have investment opportunities which cost $I = 20$ to undertake, and yield a cash flow at the end of the period of $V = 30$. The NPV of the project is thus $30 - 20 = 10$. Finally, the probability that the firm type is high is $\%H = 0.1$. The firm has no slack (or cash on hand), and therefore must issue equity to obtain the \$20. In this example we assume that the firm's managers maximize the wealth of the original shareholders.

In order to find the sequential equilibria (as defined by Kreps and Wilson (1982)) of the model we determine the payoffs to the firm for each strategy it might employ and for each possible set of investor beliefs *about* the firm's strategy. Based on the assumptions about the equity market stated in Section 1, the investors' purchase the issue at a price equal to the firm's expected value conditioned on all available information. If the firm's optimal strategy coincides with the investors' beliefs (*i.e.*, if a fixed point is achieved), then neither the investors nor the firm wish to change their strategy knowing the strategy of the other, and the strategies form a sequential equilibrium. For example, if an investor believes that the strategy of both type H and type L is to take the project (Belief Set A), and sets prices accordingly, then the payoffs to the high and low type firm are \$99.40 and \$61.20, respectively, if it issues equity and takes the project, and \$100 and \$50 if it does not. For these payoffs, it is optimal for the high type to *not* take the project, contrary to investor beliefs, and thus we see that there is no sequential pooling equilibrium for these parameter values.

The values in the payoff matrix are calculated in the following way: First, the payoff to either a high or low type firm if it doesn't take the project is just the value of that firm's assets-in-place (50 or 100 in this case). Note that in calculating these two payoffs the investors' beliefs do not matter because a firm doesn't issue equity if it doesn't take the project. Calculating the payoff for the high type firm when it takes the project is slightly more complicated. Now the firm must issue equity worth \$20 to finance the project. Investors, in return for their \$20, will receive a certain number of the firm's shares or, equivalently, a fraction of the firm's cash flows at the end of the period. The total cash flow for a given type of firm at date 1 is equal to the value of its assets-in-place plus the gross payoff of the new project:

Table 1: Separating Equilibrium

Model Parameters:		$H = 100$	$V = 30$			
		$L = 50$	$\%H = 0.1$			
		$I = 20$				
Belief Set A:				*Belief Set B:*		
Don't Take:	None			*Don't Take:*	Highs	
Take:	Both (Pool)			*Take:*	Lows	

Payoffs:		**RESPONSE**		*Payoffs:*		**RESPONSE**	
		Don't Take	Take			Don't Take	Take
	High	100	99.40		High	100	97.50
TYPE				**TYPE**			
	Low	50	61.20		Low	50	60

Total Payoff for High Firm: $100 + 30 = 130$
Total Payoff for Low Firm: $50 + 30 = 80$

Because investors are risk neutral they demand an expected payoff equal to the size of the equity issue, \$20. Because prior beliefs are that 10% of the firms are high type and 90% are low, investors will have an expected payoff of:

$$E(Payoff) = f \cdot (\overline{\theta} + V) = I \tag{1}$$

Where f is the fraction of the firm's cash flow demanded by the investors and $\overline{\theta} + V$ is the average firm value, equal to $0.1(H + V) + 0.9(L + V)$. Inserting the values from Table 1 into Equation (1) we see that the value of f in this numerical example is:

$$f = \frac{20}{0.1(130) + 0.9(80)} = \frac{20}{85}$$

With investors demanding this fraction of the firm's cash flows, the original shareholders' payoff is:

$$\begin{pmatrix} \pi_H \\ \pi_L \end{pmatrix} = (1 - f) \cdot \begin{pmatrix} H + V \\ L + V \end{pmatrix} = \begin{pmatrix} 99.40 \\ 61.20 \end{pmatrix} \tag{2}$$

These are the entries in the left hand side of the payoff matrix in Table 1.

Comparing the payoffs for the two types reveals that while the low type will take the project the high will not: their payoff is 100 if they don't take the project versus 99.40 if they do. This indicates that the investors' beliefs are not confirmed in equilibrium, showing that this is not a sequential equilibrium. So we must try another set of beliefs.

The right side of Table 1 shows the payoffs given the belief that only the low firms accept the project. The fraction f of the firm now demanded in return for the \$20 investment is:

$$f = \frac{20}{80} = 25\%$$

Substituting f into equation (2) gives payoffs of (97.50,60) for taking the project, as shown in Table 1B. With these payoffs we see that the beliefs are now credible: the lows take the project and the highs do not. This separating equilibrium is the only sequential equilibrium.[3]

It is important to note that the separating equilibrium is not *ex-ante* efficient in the sense that if before discovering its type, the firm could make a binding commitment to always issue and invest it would be better off. In the Table, note that the original shareholders' expected payoff in the separating equilibrium is $0.1 \cdot 97.50 + 0.9 \cdot 60 = 63.75$. If, however, the manager could commit to always issuing, this payoff would be $0.1 \cdot 99.40 + 0.9 \cdot 61.20 = 65.02$. The reason why this payoff will always be higher is that if the new investors believe the firm's commitment to issue, and the firm does in fact issue regardless of type, than the firm will on-average sell equity which is properly priced. Since, in this case, the original shareholders always capture the full NPV of the project, they are better off than in the separating equilibrium, where the equity which is sold is always properly priced but where the shareholders forego the project NPV is passed up 10% of the time. Of course, the problem is that without a strong commitment mechanism the policy of always taking the project is time inconsistent: at time 0 the manager will always prefer to pass up the project if the firm turns out to be a high type even if investors believe that the firm will always issue. Although the equilibrium given in Table 1 is unique, it is important to note that with different parameter values multiple equilibria can sometimes obtain. This is demonstrated in the Table 2 example where all parameters are the same except for the priors which have now been changed from 10% high to 90% high. In this case two

[3] We have assumed in this analysis that the value of the project is equally well known by the firm manager and by investors, and have shown that under these conditions there will be "underinvestment". It is easily shown that if only the project value is asymmetric information then the firm will overinvest: firms with negative NPV projects will sometimes issue and invest to take advantage of overvalued shares. Myers and Majluf argue that this overinvestment problem should not be a problem since firms can always buy securities (a zero NPV investment) rather than taking a negative NPV investment.

Table 2: Separating and Pooling Equilibria

Model Parameters:			
	$H = 100$	$V = 30$	
	$L = 50$	$\%H = 0.9$	
	$I = 20$		

Belief Set A:		*Belief Set B:*	
Don't Take:	None	*Don't Take:*	Highs
Take:	Both (Pool)	*Take:*	Lows

Payoffs:		**RESPONSE**		*Payoffs:*		**RESPONSE**	
		Don't Take	Take			Don't Take	Take
	High	100	109.20		High	100	97.50
TYPE				**TYPE**			
	Low	50	67.20		Low	50	60

equilibria exist:[4] a pooling equilibrium illustrated in the left side of Table 2, where both types take the project; and a separating equilibrium, illustrated in the right side of Table 2, where the lows accept the project and the highs do not.[5]

Both sets of beliefs are confirmed in equilibrium; if the investors believe the highs will accept the project, they will, and vice-versa. Thus both equilibria are sequential.[6]

2.2 Money Burning

In this section we extend the firm's strategy space to allow it to commit to burn money. We show that if this signal is available to the firm, then both the separating and the pooling equilibria of the last section will be "broken" in that there will not exist beliefs satisfying the Cho and Kreps (1987) intuitive criterion which support either equilibrium. Specifically we

[4] Additionally, for these parameter values a mixed strategy equilibrium exists in which the high firm takes the project with a probability α of approximately 0.17, and the payoff to a high firm whether or not he invests is 100. However, this equilibrium is unstable in the sense that if the investors behaved as if α were slightly higher than 0.17, the high firm would move from being indifferent to wanting to always take the project, which should cause the investors to revise their probability of α even further upwards, and if investors behaved as if α were slightly lower than 0.17, high type firms would never want to take the project, which should cause investors to revise the belief about α even further downwards.

[5] Cadsby, Frank, and Maksimovic (1990) also note the existence of multiple equilibria. In their experiments, they demonstrate that markets tend to converge to the pooling rather than the separating equilibrium in these cases.

[6] Additionally, with the limited strategy space here, both equilibria survive the Cho and Kreps (1987) intuitive criterion. However the beliefs supporting the separating equilibrium are not part of a perfect sequential equilibrium (Grossman and Perry (1986)) if the manager's strategy space includes the possibility of somehow indicating that he is making an out-of-equilibrium move when he takes the project.

show that for some parameter values a single equilibrium whose supporting beliefs satisfy Cho-Kreps exists. In this equilibrium both firms issue equity and finance the project and the high valued firm commits to burn money to signal its type.

We note here that when we refer to money burning we do not mean that the firm actually burns currency. Rather, we use money burning to refer to an irreversible action on the part of the firm which either causes the future revenues to drop by a fixed amount (whether the firm is a high or low type), or commits the firm or its shareholders to pay out some fixed dollar amount after project revenues are realized. We show in Section 4 that the John and Williams dividend signal and the Choe, Masulis and Nanda project delay signal fall into this category.

2.2.1 Robustness of the Underinvestment Equilibrium

In this subsection we describe why the underinvestment equilibrium will be broken when firms are able to signal their types with a commitment to burn money. Consider again the example illustrated in Table 2. In the separating equilibrium, (on the right hand side of the table) an issuing firm (being revealed as a low type) must give up 25% of its firm to obtain equity financing for its project. If, however, a high valued firm could reveal its type, it would only have to issue 15.4% of its equity to obtain the $20 financing for the project. For the investor:

$$E(\mathit{Payoff}) = f' \cdot 130 = 20 \Longrightarrow f' = \frac{20}{130} = 15.4\%$$

Hence, by credibly signalling, a high type can reduce the percentage of the firm it offers by (25% - 15.4%) = 9.6%. This reduction has a value to the original shareholders of ($130 · 9.6%) = $12.48. But if a low firm were to mimic this signal, its gain would be only ($80 · 9.6%) = $7.68, because each share of the low firm is worth less. This asymmetric benefit means that by committing to burn only $7.68 a high type firm can credibly signal its type. Really, a high type must actually commit to burn sightly more than $7.68 because new investors will not demand $f' = \frac{20}{130}$ as shown above, but rather $f' = \frac{20}{130-C}$, where C is the signalling cost. In signalling, the firm is dissipating part of its $130 value. When this factor is taken into account, the payoffs to the firm depending on their type (either H or L) and on their action (denoted by a superscript B if they commit to burn money) are:

$$\begin{aligned} \pi_H^B &= \left(1 - \tfrac{I}{H+V-C}\right)(H+V-C) & \pi_L^B &= \left(1 - \tfrac{I}{H+V-C}\right)(L+V-C) \\ \pi_H &= \left(1 - \tfrac{I}{L+V}\right)(H+V) & \pi_L &= \left(1 - \tfrac{I}{L+V}\right)(L+V) \end{aligned} \tag{3}$$

In order for committing to burn money to be a signal, incentive compatibility must be satisfied, meaning that there must exist a C for which $\pi_H^B > \pi_h$ and $\pi_L^B \leq \pi_L$,[7] with the equality holding for the most efficient signal level. The value of C which makes this an equality is:

$$C = \frac{1}{2}\left((H+V) - \sqrt{(H+V)^2 - 4I(H-L)}\right) \tag{4}$$

Substituting C, which equals \$8.21 in our numerical example, into equation (3), gives the values for the right side of Table 3. Inspection reveals that for this C, $\pi_H^B > \pi_H$. Substituting equation (4) into equation (3) shows that this inequality holds for all parameter values. However, for some parameter values, $\pi_H^B < H$, implying that a high would prefer to pass up the project rather than signal.

2.2.2 Robustness of the Pooling Equilibrium

In this subsection we argue that the pooling equilibrium can never exist if firms have the opportunity to signal their types with a commitment to burn money. In the pooling equilibrium, the investors believe that both types take the project, and price the firm's equity accordingly. The question that we now ask is whether a high type firm can make an *out-of-equilibrium* move which will not be imitated by the low type and which gives it a higher payoff than what it receives in the pooling equilibrium. Consider again the example illustrated in Table 3. The high firm in this example can improve its value by \$0.28 by committing to burn \$0.52 to signal its type if investors consider this signal credible and value the firm accordingly. The signal should in fact be credible, since the low valued firm will not find it in its interest to mimic this money burning commitment. The signal level of \$0.52 is chosen because this is the level that just deters the low type from mimicking; the Table shows that the low type's payoff does not increase from \$67.20 if it signals.

[7] Incentive compatibility requires that the payoff to the low valued firm if he mimics the high be no lower than his payoff if he does not mimic.

Table 3: Nonexistence of Pooling and Project Choice-Separating Equilibria when the Firm Can Commit to Burn Money

Model Parameters:		
	$H = 100$	$V = 30$
	$L = 50$	$\%H = 0.9$
	$I = 20$	

Belief Set A:		*Belief Set B:*	
Don't Take:	(Doesn't Matter)	*Don't Take:*	(Doesn't Matter)
Take:	Both (Pool)	*Take:*	Lows
Take & Signal:	Highs	*Take & Signal:*	Highs

Payoffs:		*Signal Cost:* 0.52			*Payoffs:*		*Signal Cost:* 8.21		
		RESPONSE					**RESPONSE**		
		Don't Take	Take	Signal			Don't Take	Take	Signal
TYPE	High	100	109.20	109.48	**TYPE**	High	100	97.50	101.79
	Low	50	67.20	67.20		Low	50	60	60

Under Kreps and Wilson's (1982) original sequential equilibrium concept, the pooling equilibrium would still be viable. It would be supported under the out of equilibrium belief that both types of firms are equally likely to commit to burn money. Given that under these beliefs neither type will burn money, the beliefs cannot be considered entirely unreasonable. Cho and Kreps (1987) analysis, however, suggests that a more reasonable out-of-equilibrium belief in this case is that the signalling firm's value is high: since it is not in the low type's interest to send this signal even if the signal results in the investor believing that the signalling firm is high, the signalling firm must be high.

The intuition behind this criterion is that a high valued firm could send a message to the investors spelling out this argument, and if the investors are rational they should believe it. The intuitive criterion essentially requires that such a message can be sent.[8] When uniform

[8] The essence of the intuitive criterion is that the beliefs supporting a sequential equilibrium should not be considered reasonable if the following message can be sent:

> I am sending you the message that my type is $t_i \in B$ and you should believe me. For I would *never* send this message if I were in $T - B$ (where T is the set of all possible types), regardless of your inference as to who is making the out-of equilibrium move. However, if sending this message convinces you that my type is any of the types in B, you can see that it is in my interest to send it.

If *any* player can send this message then the equilibrium fails the Intuitive Criterion. The pooling equilibrium here clearly fails when the manager's strategy space includes the ability to commit to burn money. The intuitive cost signalling is possible, the pooling equilibrium does not survive the intuitive criteria, but the illustrated example in which the highs pay \$0.52 to signal their type is also not a sequential equilibrium: if the highs signal, then firms that do not signal are revealed to be lows. Although a low type is only willing to pay \$0.52 to be treated as a high rather than a pool member, he is willing to pay up to \$8.21 to move from being considered a low to being considered a high.

Hence, in equilibrium, a high must commit to burn at least \$8.21 to credibly reveal his type. No other sequential equilibrium satisfies the intuitive criterion.[9][10]

Also, note that although the sequential pooling equilibrium never survives the intuitive criterion if the firm can commit to burn money, the unique equilibrium which *does* survive need not involve committing to burn money. If, for example, V is reduced from 30 to 28, the "no-mimic" signal level of \$8.26 is higher than the NPV of the project, so the highs prefer to pass up the project. The only equilibrium surviving the intuitive criterion is then one in which the high firm chooses to forego the project.

The results of this section are summarized in the following proposition:

Proposition 1 *In a setting where firms have the ability to commit to burn money and beliefs are governed by the Cho and Kreps intuitive criterion,*

1. *A single equilibrium will exist in which the high type either commits to burn money, or passes up the project.*

2. *If, without money burning, a unique pooling equilibrium exists, then a high valued firm will commit to burn money, issue equity, and take the project.*

3. *If, without money burning, either a unique separating equilibrium or multiple equilibria exist, a high firm will commit to burn money, issue equity and take the project if* $(H + I)(V - I) - I(H - L) > 0$,[11] *and pass up the project otherwise.*

criterion also breaks the equilibrium in which the high firm chooses not to invest.

[9] Any equilibrium with a signalling cost higher than \$8.21 cannot survive because investors must believe that a signal of \$8.21 indicates a firm is high, and any equilibrium with a signalling cost lower than \$8.21, which must be a pooling equilibrium to be sequential, will fail because, based on the same belief, a high can and will signal his type by committing to burn \$8.21.

[10] Note that the money burning equilibrium is Pareto dominated by the sequential pooling equilibrium in this example. This means that the beliefs supporting the money burning equilibrium cannot be part of a perfect sequential equilibrium (Grossman and Perry (1986)). Also, see footnote 6.

[11] *This condition comes from the requirement that* C *in equation* (4) *be less than* $V - I$.

2.2.3 The Equilibrium when the Manager has Private Information about the Firm's Investment Opportunities

In order to conform to the literature, our analysis up to this point has assumed that the manager's private information concerns the value of the firm's asset's in place. However, Narayanan (1988) has suggested that managers are more likely to have private information about the value of the firm's investment opportunity than about the value of the firm's assets in place. In this setting, Narayanan shows that all firms with project's NPV is above a certain cutoff level will issue and invest.[12] Additionally, he shows that this cutoff level will be less than zero: in contrast to the Myers and Majluf equilibrium, this equilibrium will be characterized by "overinvestment," or firms investing in negative NPV projects.[13]

To see the intuition behind this result, consider a candidate equilibrium in which only firms with positive NPV projects issue and invest. In this setting, a firm which had a slightly negative NPV project would still wish to issue, because even though the firm loses on taking on the negative NPV project, the original shareholders in the firm benefit when the firm sells overvalued securities; the securities will be overvalued for this firm because the market price of the securities is based on the fact that, in the candidate equilibrium, only firms with positive NPV projects issue. So we see that the candidate equilibrium is not, in fact, an equilibrium, and that the equilibrium must be one in which some firms with negative NPV projects issue and invest. Similar reasoning reveals that, in equilibrium, the average project NPV of the issuing firm must positive.

A difficulty with Narayanan's analysis is its counterfactual implication that the firm's share price will rise on the announcement of an equity issue (provided there are some firms for which the project is sufficiently unprofitable that they pass it up).[14]

[12] Narayanan also shows that this result holds for either risky debt or equity

[13] Myers and Majluf challenge this conclusion on the basis that a firm can always invest any excess funds in securities (which are a zero NPV investment), and therefore would never need to take on negative NPV projects. They also show that under this assumption the share price reaction to an equity issue should always be negative.

[14] Actually, Narayanan shows that, given his assumptions the firm will not issue equity. However, were the firm constrained to issue equity, this would be the implication. We discuss these aspects of Narayanan's paper in Section 5.1

2.3 Generalization to a Model with a Continuum of Types

Having developed the intuition for our model by examining the case with two types, we now extend the model to allow for a continuum of types with values of their assets-in-place denoted by θ, where $\theta \in [\underline{\theta}, \overline{\theta}]$. We derive the equilibrium signalling schedule and show that this equilibrium uniquely survives the intuitive criterion. In the proposed equilibrium the amount that the firm commits to burn, C, is a monotonic function of θ. By inverting this function, investors can infer the value of the firm's assets-in-place as $\hat{\theta}(C)$. The firm solves the problem choosing c to maximize the value of the original shares:

$$\max_C \quad \underbrace{\left(1 - \frac{I}{(\hat{\theta}(C) + V - C)}\right)}_{\text{fraction of the firm retained by the original shareholders}} \quad \times \quad \underbrace{(\theta + V - C)}_{\text{total value of the firm}}$$

Assuming that $\hat{\theta}(C)$ is differentiable, the sufficient first order condition with respect to the signalling cost, after some manipulation, is:

$$(\hat{\theta}'(C) - 1)\frac{(\theta + V - C)I}{(\hat{\theta}(C) + V - C - I)(\hat{\theta}(C) + V - C)} = 1 \tag{5}$$

To obtain the equilibrium solution, we set $\theta = \hat{\theta}(C)$, which yields:

$$\frac{\hat{\theta}'(C) - 1}{\hat{\theta}(C) - C + V - I} = \frac{1}{I}$$

which can be solved to yield:

$$\hat{\theta}(C) = Ke^{\frac{C}{I}} + C + I - V \tag{6}$$

where K is a constant of integration. Because a firm with the lowest θ has no incentive to signal, we have the requirement that $\hat{\theta}(0) = \underline{\theta}$, implying that $K = \underline{\theta} + V - I$. Additionally note that for the highest type firms with $\theta > \theta_c = (\underline{\theta} + V - I)e^{\frac{V-I}{I}}$ the signalling cost C exceeds $V - I$, the NPV of the project. Hence, as in the Myers and Majluf model, these types pass up the project. Note that in this signalling equilibrium the cutoff value depends only on the lower bound of the type distribution $\underline{\theta}$, while in the Myers and Majluf equilibrium without signalling the cutoff depends on the actual shape of the type distribution.

Based on this development, we can state the following proposition:

Proposition 2 *Given that the value of a firm's assets-in-place, known by the firm's managers, is believed by investors to be distributed on $\theta \in [\underline{\theta}, \overline{\theta}]$ with a continuously increasing distribution function $F(\theta)$, the only sequential equilibrium in which each issuer is uniquely identified by his signal has the property that:*

1. *The firm takes the project and commits to burn C dollars if $\theta < \theta_c$, where*

$$\theta_c = (\underline{\theta} + V - I)e^{\frac{V-I}{I}}.$$

2. *C reveals the firm's θ and can be implicitly defined from the following signalling schedule:*

$$\hat{\theta}(C) = (\underline{\theta} + V - I)e^{\frac{C}{I}} + C + I - V. \tag{7}$$

This equilibrium uniquely satisfies the Cho-Kreps intuitive criterion.[15]

Proof: See Appendix.

Is is interesting to consider the efficiency of this signalling equilibrium relative to an equilibrium where signalling is prohibited. Without signalling a semi- separating equilibrium will usually obtain in which all types with assets-in-place worth less than a certain cutoff value will take the project, and all others will pass it up (*i.e.*, a Myers and Majluf type equilibrium). However, depending on the distribution of types, there may be multiple equilibria of this type, just as in the discrete type example illustrated in Table 2. Hence the relative efficiency of the signalling and non-signalling equilibria will depend on both the distribution of types and the particular non-signalling equilibrium chosen.

Additionally, we state without proof the following proposition for a Narayanan (1988) type setting where the manager has private information about the value of the project (V), but where θ is common knowledge.

Proposition 3 *Given that the the value of a firm's assets in place is common knowledge and that the value of a firm's investment opportunity, known by the firm's managers, is believed by investors to be distributed on $V \in [\underline{V}, \overline{V}]$ where $\underline{V} \leq I$, with a continuously increasing distribution function $F(V)$, the only sequential equilibrium in which each issuer is uniquely identified by his signal has the property that:*

1. *The firm takes the project and commits to burn C dollars if and only if $V \geq I$,*

2. *C reveals the firm's V and can be implicitly defined from the following signalling schedule:*

$$\hat{V}(C) = \theta e^{\frac{C}{I}} + C + I - \theta. \tag{8}$$

[15] *It is also possible to show that no equilibrium, including the one here, is a perfect sequential equilibrium (Grossman and Perry (1986)) if $F'(\underline{\theta}) < \infty$.*

This equilibrium also uniquely satisfies the Cho-Kreps intuitive criterion.[16]

Note that in this setting, the firm *always* issues and invests if the project NPV is greater than zero. Thus investment is perfectly efficient. The intuition for this result is as follows: First, since the equilibrium is fully revealing for all types which issue, it cannot be the case that a type with a negative NPV project would issue. Also, the lowest type which issues cannot be a firm with a positive NPV project, because then types with lower project values would mimic this firm.

What we conclude is that the overinvestment problem (when only project value is asymmetric information) is completely eliminated

through money burning, but that money burning cannot completely solve the underinvestment problem (when only the value of the assets-in-place is asymmetric information). Additionally, one can show that a signalling schedule will obtain in the case where both θ and V are asymmetric information, and that this signalling schedule is described by $\hat{u}(C) = \underline{u}e^{\frac{C}{I}} + C$ where $u = \theta + V$ is the firm value contingent on issuing and where $\underline{u}$ is the lowest u firm which will issue in equilibrium.[17] In this equilibrium, firms will only issue if the project is positive NPV and if the signalling cost C is less than the NPV of the project $(V - I)$.

2.4 Equity Financed Money Burning

In the preceding analysis the money that was burned as a signal came out of project revenues rather than from the equity issue. This distinction turns out to be critical. We now show that while an equilibrium in which money is burned from the proceeds of the equity issue is possible, such a signal is very inefficient. In such an equilibrium the high valued firm is indifferent between signalling and not, and the low valued firm is indifferent between mimicking and not. Why can't equity financed money burning be an efficient signal in this setting, while committing to burn money out of project revenues is? Burning money in the current period uses up resources and increases the amount of capital required by the firm to fund its investment and therefore the size of its equity issue. This larger equity issue increases the benefit to the low type because now, in mimicking, he can sell more overpriced stock, and as a result the high type

[16] *And again, it is possible to show that no equilibrium, including the one here, is a perfect sequential equilibrium if $F'(\underline{\theta}) < \infty$.*

[17] Given certain distributional assumptions, this will be the lowest valuation firm with a positive NPV project.

must burn still more money to satisfy incentive compatibility. As we show in the proposition below, this extra benefit to the low type exactly offsets the above mentioned benefit to the high type from signalling. As a result, the amount that must be burned to prevent mimicking is such that a firm gain nothing and is thus indifferent between signalling and not signalling. We state this formally in the following proposition:

Proposition 4 *In an equilibrium in which equity financed money burning is used as a signal, the costs and benefits of signalling are equal for both high and low types. Both firm types are therefore indifferent between signalling and not signalling. A strong preference for signalling requires that burned money come out of project revenues.*

Proof:

Again we can construct a payoff matrix for a two type model assuming a separating equilibrium. Here the subscript of either H or L indicates the firm type, and a superscript of U indicates that this is the payoff if the firm burns money raised through an equity issue:

$$\begin{aligned} \pi_H^U &= \left(1 - \tfrac{I+C}{H+V}\right)(H+V) \qquad & \pi_H &= \left(1 - \tfrac{I}{L+V}\right)(H+V) \\ \pi_L^U &= \left(1 - \tfrac{I+C}{H+V}\right)(L+V) \qquad & \pi_L &= \left(1 - \tfrac{I}{L+V}\right)(L+V) \end{aligned} \tag{9}$$

Again, incentive compatibility requires that $\pi_L^U \leq \pi_L$, and therefore that $\frac{I+C}{H+V} \geq \frac{I}{L+V}$. If C is large enough to satisfy this condition, the high firm's payoff from signalling is seen to be less than or equal to that from not signalling. The equilibrium signal level is therefore $C = I\left(\frac{H-L}{L+V}\right)$, and therefore $\pi_H^U = \pi_H$ and $\pi_L^U = \pi_L$. The high firm is indifferent between signalling and not and the low firm is indifferent between mimicking and not. $\|$

Note that the proposition also implies that slack cannot be burned as a signal: high and low types equally benefit from this action because it increases the size of the equity issue. Another implication is that no pooling equilibrium can be broken by an equity financed money burning signal. Also, note that this signal is functionally equivalent to simply increasing the fraction of the firm given to the new shareholders from $\frac{I}{L+V}$ to $\frac{I+C}{H+V}$: instead of burning the money, the managers give it to the new investor by underpricing the issue. Proposition 2 implies that underpricing will not be an efficient signal. However we shall show in section 4.1 that with a minor change in the assumptions we can obtain an equilibrium, similar to Welch (1989), in which firms with favorable information underprice.

3 Further Extensions of the Model Strategy Space

3.1 The Price Setting Signal

If the Myers and Majluf strategy space is enlarged to allow the manager to set the offering price of the issue, an equilibrium arises in which the high type firm sets an offering price above the firm's *ex-ante* market price, but equal to the full-information value of the firm. In the two type model, high price offers have some probability of being rejected, but low price offers never fail. In equilibrium, the probability of rejection of a high price offer, α, is just high enough to keep the low type firms from setting a high price. The investor is indifferent between accepting and rejecting the offer because the equity issue is a zero-NPV investment, so any probability of acceptance is an optimal strategy for him.

The price setting equilibrium presented here is essentially the same as the equilibrium presented in Giammarino and Lewis (1988). In the Giammarino and Lewis model, however, the firm proposes a price to an investment banker, who rejects the offer with a certain probability. In our model, the firm sets an issue price, and the *market* issue goes through (*i.e.,* is fully subscribed) with a certain probability. In addition to the model differences, our analysis is slightly different. Giammarino and Lewis present a set of mixed strategy equilibria, each characterized by the probability that a low type mimics. We consider only one member of this set, that for which the low type never mimics. This equilibrium dominates the others in the sense that all others can be eliminated by a modified intuitive criterion which we will discuss in Section 3.1.1. Additionally this no-mimic equilibrium Pareto dominates the other members of the set.

In Section 3.1.1 we discuss the price setting model with observable project values and show that price setting dominates committing to burn money as a signal. However, we show in Section 3.1.2 that if the project's cash flow is significantly larger for the high type firm, committing to burn money is the more efficient signal.

3.1.1 Price Setting when Project Value is Observable

Equation (10) gives the payoffs to a firm of type t (either H or L) given that it sets an offering price p (either *h*igh or *l*ow), and given that the investor believes that a firm which sets a high price is indeed a high type firm:

$$
\begin{aligned}
\pi_{t=H}^{p=h} &= H + \alpha(V - I) \qquad & \pi_L^h &= L + \alpha(V - I\tfrac{L+V}{H+V}) \\
\pi_H^l &= H + V - I\tfrac{H+V}{L+V} \qquad & \pi_L^l &= (L + V - I)
\end{aligned}
\tag{10}
$$

In equilibrium, α will be set so that $\pi_L^h = \pi_L^l$ and incentive compatibility is satisfied. This leads to the following equation for α:

$$\alpha = \frac{V - I}{V - I\frac{L+V}{H+V}} \tag{11}$$

Note that α is always between zero and one for $H > L$ and $V > I$. Additionally, substituting this definition for α into equation (10) reveals that $\pi_H^h > \pi_H^l$, and thus a mixed strategy equilibrium exists for all parameter values.

Unlike the money burning equilibrium, where for some parameter values the high type chooses to pass up the project, the high type in this equilibrium always signals and takes the project if he receives funding.

Table 4 shows the payoffs to the two types of firms for each possible action under two sets of investor beliefs. Notice that the payoffs in the columns labeled "Reject," "p = pool," and "p = low," are the same as given in Table 3. The left side of the table shows that if a firm can set a price for its equity, a sequential pooling equilibrium will not exist if beliefs must satisfy the modified intuitive criterion which is discussed below. This will be true for any parameter values, again provided the project's value is common knowledge. Comparing the payoffs here with the payoffs in Table 3 shows that the mixed strategy equilibrium Pareto dominates the money burning equilibrium.

As was discussed earlier, the viability of a sequential equilibrium is critically dependent on the admissibility of the investor's out-of-equilibrium beliefs. Section 2.2.2 showed that the out-of-equilibrium beliefs required to support a sequential pooling equilibrium do not satisfy the intuitive criterion when money burning is possible. However, the price setting signal will not break the pooling equilibrium if out-of-equilibrium beliefs are governed by the intuitive criterion.

For a signal to break an equilibrium under the standard intuitive criterion, the signal must pass a test: some type (the low, in this example) must never wish to send the signal *no matter what the investor's anticipated response to the signal,* provided the investor's response to the signal is optimal for *some* belief about who is signalling. In the preceding example, accepting the high price offer with certainty is an optimal response if the investor believes that only the high type would select the high price. However, the low type would have an incentive to also price high if this were the investor's response, so the pooling equilibrium cannot be broken with this signal.

It does seem reasonable that the pooling equilibrium should be eliminated by the the price-setting signal since the issuing firm strictly prefers the signalling equilibrium to the pooling equilibrium, and would therefore take actions, if it can, to insure that the separating beliefs prevail. However, to kill the pooling equilibrium with the price setting signal, we need a stronger refinement which eliminates certain possible best-responses by the investor.

To eliminate the pooling equilibrium with the price-setting signal, we propose a refinement we call *the modified intuitive criterion*, which requires that the price-setting signal be met by a specific mixed response by the investor. The modified intuitive criterion we propose limits the investor's best responses but not his beliefs. If the investor believes the firm making the out-of- equilibrium move has a high value, and as a result of this belief he is indifferent between a set of best responses, he must if possible choose a best-response which would make the low not want to mimic.[18]

In this specific model, if the investor makes the inference that it is the high type that is setting a high price, *any* rejection probability is a rational response, since the issue is properly priced and thus is a zero NPV investment.[19] However, a small rejection probability will not deter a low type from mimicking, and if the low type mimics, the investor will, on average, pay too much for the equity issue. The modified intuitive criterion therefore states that the

[18]The modified message analogous to that in footnote 8 is the following:

> I am sending you the out of equilibrium message m, indicating that my type is $t_i \in B$ and you should believe me. For I would never send this message if I were in $T - B$ regardless of your inference as to who is making the out of equilibrium move, assuming your response meets the condition specified below. However if sending this message convinces you that my type is any of the types in B, then you can see that it is in my interest to send it.
>
> The condition I place on your response is that if an inference that I was in B led you to be indifferent between a number of responses, but your utility would be lower were I in fact some type in $T - B$, then you must, if possible, choose a response which would make any member of $T - B$ not wish to send the message.

[19]If the investor believes that it is the low type *with any probability*, the only rational response is to always reject the issue.

investor *must* respond with a probability large enough to keep the low type from mimicking.

The price setting signal is somewhat less intuitive than the other signals considered here since it requires investors to reject issues at specific probabilities even though, within the equilibrium, they are indifferent between rejecting and accepting the issues. As a modeling convenience we assume that investors in a sense flip a coin in this equilibrium to decide whether to purchase an issue or not. Although this is clearly unrealistic, we think it approximates a more complex model where the issuing firm does not know investors' reservation price exactly. For example, the firm may not know the investors' full information set and may instead observe only some (common knowledge) distribution on the probability of the issue's success as a function of the issue price. In this setting, the high type firm would set a price high enough so that, even if the investors believe the firm is a high type, there is still some significant probability that the issue is rejected.[20] In this setting the firm is in essence specifying the desired probability of rejection by the price it sets. Therefore the standard intuitive criterion would suffice to eliminate other equilibria when the price setting signal is available. However, this model would be far less tractable and would add little additional insight to the problem. We therefore choose to model the investor's decision as a mixed strategy and consequently need the modified criterion.

To see why the price setting signal works, consider the costs and benefits it creates for each of the two types.[21] The benefit is asymmetric just as with committing to burn money since the firm gives up a fixed percentage of its cash flows in exchange for the required investment funds. If a high value firm signals its type, this percentage drops from $\frac{I}{L+V}$ to $\frac{I}{H+V}$. We have shown that a percentage drop is more valuable to a high value firm than to a low value firm. Of course, the high type firm only gets this benefit when the issue succeeds, which it does a fraction α of the time. The benefit from signalling is therefore:

$$\alpha\left(\frac{I}{L+V}-\frac{I}{H+V}\right)(\theta+V), \tag{12}$$

where θ is the firm's type (H or L). Recall that the cost of committing to burn money is the same for all types: the cost of burning a dollar of project revenues is a dollar whether you

[20] This argument, in a slightly different context, is in the Appendix of Hirshleifer and Titman (1990), and is similar to the intuition behind the model of Jegadeesh and Chowdhry (1994).

[21] Note that the absolute cost and benefit here are arbitrary and are defined to facilitate comparison with the money burning equilibrium. However the benefit minus the cost must equal $\pi_\theta^h - \pi_\theta^l$.

Table 4: Price Setting as a Signal

Model Parameters:		
	$H = 100$	$V = 30$
	$L = 50$	$\%H = 0.9$
	$I = 20$	

Price Setting:		*Price Setting Equilibrium:*	
Reject:	(Doesn 't Matter)	*Reject:*	(Doesn' t Matter)
High Price:	High	*High Price:*	High
Pool Price:	Both(Pool)	*Low Price:*	Low

Payoffs:				$\alpha = 0.972$
		RESPONSE		
		Reject	$p =$ high	$p =$ pool
TYPE	High	100	119.44	109.48
	Low	50	67.20	67.20

Payoffs:				$\alpha = 0.565$
		RESPONSE		
		Reject	$p =$ high	$p =$ low
TYPE	High	100	105.65	97.50
	Low	50	60	60

are a high or a low type. In the Giammarino and Lewis model though, the cost of signalling is asymmetric. With the benefit defined in equation (12), the cost of signalling for the low firm is $(1-\alpha)(V-I)$: the low firm, in mimicking, gives up the NPV of the project $(V-I)$ a fraction $(1-\alpha)$ of the time. For a high type though, the cost of signalling is lower since it has to sell undervalued equity and thus does not capture the full NPV of the project when it obtains financing without signalling. The cost of issuing undervalued equity to fund the project is $I\frac{H+V}{L+V}$, and the captured NPV is therefore $(V - I\frac{H+V}{L+V})$. This makes the high type's cost of signalling:

$$(1-\alpha)(V - I\frac{H+V}{L+V}) \tag{13}$$

Since $H > L > 0$ and $V > 0$, this is less than the low type's cost of signalling. Extending this analysis of costs and benefits, it is clear that the price setting signal is more efficient than the money burning signal, meaning that a high type's payoff will be higher if he sets a high price than if he burns money. This means that if both money burning and price setting are included in the strategy space, and if beliefs are required to satisfy the modified intuitive criterion, no equilibrium in which money is burned will exist.[22] Formally, we have the following proposition:

[22]We also note that the price setting equilibrium Pareto dominates an equilibrium in which money is burned, since the high type's payoff is higher and the low type's payoff is the same.

Proposition 5 *Given that firms can specify a price for their equity or can commit to burn money, and given that project NPV is known to investors, a sequential mixed strategy equilibrium always exists in which all types sell equity at the full information price, the low type's offer is always accepted by the investor and the high type's is accepted with probability $\alpha < 1$. This equilibrium uniquely survives the modified intuitive criterion.*

Proof: See Appendix

3.1.2 The Price Setting Signal when the Project Value is Unobservable

As is indicated, the above Proposition holds when the only asymmetric information concerns the value of the firm's assets-in-place. However, to the extent that the value of the firm's assets-in-place is better known by the firm's manager so should the NPV of the project. Also, it is reasonable to assume that the project NPV should be positively correlated with the value of the assets-in- place: to the extent that type is a proxy for management ability or for the general prospects of the firm, a higher type should have a higher value of both ongoing assets and new projects.

The following Proposition demonstrates that if the NPV of the high type's project is sufficiently high relative to the NPV of the low type's project, committing to burn money will dominate price setting as a signal. The intuition for this result is that if the high type has a more valuable project, the cost of foregoing this project is higher. If the cost is high enough, the high prefers to commit to burn money rather than run the risk not obtaining financing. This result is formally stated in the following proposition:

Proposition 6 *If the NPV of the high type's project is sufficiently greater than the NPV of the low type's, committing to burn money will be a more efficient signal than price setting, and the price setting equilibrium will fail to survive the intuitive criterion.*

Proof: See Appendix

Note that if the project NPV is inversely related to the value of the firm's assets-in-place, the result will be reversed: the price setting signal will be even more efficient, in the sense that the cost to the high type of achieving separation from the low type is lower if the project scaling signal is employed. However, for the reasons discussed above, our intuition suggests the correlation is more likely to be positive.

3.2 The Project Scaling Signal

Krasker (1986) extended the Myers and Majluf strategy space to allow the firm to scale back the size of the investment project and thus issue fewer shares. He showed that in this setting the amount of equity issued, or alternatively the amount by which the project is scaled back, serves as a signal of the value of a firm's assets-in-place. This section provides conditions under which the project scaling signal will be a more or less efficient signal than committing to burn money or price setting. As in Section 3.1, we begin by analyzing a model in which all project details are observable. With this restriction project scaling, like price setting, is a more efficient signal than committing to burn money because scaling the project decreases the size of the firm's equity issue and thus decreases the benefit to the low firm of mimicking. However if project value is related to the unknown value of the assets-in-place, project scaling will not be as efficient a signal.

In the following example, we assume that if the firm invests x fewer dollars (for a total investment of $(I - x)$), its payoff will be reduced by γx dollars, for a total cash flow from the project of $(V - \gamma x)$ dollars. Note that if γ is (less than, equal to, greater than) $\frac{V}{I}$, the project is (decreasing, constant, increasing) returns to scale. We again assume that everything is known except θ, the value of the firm's assets-in-place, which is known only to the managers.

The equilibrium payoffs to the high and low types, given a decrease in investment of x, are given in the following payoff matrix. The payoffs are calculated assuming that the investor believes that a firm which scales is a high type.

$$\begin{array}{ll} \pi^S_{t=H} = \left(1 - \frac{I-x}{H+V-\gamma x}\right)(H + V - \gamma x) & \pi^S_{t=L} = \left(1 - \frac{I-x}{H+V-\gamma x}\right)(L + V - \gamma x) \\ \pi_H = (1 - \frac{I}{L+V})(H + V) & \pi_L = (L + V - I) \end{array} \tag{14}$$

Subtracting π^S_L from π_L gives a benefit minus cost from signalling of:

$$I\left(1 - \frac{L + V - \gamma x}{H + V - \gamma x}\right) - x\left(\gamma - \frac{L + V - \gamma x}{H + V - \gamma x}\right) \tag{15}$$

In equilibrium, x is chosen so that this is equal to zero. Similarly, the benefit minus cost for the high type is:

$$I\left(\frac{H + V}{L + V} - 1\right) - x(\gamma - 1)$$

The high type's benefit minus cost from signalling is seen by comparison with equation (3) to be equal to the cost of committing to burn $x(\gamma-1)$ dollars.[23] However, for the low type the cost is considerably higher,[24] indicating that at least for small values of γ, project scaling is more efficient than committing to burn money. In fact, for $\gamma = 1$, when the marginal return on additional investment is zero the cost to the high type of reducing his investment is zero since decreasing the project size does not change the NPV of the project. The cost to the low is not zero, however, because as the project size is decreased, the low cannot sell as much overpriced equity.

If the project has constant returns to scale (*i.e.* $\gamma = \frac{V}{I}$), the scaling signal is similar to the price setting signal. The cost of the price setting signal is that the firm gives up the entire project NPV a certain fraction of the time in exchange for a higher price for its equity.[25] When project scaling is used as a signal, a fraction of the NPV of the project is given up all of the time. Although these signals appear to be equivalent, price setting in this case is slightly more efficient than project scaling.

The intuition for this is the following:[26] The high type's cost of signalling is lower if he can reduce the degree of asymmetric information about the firm, the value of which is composed of two parts: the cash flow from the assets-in-place, θ, which cannot be directly observed except by management, and the cash flow from the project, V, which is known. Scaling back the project reduces the portion of the firm's cash flow that investors are informed about, and therefore increases the asymmetry of information and increases signalling costs. While in the price setting equilibrium, the total firm value when the equity issue succeeds is $\theta + V$, in the scaling equilibrium the total firm value is $\theta + \alpha V$; a larger fraction of the firm is composed of the uncertain assets-in-place. Because of this more severe asymmetric information problem, the cost of signalling is higher, and the net payoff to the high firm is lower. As one would expect based on this intuition, the difference between the payoffs using the two signals approaches zero for $\theta \gg V$.

[23] From (3), $\pi_H^B - \pi_H = I(\frac{H+V}{L+V} - 1) - C$.

[24] This is because the fraction $\frac{L+V-\gamma x}{H+V-\gamma x}$ is always less than one.

[25] In Section 3.1 we show that a fraction $(1-\alpha)$ of the time, the high's equity issue fails, in which case the firm fails to capture the NPV of the project. However, when the issue succeeds, the high value firm sells equity at a price which is a factor $\frac{H+V}{L+V}$ higher.

[26] Our thanks to David Hirshleifer for suggesting this interpretation.

For large returns to scale, project scaling is a less efficient method of signalling firm value. Intuitively, this is because it is now more costly for the high type to reduce the size of the equity issue. However, as long as the project's value is known, project scaling is always more efficient than committing to burn money. As $\gamma \to \infty$, scaling the project by some infinitesimal amount reduces the cash flow from the project without appreciably decreasing the size of the equity issue, and hence, is equivalent to committing to burn money.

We can now state the following proposition. Note again that with any of the three signalling mechanisms in the strategy space, the equilibrium in which the most efficient signal is used will be the only one which will survive the modified intuitive criterion:

Proposition 7 *The following are characteristics of the project scaling equilibrium in the case where the project NPV is observable and where project scaling, price setting, and committing to burn money can be used as signals:*

1. *For a constant returns to scale project, project scaling is a less efficient signal than price setting, and the project scaling equilibrium does not survive the modified intuitive criterion*
2. *For $\gamma < \gamma^*$, project scaling dominates price setting as a signal, and the price setting equilibrium does not survive the intuitive criterion, where γ^* is defined by the system of equations:*
$$\gamma^* x = (V - I)\left(1 - \frac{(V-I)}{V - I\frac{L+V}{H+V}}\right)$$
$$I\left(1 - \frac{L+V-\gamma^* x}{H+V-\gamma^* x}\right) = x\left(\gamma^* - \frac{L+V-\gamma^* x}{H+V-\gamma^* x}\right)$$
3. *Project Scaling is always a more efficient signal than committing to burn money, and the money burning equilibrium never survives the intuitive criterion.*
4. *As $\gamma \to \infty$, the efficiency of project scaling approaches that of committing to burn money.*

Proof: See Appendix

3.3 An Optimal Combination of Signals

Until now we have made the assumption that a high firm would signal using only one of the three available signalling mechanisms. However, under certain conditions a combination of scaling, committing to burn money, and price setting will be the most efficient signal and hence the only signal observed in an equilibrium which satisfies the modified intuitive criterion.

As an example, when $H - L$ is large and the project's production function $V(I)$ is identical for the two types, and is everywhere concave with $V'(\infty) < 1$ and $V'(0) = \infty$, the optimal signal will be a combination of project scaling and price setting. The intuition for this is as follows: We have already shown that if the project's marginal return on investment is zero (*i.e.,* if $V'(I) = 1$), then a marginal scaling of the project imposes a cost of mimicking on a low type, and no signalling cost on a high. Because of this, a high type will always do some scaling. But, as we showed in Section 3.2, as investment decreases, so does the efficiency of the scaling signal. Therefore, after some amount of scaling, the marginal cost to a high of further scaling becomes higher than the cost of increasing the probability of having the issue rejected.[27] In this case, a high can minimize its signalling cost if it utilizes a combination of scaling and price setting as a signal.

Similarly, Appendix B shows that if the project's production function is different for high and low type firms, then a combination of two of the three, or of all three signals may be the most efficient way for a high valued firm to convey its type. Based on the derivation presented in Appendix B, we present the following proposition.

Proposition 8 *In an equilibrium in which beliefs are governed by the modified Cho-Kreps intuitive criterion, there exist production functions for which the equilibrium signal is either price setting, committing to burn money or project scaling, or a combination of either two or three of the signals.*

Ambarish, John and Williams (1987) is a special case of this analysis in that they do not allow the firm to set its issue price, but do allow it to vary its investment level and commit to burn money (by paying taxable dividends). Based on their assumptions, they find that for small $H - L$, a high valued firm will signal by scaling, but for large $H - L$, a combination of the two signals will be optimal. Their model assumes that the high type's production function is everywhere more concave than the low type's, so that the high type's marginal return on investment changes faster with a change in investment than the low type's. Therefore, though the scaling signal is always more efficient at the high's full information investment level, if the high firm decreases its investment level far enough, scaling becomes a less efficient signal than committing to burn money.[28]

[27] Where "marginal cost" is defined as the cost to the high of imposing an additional \$1 cost of mimicking on the low

[28] In Ambarish, John and Williams the present value of the investment opportunity for a type j firm, $F_j(I)$ is

4 Money Burning Models in the Finance Literature

As we stated in the introduction, a number of papers in the literature analyze signals that are either equivalent to burning money or committing to burn money. By "equivalent" we mean that the cost and benefit for each type is the same. In this section we discuss a number of these models. These include the underpricing models of Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989), which feature a signal that is equivalent to equity financed money burning, and the dividend signalling model of John and Williams (1985) and the project delay model of Choe, Masulis and Nanda (1993), which employ methods of committing to burn money.

4.1 IPO Underpricing Models

In equity financed money burning, firms must sell extra equity to get the money that they burn. If they were to take the extra money obtained from the equity issue and, instead of burning it, return it to the new shareholders, the effect would be the same. This latter action is equivalent to underpricing the issue.

The interpretation of equity financed money burning as underpricing yields insights into why equity financed money burning does not work as a signal. To satisfy incentive compatibility in a separating equilibrium, the high type would have to set a share price for his IPO which would be at or below the share value of the low type. However, this could not be an equilibrium because the high would always prefer pooling with the low, and receiving the average of the high and low share values.

In several recent papers, IPO underpricing does serve as a signal. However, these models either provide an additional benefit to the higher valued firms from signalling, or alternatively impose an additional cost on the low. In Welch (1989), for example, the project NPV is negative for the low type and it is assumed that to mimic, the low type must both issue equity

equal to $a_j + b_j G(I)$, where $G'(I) > 0$, $G''(I) < 0$, $G(O) = 0$, and $G'(0) = \infty$. They show that when $a_2 > a_1$ and $b_2 > b_1$ a type 2 firm will combine underinvestment with payment of a dividend as a signal of firm type. In their model, as the investment decreases, $b_j G'(I)$ (the marginal return on investment) increases, and project scaling becomes a less efficient signal. Also, because $b_2 > b_1$, scaling the project costs the high (type 2) firm more than it cost the low firm in revenues from the project. So below some investment level, project scaling is less efficient than burning money. Thus a high firm finds it optimal to signal its type through a combination of project scaling and money burning.Note that they also require that $\frac{a_2}{a_1} > \frac{b_2}{b_1}$. If this condition is not satisfied the high firm will find overinvestment a superior signal to underinvestment.

and take on the project. The low thus incurs a cost by taking on the project. If this cost outweighs the benefit from selling overpriced equity, the low will choose to not take the project so that the high type will not have to signal. However, if the cost to a low is not quite high enough to induce separation in this way, a high can underprice slightly, and reduce the benefit to mimicking just enough so that a low would not be willing to mimic. This rationale for underpricing an issue can easily be illustrated within the context of our model as we do in the following proposition:

Proposition 9 *Assume that firms cannot commit to burn money and cannot scale back their projects. Then, if the project has a negative NPV for the low type ($V_L < I$) and a positive NPV for the high types ($V_H > I$), with assets-in-place of $\theta_H > \theta_L > 0$:*

1. *If $V_L < I\left(\frac{\theta_L}{(\theta_H + V_H - I)}\right)$ a pure strategy equilibrium exists in which:*
   - *The high type issues equity at the market price, which is equal to the full information value, and takes the project.*
   - *The low type does not issue equity and does not take on the project.*
2. *If $\frac{\theta_L}{(\theta_H + V_H - I)} < V_L < V_H\left(\frac{\theta_H + V_H}{\theta_L + V_L}\right)$ an equilibrium exists in which:*
   - *The high type issues equity at less than the full information value (underprices) and takes the project.*
   - *The low type does not issue equity and does not take on the project.*
3. *If $V_L > V_H\left(\frac{\theta_H + V_H}{\theta_L + V_L}\right)$ an equilibrium exists in which:*[29]
   - *Neither high nor low type issues equity nor takes on the project.*

All of these are sequential equilibria, and satisfy the intuitive criterion if the firm is unable to commit to burn money. If the firm has the ability to commit to burn money, the second and third equilibria will not satisfy the intuitive criterion.

Proof: See Appendix

In the Welch model V_L is constrained to be zero, so an underpricing equilibrium cannot be obtained without a more complicated structure. In the Grinblatt and Hwang (1989) and Allen and Faulhaber (1989) models the project NPV is positive for both firm types, so these models also require somewhat more complicated structures to get underpricing results. However in all three models as in the above proposition, committing to burn money in the future, if feasible, would be a more efficient signal and would dominate underpricing.

[29] *We also note that, although this equilibrium satisfies the intuitive criterion, there always exists either a pooling or a mixed strategy equilibrium which Pareto dominates it.*

It is important to note that, in terms of maximizing social welfare, the underpricing signal may be optimal, in that it is a simple transfer while the project scaling, and price-setting signals both result in a mis-allocation of resources. Burning money may also be optimal in this sense if the manager transfers money to some party rather than wasting resources. However, keep in mind that the notion of efficiency we are employing here is concerned only with the old shareholders' welfare.[30]

4.2 Dividend Signalling Models

As we mentioned earlier, dividends in the John and Williams model are paid out of the equity issue. This seems to contradict Proposition 2, which shows that the money burned, or in this case the taxes paid on the dividends, must come out of project revenues. However, in the John and Williams model both the firm and the shareholder sell shares: the firm sells new equity to raise money for the investment opportunity and the original shareholders sell shares to fulfill their liquidity needs. When a dividend is paid to the original shareholders, their liquidity needs decrease by the amount of the dividend, and thus the net amount of equity sold by the firm and the original shareholders does not change. It should be noted that this assumes that the shareholder's tax liability is not incurred until after the firm's type is revealed. If the taxes were due immediately the shareholders would have to sell additional shares to raise the money necessary to pay the taxes on the dividend. This would mean that the total amount of equity sold would increase with the dividend, making the signal equivalent to equity financed money burning. As we showed earlier, dividends would not be an effective signal if that were the case. However, the taxes on the dividends in John and William's model are not required to be paid until after the project's revenues are realized. So the dividend tax is actually modeled as a commitment to burn money in the future, which as we have shown, is in fact an effective signal in this setting.[31]

[30] Perhaps, it is possible for the firm to ex ante sell the right to receive undervalued shares in the future. If they could do this, we might conclude that the underpricing signal is ex ante efficient as well as socially beneficial.

[31] In Ambarish, John, and Williams (1987), L denotes the after-tax demand for liquidity, and is an arbitrary function of the dividend D. AJW show that if $L_D = 0$, dividends cannot function as a signal of firm value, and the model becomes a Myers and Majluf like model in which high value firms underinvest.

4.3 A Model of Project Delay

A recent paper by Choe, Masulis and Nanda (1993) proposes a theory to explain why there are more equity issues in economic expansions than in contractions. A crucial feature of the model is that all firms have the option to delay investment. Delay is costly in the sense that it reduces the present value of the project revenues, but does not increase the investment required to fund the project. What Choe, Masulis and Nanda show is that although high value firms may choose to delay the project to a good period, low value firms will issue and invest when the project first becomes available.

A somewhat modified version of the Choe, Masulis and Nanda model is as follows: Assume that all details of the model are the same as presented in Table 1 in Section 2.1, except that now the firm can issue and invest in the project at either $t = 0$ or $t = 1$, and the cash flow from both the assets-in-place and the project are realized at $t = 2$. If the firm invests at $t = 0$, then $V = 30$ (and $NPV = 10$), as in Table 1, but if the firm delays and invests at $t = 1$, V falls to 21 (*i.e.*, the project NPV falls to \$1). It is easily shown that the only equilibrium whose beliefs satisfy the intuitive criterion is one in which the low valued firm issues and invests at $t = 0$ and the high valued firm issues and invests at $t = 1$.

This equilibrium can easily be understood in relation to the concept of burning money. Delaying the project in this model is precisely equivalent to committing to burn money in that it wastes future resources but costs nothing extra at the time of the equity issue. However, the way in which the delay affects the NPV of the project is crucial in the model. If instead of decreasing project revenues delaying the project increased the required investment and left the project's cash flows unchanged, the project delay would be equivalent to money burning in the current period (or underpricing), and delay would then not then be observed in equilibrium.

Additionally, it seems reasonable that delay should cost the high type more than it costs the low type; this will be the case if a high type's project is more valuable and the cost of delay is proportional to the value of the project. If there is a substantial cost differential, project delay will not function as a signal.

5 Debt and the Pecking Order Hypothesis

The literature that we have discussed so far assumes that the firm must issue equity. If the firm can issue debt, many of our earlier conclusions change significantly. Myers and Majluf consider the implications of adding the option of issuing debt to the firm's strategy space, and conclude that there exists a "pecking order" in the issuance of securities;[32] the securities whose payoffs depend least on the manager's private information should be issued, because by issuing these securities the manager minimizes the adverse selection problem. Therefore, when the manager has private information about the level of the cash flows from the project, the firm will always issue riskless debt if possible. However, a more likely scenario is one in which there is some uncertainty, even from the manager's perspective, as to what the firm's future cash flows will be, and probably enough uncertainty so that the firm cannot issue perfectly risk-free debt. In this scenario, as we show below, the firm will sometimes find it optimal to issue equity instead of risky debt.

This section is divided as follows: In subsection 5.1 we analyze the setting in which managers have information which investors do not about the mean of the project's cash flows distribution, and in subsection 5.2 we explore the possibility that managers have extra information about the variance of the project's cash flows. In subsection 5.3 we investigate the setting where the firm has outstanding risky debt. Brennan and Kraus (1988) shows that firms in this category can costlessly signal their type though their financing decisions. The final subsection discusses the relation between the Myers and Majluf pecking order hypothesis and the Modigliani and Miller capital structure irrelevance theorem.

5.1 Financing when the Project's Cash Flows are Uncertain

If a firm can issue riskless debt, the adverse selection problem disappears, and only firms with positive NPV projects issue riskless debt and invest. In the basic Myers and Majluf model where cash flows are certain, debt would be riskless and there would efficient investment. However, in a more realistic setting the firm manager will not know the firm's future cash flows with certainty but will know certain properties of the cash flow distribution better than investors. In this setting, there can be an adverse selection problem whether the firm issues

[32] See also Myers (1984)

equity or risky debt.

Narayanan (1988) suggests that in a setting where the manager has private information about the mean of the distribution of the cash flows from the firm's investment opportunity but where the variance of this distribution is common knowledge, the Myers pecking order will still apply in the sense that the firm will always issue risky debt rather than equity. Narayanan demonstrates his result by comparing a pooling equilibrium in which firms only issue risky debt to one in which firms only issue equity. In the debt equilibrium, security values are less affected by the manager's private information, so the higher value firms lose less due to adverse selection. Narayanan concludes from this that only debt would be issued in this setting.[33]

Noe (1988) extends Narayanan's model to a three type example, and shows that an equilibrium may obtain in which low (L) and high (H) value firms issue debt and medium (M) value firms issue equity. In this example, type L issues debt because there is a substantial probability that it will default, and since the debt is also issued by a type H firm, the yield on the debt is sufficiently low to make the type L prefer debt to equity. The type M firm is less likely to default on the debt, so the yield is too high from its perspective so it therefore prefers to issue fairly priced equity. For the type H firm, the debt yield is also too high, but the equity would be even more mispriced since the cost of pooling with the type M firm would be still higher.

5.2 Financing When Managers Have Private Information About Cash Flow Variance

Both Noe and Narayanan assume that the mean of the distribution of project's cash flows is symmetric information, but that the variance of the distribution is common knowledge. However as Giammarino and Neave (1982) have shown, if manager's have private information about only cash flow variance then the pecking order will be reversed: firms will never issue debt.

Table 5 provides a numerical example that illustrates this concept. The firm has an invest-

[33]To be somewhat more rigorous about Narayanan's argument, note that a sequential equilibrium may in fact exist in which only equity is issued: this equilibrium would have to be supported by the belief that a firm which issued debt was low valuation. Given this belief, an investor's response to the out-of-equilibrium move of issuing debt would be to pay only a low price for the debt, and based on this firms would therefore always issue equity. While this equilibrium would survive the Cho-Kreps refinement, it would not survive be a perfect sequential equilibrium (Grossman and Perry (1986)).

ment opportunity which requires investment of 30 and has an (expected) NPV of 10. There are two equally probable states of nature, U and D, in which the cash flows from the project take on the values specified in Table 5. It is seen that the value of the assets in place as well as the expected value of the project's cash flows are the same for the two types. The only thing that now distinguishes high and low types is the variance of the project's cash flow: for type H's project cash flow variance is 100, and type L's is 1600. The probability that a firm is type H is 0.5.

It is straightforward to show that if the firm were to issue equity there would be no adverse selection problem because the full-information value of a share of a type L's equity would be the same as a type H's (given our assumption of risk neutrality). Therefore, the original shareholders of both types would issue equity at the full-information value and therefore capture the full NPV of the project, and the value of the original shares would be $V_{\text{old}} = 20$.

However, let's consider the situation where the firm chooses to issue debt. Here the full-information value of a type L bond would be lower because a type L would default more often. In this example, if the type were observable, a type L would have to incur an obligation of 50 to raise the 30 necessary to fund the project: this is because 50% of the time the low type would default on the bonds and the bondholders would only receive 10, giving the bonds a total value of $(0.5 \cdot 10 + 0.5 \cdot 50) = 30$.

In the setting where the investor does not know the firm's type there exists a pooling equilibrium (supported by Belief Set A) in which the unknown type firm would have to incur a total debt obligation of 36.67 to raise the necessary \$30. To see that this is the equilibrium obligation, note that in the pooling equilibrium, if state D occurs a type L firm will default and the debtholders will receive only 10, otherwise a type L pays the full obligation. Since a type H never defaults, the value of the bonds is $(0.25 \cdot 10 + 0.75 \cdot 36.67) = 30$.

The right hand side of Table 5 shows that there also exist investor beliefs which support a separating equilibrium in which a type H firm does not issue. However, note that the payoff to the type H firm in both of the pooling and separating equilibria is lower than it would be if the firm issued debt. Thus, we can argue that, under reasonable restrictions on out-of-equilibrium beliefs, the type H firm would always issue equity: debt will be the dominated security.

Table 5: Debt-Issuance Equilibria

Model Parameters:	
$H = L = 10$	$V_H^u = 50, V_H^d = 30$
$I = 30$	$V_L^u = 80, V_L^d = 0$
$E(V_H) = E(V_L) = 50$	$\%H = 0.5, \pi_u = \pi_d = 0.5$

Belief Set A:		*Belief Set B:*	
Don't Take:	None	*Don't Take:*	Highs
Take:	Both(Pool)	*Take:*	Lows

Payoffs:		**RESPONSE**		*Payoffs:*		**RESPONSE**	
		Don't Take	Take			Don't Take	Take
TYPE	High	10	13.33	**TYPE**	High	10	5
	Low	10	26.67		Low	10	10

5.3 Signalling by Choice of Financing

Brennan and Kraus (1987) examine the case where the firm has existing senior debt and the project's cash flows are uncertain. In this model in which firms can sometimes costlessly signal their type by appropriate choice of financing.[34] They show that when the firms' managers have private information about the mean of their projects' cash flow distributions and the variances are common knowledge, high value firms can signal their type by repurchasing outstanding risky debt. Additionally, they show that when the manager has private information about the variance (and the mean is common knowledge), high value (meaning high variance) firms will issue subordinated debt, and low value firms will issue equity.[35]

The intuition for Brennan and Kraus' first result is as follows: the low type's debt is less valuable because the low is more likely to default. Therefore a high can signal its type by repurchasing some of its outstanding debt at its full information value: this action is costless for the high type, but is costly for the low type. If the cost differential is large enough, it outweighs the gain from mimicking and a separating equilibrium will obtain.

[34] Constantinides and Grundy (1989) present a model with many of these same features.

[35] Heinkel and Zechner (1990) combine elements of Narayanan (1988) , Brennan and Kraus (1987), and Myers (1977) into a model in which in which firms have an opportunity to take on a risky project. As Narayanan shows, firms in this setting will overinvest in the project. However, in the Heinkel and Zechner model firms have an opportunity to issue risky debt before the firm's manager's know the project value (before they gain their information advantage). The reason they have an incentive to issue risky debt is in order to set up a Brennan and Kraus situation: with risky debt in place the firm can costlessly signal its type by repurchasing risky debt, and thus can avoid the over/underinvestment problem.

Table 6: Brennan and Kraus Equilibria

Costless Signalling with Outstanding Debt

Model Parameters:				
$H = L = 10$		$V_H^u = 50,\ V_H^d = 0$		
$I = 5,\ D = 10$		$V_L^u = 40, V_L^d = -10$		
$E(V_H) = 25;\ E(V_L) = 15$		$\pi_U = \pi_D = 0.5$		
Belief Set:				
Forego:		None		
Take:		Low		
Take/Repurchase Debt:		High		
Payoffs:		**RESPONSE**		
		Forego	Take	Repurchase
TYPE	High	0	18.75	20
	Low	0	15	14.29

Costless Signalling with Junior Debt

Model Parameters:				
$H = L = 10$		$V_H^u = 25,\ V_H^d = -5$		
$I = 5,\ D = 10$		$V_L^u = 10, V_L^d = 10$		
$E(V_H) = E(V_L) = 10$		$\pi_u = \pi_d = 0.5$		
Belief Set:				
Forego:		None		
Take/Issue Equity:		Low		
Take/Issue Junior Debt:		High		
Payoffs:		**RESPONSE**		
		Forego	Equity	Debt
TYPE	High	0	6.25	7.5
	Low	0	5	0

The left side of table 6 gives a two-type example of such a separating equilibrium. In this example types H and L both have assets-in-place worth 10 and outstanding debt with a face value (D) of 10. Each type has an opportunity to take on a project which requires an investment of 5. The payoff to the project (V_θ^s) is dependent on both the firm type ($\theta \in \{H, L\}$) and on the state of nature ($s \in \{u, d\}$). Type H's project has a NPV which is \$10 greater than that of the type L firm, but the variance of the project cash flow is the same for the either type. In this equilibrium, the type H firm sells 42.8% of the firm's equity for \$15; it uses \$10 to repurchase debt at face value and \$5 to fund the new project. The type H loses nothing by repurchasing the debt at face value since the face value is equal to the full information value, but a type L would lose \$5 by repurchasing debt (because the full information value of L's debt is only \$5). Since the cost of mimicking is less than the benefit, a type L does not mimic, and instead sells 25% of the firm's equity for \$5. The payoff diagram at the bottom of the table shows that the incentive compatibility conditions are satisfied.[36]

Brennan and Kraus' second result concerns a setting where the different types have the same expected cash flows, but where type H's cash flow variance is higher. In this setting, a type H wants to show that it is risky before it undertakes an equity issue, because the riskier

[36]We note that when the type distribution is continuous, a fully revealing equilibrium will obtain in which higher value firms will repurchase more debt, and all debt will be repurchased at the full-information value (*i.e.*, at less than the face value).

it is, the lower the value of the outstanding debt and hence the greater the value of its equity. Here, the type H firm can signal its type by issuing subordinated debt. Since the type H firm is characterized by higher cash flow variance and hence a higher probability of default (and lower price) on new subordinated debt, the type H can issue debt at this low price, and the low will be unwilling to mimic.

A simple numerical example of this equilibrium with two types is provided as the right side of Table 6. Here, the firm has assets in place with a value of 10, outstanding senior debt with a face value of 10, and a project with a required investment of 5 and an expected cash flow of 10. However if the firm is type H the project pays a *certain* 10, while if it is type L the project will pay either -5 or 25 (0.5/0.5 probability). In equilibrium, type H issues junior debt with a face value of 10, which sell at a price of 5. The only way that a type L can obtain financing in this setting is to sell undervalued subordinated debt, or to issue equity at the full information value, which it does.[37]

5.4 The Pecking Order Hypothesis and Capital Structure Irrelevance

Up to now, we have assumed that managers act to maximize the value of the firm's shares. Assuming that managers maximize share prices is equivalent to assuming that they act in the interests of "passive" shareholders, who do not update their portfolios in response to the firm's investment and financing decisions. While this type of behavior seems plausible, the passive shareholders are clearly not optimizing their portfolios. In this section we examine the implications of having managers act in the interests of "active" shareholders who do optimize.

To begin this consideration it is instructive to compare the Myers and Majluf pecking order hypothesis with the original Modigliani and Miller irrelevance proposition. Like Modigliani and Miller, Myers and Majluf assume that there are no taxes and no transaction costs, yet the general conclusions of the two papers are very different: Modigliani and Miller conclude that the firm's capital structure choice is irrelevant while Myers and Majluf conclude that firms will prefer to issue debt.

To understand the difference between the two models, recall the well-known proof of the

[37] For a continuum of types, this model extends to an equilibrium in which firms issue either a combination of more subordinated debt and less equity (higher types issuing more subordinated debt and less equity) or, as Brennan and Kraus suggest, convertible subordinated debt with a higher face value and a lower conversion ratio.

Modigliani and Miller theorem which demonstrates the equivalence of "corporate" leverage (the firm borrowing on its own account) and "homemade" leverage (the investors borrowing on their personal accounts). With perfect markets, shareholders are unaffected by increases or decreases in the debt ratio of a firm since they can "undo" the leverage change in their personal portfolios, keeping both their fractional holdings of the firm's assets and their net debt level unchanged.

In contrast to the Modigliani and Miller framework, the Myers and Majluf pecking order hypothesis considers the capital structure choice from the perspective of "passive" shareholders who do not rebalance their portfolios when the firm issues new securities. When the firm issues equity to finance investment, the passive shareholders' fractional holdings of the firm's assets decrease, causing them to suffer losses (from dilution) if the shares are undervalued. However, if the firm instead issues riskless debt, the shareholders' fractional holdings remain constant so that their interests are not diluted. Based on this logic, Myers and Majluf conclude that under these circumstances firms should undertake all positive NPV projects and should always use debt to finance them.

Myers and Majluf recognize that in theory, with frictionless markets, shareholders should be "active" in the Modigliani and Miller sense: they should keep the risk of their portfolio constant and respond to a debt issue by selling equity and purchasing some of the newly issued debt. Such active shareholders' post-issue fractional holdings of the firm's assets would be independent of the financing choice: the shareholders would respond to a debt financed investment by selling shares and adding the risk-free asset to their portfolios, and as a result their fractional holdings of the firm's equity will not be a function of whether the firm uses debt rather than equity to fund the project. Managers acting in the interest of these shareholders might therefore pass up positive NPV debt financed investments, since doing so could lead the shareholders to sell undervalued shares.

Although Myers and Majluf consider the possibility that shareholders might be active with respect to capital structure changes they ignore the possibility that shareholders might also be active with respect to equity issues. They assume, as indeed we have throughout the chapter, that the original investors do not allocate additional funds to the firm when it undertakes a new investment that expands its capital base. This assumption is not consistent with most equilibrium models which suggest that the original investors will in fact purchase some of the new equity. For example, if the model were embedded into the capital asset pricing model (CAPM), each investor would purchase a *pro-rata* share of any new debt *or* equity issue, thereby eliminating the adverse selection costs associated with issuing underpriced shares to finance a new project. The implication of this is that a manager who wished to maximize the original shareholders' wealth would invest whenever the firm has a positive NPV investment opportunity, regardless of the firm's current share price, even if the investment required equity financing.

The question that arises at this point is which assumption is most reasonable? Do corporate managers act to maximize wealth for the passive shareholders, do they maximize wealth for what Myers and Majluf call the active shareholders who undo capital structure changes but do not add shares when the firm issues new equity, or do they maximize wealth for the even more active shareholders who buy the new equity on a *pro-rata* basis? The debate on this issue is quite important since it has implications for all of the signalling models discussed in this chapter. If managers act in the interests of shareholders that purchase new equity issues on a *pro-rata* basis, the incentive to take any of the costly actions described in this chapter for the purpose of increasing stock prices is eliminated.

Since maximizing the wealth of the passive shareholders is equivalent to maximizing the firm's share price one might argue that this is the most plausible objective function. Of course, the disadvantage with having managers act to maximize share price is that it either leads to underinvestment or dissipative signalling costs. Ex ante, shareholders would like their managers to have the incentive to accept all positive NPV projects that occur in the future since such a policy maximizes the current value of the firm. To give the managers this incentive, they should be compensated to act in the interests of the most active shareholders who purchase new equity issues on a *pro-rata* basis, as pointed out by Dybvig and Zender (1991).

As we have seen, compensating them in this way would, in theory, be straightforward: managers will take all positive NPV projects as long as they are paid a fixed salary and are required to keep their percentage ownership of the firm constant. However, in reality, managers have other considerations that might make a solution to the underinvestment problem more complex. Consider the situation where the perceived market value of a firm influences its full information intrinsic value. For example, Apple computer might find that its customers, who may be concerned about the future viability of the Macintosh operating system, will view the firm's products more favorably if Apple's stock price is higher.[38] As a result, there will be a tendency to take costly actions to signal favorable information even when managers are compensated in a way that makes them act in the interests of the most active shareholders. While it is still possible, in theory, to come up with a compensation package which induces the manager to take on all positive NPV projects, this compensation package would be very complex and may not be implementable in reality.

5.4.1 The effect of a manager's shareholdings on the investment decision

We believe that future research on the topics considered in this chapter will take management incentive issues much more seriously. Rather than starting with the assumption that managers either maximize share prices or maximize the expected wealth of active shareholders, future work is likely to determine these incentives from more basic principles. A key issue that will have to be resolved in this work relates to the kinds of constraints that are placed on managers in regards to how they trade in their firm's shares. As a first step along these lines it makes sense to think about how an unconstrained owner/manager's portfolio choice interacts with project selection choices in a Myers and Majluf setting.

As we mentioned above, if the manager does not participate in equity issues, then there will be an underinvestment problem. However, if the manager is forced to buy additional stock in proportion to his original holdings, then there will be no underinvestment problem. The question that we wish to address in this section is what happens when managers are not constrained in the amount of their firm's stock they buy or sell, either before or after the investment is taken. In this case, as we argue below, the Myers and Majluf underinvestment problem may be eliminated.

To understand this, consider a firm that owns property on which a gold mine was recently discovered. Only the firm's manager knows about this gold mine, and as a result the firm's shares are undervalued (from his perspective) which means that he has an incentive to purchase shares for his own account. A risk-neutral manager would buy as many shares as possible, up

[38] See for example, Titman (1984) and chapter * in this volume.

to the point where he is financially constrained. At this point, for the risk-neutral manager, the marginal value of a share still exceeds the market price because the marginal value of the a shares to him is not affected by the size of his holdings.

However if the manager is risk-averse, this will not be true. Now, since each additional share the manager acquires increases the covariance between the returns of his portfolio and the firm's stock, the stock's marginal value to him decreases as he accumulates more shares. In the absence of financing constraints, the risk-averse manager will continue acquiring shares only up to the point where the marginal value to him of an additional share falls to the market price of a share of the firm's stock.[39] At this point, though the share price is equal to the a share's marginal value for the manager, other investors would still find them underpriced if they had the manager's information.

Consider now what happens when the manager has the opportunity to issue additional shares at the prevailing price to fund a risk-free positive NPV project. To the constrained, risk-neutral manager, the firm's stock is undervalued, so from his perspective when the firm issues shares it is giving away a positive NPV project to the new shareholders. For this reason the risk neutral manager might pass up a project that requires an equity issue.

However from the risk-averse manager's perspective, the shares that would be issued to fund the project would be priced at their fair value; they are a zero NPV investment. In contrast, the NPV of the project is positive. Therefore it would be in the manager's interest to sell some of his shares in the firm and use the proceeds to purchase shares in the new project. This is what the firm, in essence, does for him when it sells equity to fund the new project. What this means is that as long as the manager has optimized his portfolio, he will make appropriate choices with regards to the selection of risk-free projects.

When the project is risky the analysis becomes somewhat more complicated. If the returns of the project can be spanned by the returns of other traded securities the answer is the same. In essence, a positive NPV project that can be spanned by existing securities is equivalent to a risk-free positive NPV project since all of its risk can be effectively hedged. However, if the project contains risks that are specific to the firm's stock, then in taking on the project the

[39] Note that we are assuming here both that the manager is risk averse and that markets are not complete. Both these conditions are necessary to obtain an interior optimum.

manager would be increasing the riskiness of his own portfolio.[40] He would therefore pass up projects that (from the outside shareholders' perspective) have positive NPVs. An example of this would be a project that simply increased the firm's scale of operations. This would be a positive NPV project if it yielded, for example, 15% when its cost of capital (from the perspective of outside shareholders) was 12%. However, if the unknown gold mine implied that the shares would have an expected return of 18%, then the manager would prefer to pass up the project. Since most projects do have some firm specific component, one must conclude that in general there will be some tendency to underinvest.

On the other hand, if the project cash flows are negatively correlated with the firm specific cash flows, the manager might take on the project even if it has a negative NPV. Taking on such a project lowers the portfolio's expected return but also diversifies the portfolio. For these projects there could be a tendency to overinvestment.

All of this suggests that a manager who is unconstrained in his portfolio choice is likely to have a tendency, unless compensated otherwise, to overinvest in diversifying projects and underinvest in projects in the core business of the firm.

6 Conclusion

In this chapter we examined the incentives of firms to signal their values prior to making a new equity offering. By analyzing this issue within a simple framework that encompasses a number of models in the literature, we were able to judge the relative efficiency of various signals that have been proposed.

Although we believe that the signalling literature examined in this chapter offers valuable insights, our analysis suggests that in a number of respects the models are not robust. For example, a number of signals appear to be relatively inefficient, so that their use requires that more efficient signals be unavailable to the firm. Additional required restrictions relate to the type of securities the firms can issue and the way in which management is compensated.

The limited robustness of these signalling models suggests that additional research is warranted. It is important to understand, for example, whether or not plausible conditions exist

[40] What we mean by firm specific cash flows is that component of the cash flows out of the firm's assets-in-place which cannot be hedged by buying and selling other assets in the economy.

under which managers will be compensated in such a way that they will have the incentives described in Myers and Majluf and the related literature. Perhaps, compensation contracts with this feature arises endogenously as a result of other agency problems that may exist between managers and shareholders.

A second area of future research relates to motivating why firms issue equity in situations where debt financing greatly reduces the adverse selection problem. Presumably, other debt related costs (*e.g.* bankruptcy costs) can preclude the use of debt financing and force the firm to issue equity. In such a setting we would expect firms to issue securities that minimize both adverse selection costs as well as expected bankruptcy costs. This might explain, for example, the use of preferred stock, which receives a fixed dividend but which cannot trigger default. Signalling efficiency could be especially important in settings where bankruptcy is costly. Perhaps firms that can signal their values very efficiently will prefer raising capital with equity issues, while those firms that find signalling very costly will prefer the debt markets, running a greater risk of bankruptcy. We expect that in the future, these information issues will play a larger role in the literature on the determination of optimal capital structures.

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Appendix A - Proofs of Propositions

A. Proof of Proposition 2: The proof that this equilibrium is the unique sequential equilibrium in which each issuer is uniquely identified by his signal level is partly taken from Riley (1979). Riley shows that if six assumptions are met, then the only informationally consistent signalling schedule must be a solution to the differential equation (5).

Riley shows that the only admissible solutions are those for which $K \leq \underline{\theta} + V - I$. That K must be equal to $\underline{\theta} + V - I$ comes from the fact that the equilibrium must be sequential. If the equality did not hold the lowest type ($\underline{\theta}$) would have to signal to receive a fair price for his equity. Clearly this equilibrium is not sequential, because the investor *cannot* pay less than the value of the lowest type for shares of a firm which does not signal.

To prove that the equilibrium satisfies the intuitive criterion requires that we consider two cases:

1. All types signal and take the project. This is the case if $\overline{\theta} < \theta_c = (\underline{\theta} + V - I) \exp \frac{VI}{I}$.
2. All types $\theta < \theta_c$ take the project and signal according to the schedule in equation (7). All types for which $\theta_c < \theta < \overline{\theta}$ do not issue and pass up the project.

In case 1, the highest type burns some amount as a signal, say $\overline{C}$. No type is willing to burn more than this because, under the sequential equilibrium criterion, the highest value the investor could place on any firm which did this would be $\overline{\theta}$. Since they can achieve the same valuation by burning only $\overline{C}$, no type wishes to make this out-of- equilibrium move. The only other out-of-equilibrium move available to firms is to pass up the project, which we have already shown is not maximizing behavior in this case.

In case 2, the highest signal level is burning $\overline{C} = V - I$, in other words burning up the entire NPV of the project. Clearly no firm would be willing to make this out-of-equilibrium move if they are not willing to burn $\overline{C}$. Thus in both cases, the equilibrium satisfies the intuitive criterion. $\|$

B. Proof of Proposition 4:

Given that $V > I > 0$, $H > L > 0$, and that the signalling cost in the money burning equilibrium is always positive ($C > 0$), the following relationship must hold:

$$\frac{V - I}{V - I\frac{L+V}{H+V}} < 1 < \frac{H + V}{H + V - C}$$

Multiplying each side by $\frac{I(H-L)}{H+V}$, and noting that the left side of the above inequality is equal to α (by equation (11)) gives the following:

$$\alpha\frac{I(H - L)}{H + V} < \frac{I(H - L)}{H + V - C}$$

The left side of this inequality is equal to $(V - I)(1 - \alpha)$, using the definition of α given in equation (11). Multiplying each side by -1 and adding $(H - L)$ to each side gives the following:

$$(H - L) - (V - I)(1 - \alpha) > (H - L)\left(1 - \frac{I}{H + V - C}\right)$$

Equations (9) and (10) reveal that the left side of this inequality is equal to $\pi_H^h - \pi_L^h$, the difference between the high and low type's payoffs in the price setting equilibrium if the low mimics, and that the right side of the inequality is equal to $\pi_H^B - \pi_L^B$, the difference between the high and low types payoffs in the money burning equilibrium if the low mimics. By the fact that the incentive compatibility condition is binding in equilibrium, we have that $\pi_L^h = \pi_L^B = L + V - I$ (the low's payoff must be the same whether he mimics or not), and therefore $\pi_H^h > \pi_H^B$: the payoff to the high type in the price setting equilibrium is higher than in the money burning equilibrium. Therefore a high type firm would always want to overprice and, under the modified intuitive criterion, the investor would have to respond with a sufficiently high probability of rejection so that a low type would never mimic. The money burning equilibrium, as well as the Myers and Majluf pooling and separating equilibria, therefore fail the modified intuitive criterion. Note also that, since the payoff to the low type in each of the two equilibria is the same, the price setting equilibrium Pareto dominates the money burning equilibrium. ||

C. Proof of Proposition 5:

Let V_H and V_L denote the cash flow from the project for the high and low types, respectively. Appropriate modification of the payoffs given in equation (10) and application of the incentive compatibility condition gives the following equation for the fraction of the time the issue must be accepted in equilibrium, which we will denote α^*:

$$\alpha^* = \frac{V_L - I}{V_L - I\frac{L+V_L}{H+V_H}} \tag{16}$$

In the limit as $V_H \to \infty$, $\alpha^* \to \frac{V_L - I}{V_L}$, and the payoff to the high approaches $\frac{V_L I}{V_L} V_H$.

In the money burning equilibrium equation (4), which defines the amount of money that must be burned, becomes:

$$C^* = \frac{1}{2}\left((H + V_H) - \sqrt{(H + V_H)^2 - 4I(H - L + V_H - V_L)}\right)$$

so that as $V_H \to \infty$, $C^* \to I$. Thus, as $V_H \to \infty$, the payoff to the high type approaches V_H, and for large enough V_H *the payoff is higher in the burning money equilibrium than in the price setting equilibrium.* It follows then that in an price setting equilibrium, the high could increase his utility by burning money rather than price setting. The low would not wish to mimic, no matter what the investor's response. Therefore the investor would accept the issue all the time, and the price setting equilibrium fails the intuitive criterion. To show that price setting is not a credible signal for high enough V_H, note that the payoff to the high type if he sets a low issue price is:

$$\left(1 - \frac{I}{L + V_L}\right)(H + V_H)$$

In the limit as $V_H \to \infty$, this is equal to:

$$\left(\frac{L + V_L - I}{L + V_L}\right) V_H$$

If he sets a high issue price, his payoff is $H + \alpha^*(V_H - I)$ which, using equation (16) above, is equal to

$$\left(\frac{V_L - I}{V_L}\right) V_H$$

as $V_H \to \infty$. Since $\frac{L+V_L-I}{L+V_L} > \frac{V_L-I}{V_L}$, the high type cannot use price setting as a credible signal as $V_H \to \infty$. ||

D. Proof of Proposition 6:

Part 1:

For constant returns to scale, if the project is cut back to a fraction β of its original size, the payoffs to the high and low type *if they scale* are:

$$\pi_H^S = \left(1 - \frac{\beta I}{H + \beta V}\right)(H + \beta V) = H + \beta(V - I)$$

$$\pi_L^S = \left(1 - \frac{\beta I}{H + \beta V}\right)(L + \beta V) = L + \beta\left(V - I\frac{L + \beta V}{H + \beta V}\right)$$

The payoffs to the high and low types *if they set a high price* are given by equation (10) as:

$$\pi_H^h = H + \alpha(V - I)$$

$$\pi_L^l = L + \alpha\left(V - I\frac{L + \beta V}{H + \beta V}\right)$$

Because the incentive compatibility condition will be binding in both equilibria, π_L^h must equal π_L^S (which equals $L+V-I$):

$$\alpha\left(V - I\frac{L+V}{H+V}\right) = \beta\left(V - I\frac{L+\beta V}{H+\beta V}\right)$$

Since $H > L > 0$ and $\beta < 1$, it follows that:

$$V - I\frac{L+\beta V}{H+\beta V} > V - I\frac{L+V}{H+V}$$

This implies that $\alpha > \beta$, and that $\pi_H^h > \pi_H^S$. Therefore the high, in a scaling equilibrium, can make the out of equilibrium move of setting a high price and be better off. The low would never want to set a high price because, under the modified intuitive criterion, the largest probability of acceptance the investor could respond with would be α. Therefore the scaling equilibrium does not survive the modified intuitive criterion.||

Part 2:

The first of the two equation defines the γx for which the high firm is indifferent between scaling and price setting. Recall that γx is the decrease in the cash flow out of the project when the firm scales. This relation is derived from equations (10), (11), and (14). The second equation is just equation (15), the incentive compatibility condition for the scaling equilibrium. From the second equation, it is clear that if γ is large, then γx will be large, and the scaling equilibrium will be inefficient. So for $\gamma < \gamma^*$, the payoff to the high in the scaling equilibrium is greater than the payoff in the price setting equilibrium, and the price setting equilibrium will fail the intuitive criterion. By the same argument, the scaling equilibrium will fail the modified intuitive criterion for $\gamma > \gamma^*$. ||

Part 3:

Again by the fact that the incentive compatibility constraint is binding in both the money burning and scaling equilibria, we know that the payoff to the low firm in both equilibria must be the same:

$$\left(1 - \frac{I-x}{H+V-\gamma x}\right)(L+V-\gamma x) = \left(1 - \frac{I}{H+V-C}\right)(L+V-C)$$

This can be rearranged to yield:

$$\frac{H+V-I-C}{H+V-I-(\gamma-1)x} = \left(\frac{L+V-\gamma x}{H+V-\gamma x}\right)\left(\frac{H+V-C}{L+V-C}\right)$$

Given the usual parameter restrictions, we can see that the left side of the above equation is greater than one if and only if $C < (\gamma-1)x$, and the right side of the equation is greater than one if and only if $C > \gamma x$. These two conditions are incompatible, since $x > 0$, and therefore both sides of the equation must be less than one, and the following condition must hold:

$$(\gamma-1)x < C < \gamma x$$

Based on this inequality, it is clear that:

$$\frac{H+V-I-(\gamma-1)x}{H+V-\gamma x} > \frac{H+V-I-C}{H+V-C}$$

and that:

$$\left(1 - \frac{I-x}{H+V-\gamma x}\right)(H-L) = \left(1\frac{I}{H+V-C}\right)(H-L)$$

Adding the left and right sides of equation (15) to the same sides of the equation directly above gives the following:

$$\left(1 - \frac{I-x}{H+V-\gamma x}\right)(H+V-\gamma x) > \left(1 - \frac{I}{H+V-C}\right)(H+V-C)$$

The left side of this equation is π_H^S and the right side is π_H^B. So we see that, for $\gamma \in [1, \infty)$, the payoff to the high type is higher in the scaling equilibrium, and by the binding incentive compatibility constraint the payoff to the low type in the two equilibria is the same, therefore the money burning equilibrium fails to satisfy the intuitive criterion in the presence of the money burning equilibrium. ||

Part 4:

Following the argument above (Part 2 of this proof), and then taking the limit as $\gamma \to \infty$ shows that $\gamma x \to C$ and $\pi_H^S \to \pi_H^B$ as $\gamma \to \infty$. || ***E. Proof of Proposition 9:***

Part 1:

The payoffs to the two types of firm, depending on whether they issue equity (and take the project) or not, are:

$$\pi_{t=H}^I = \left(1 - \tfrac{I}{\theta_H + V_H}\right)(\theta_H + V_H) \qquad \pi_{t=L}^I = \left(1 - \tfrac{I}{\theta_H + V_H}\right)(\theta_L + V_L)$$

$$\pi_H^{NI} = \theta_H \qquad \pi_L^{NI} = \theta_L$$

In order for the low to choose to issue in equilibrium, π_L^{NI} must be greater than π_L^I. This will only be the case if $V_L < I\left(\frac{\theta_L}{(\theta_H + V_H - I)}\right)$. The belief which supports this equilibrium as a sequential equilibrium is that any firm which issues is high. This belief is admissible under the intuitive criterion. ||

Part 2: If $V_L > \frac{\theta_L}{(\theta_H + V_H - I)}$, then the low will have an incentive to mimic if the high simply issues. Therefore, the high has to burn money as well as issue to credibly signal his type. The payoffs to the two types, if they (1) burn C, issue and invest or (2) don't burn money and don't invest, are:

$$\pi_{t=H}^B = \left(1 - \tfrac{I+C}{\theta_H + V_H}\right)(\theta_H + V_H) \qquad \pi_{t=L}^B = \left(1 - \tfrac{I+C}{\theta_H + V_H}\right)(\theta_L + V_L)$$

$$\pi_H^{NI} = \theta_H \qquad \pi_L^{NI} = \theta_L$$

Again, the amount burned is set so that the incentive compatibility constraint is binding. This will be the case when:

$$C = \left(\frac{\theta_H + V_H}{\theta_L + V_L}\right) - I \tag{17}$$

Substituting this into the payoff equation for the high and rearranging, we see that the high still wishes to issue if:

$$V_L < V_H\left(\frac{\theta_H + V_H}{\theta_L + V_L}\right)$$

This two inequalities define the condition in Part 2. The beliefs which support this sequential equilibrium are that any firm which burns less than C and issues is low type, and any firm which burns C or more is high type. These beliefs are admissible under the intuitive criterion. ||

Part 3:

The development above shows that if the specified condition is met, the high type must burn so much to satisfy the incentive compatibility constraint that he no longer wishes to issue. Therefore a sequential equilibrium exists in which neither type issues in equilibrium. The investor beliefs which support this are that a type which issues and burns less than C (as defined in equation (17)) is low, and that any type which burns C or more is high. These beliefs are admissible under the intuitive criterion. ||

Appendix B - The Efficient Combination of Signals

In this Appendix we present the maximization problem the solution of which is the optimal signal. We prove, by example, that parameter values exist for which any combination of the three signals may be optimal.

The choice of the high type's optimal signal is represented by the following maximization problem:

$$\max_{\alpha,C,I} H + (1-\alpha)(V_H(I) - I - C)$$

subject to the incentive compatibility constraint for the low type:

$$\alpha L + (1-\alpha)\beta = L + V_L(I^*) - I^*$$

where β is defined as the profit to the low type if he mimics and the equity issue is successful:

$$\beta \equiv \left(1 - \frac{I}{H_V}\right) L_V$$

and H_V and L_V are defined as the value of the high and low type firms given that the firm issues and invests I and burns C (*i.e.,* $H_V \equiv H + V_H(I) - C$ and $L_V \equiv L + V_L(I) - C$). I^* is the full-information investment level for the low type (*i.e.,* $I^* = argmax(V_L(I) - I)$).[41] Additionally, the three constraints which define the firm's strategy space are $0 \leq \alpha \leq 1$, $C \geq 0$, and $I \geq 0$. Note that both $I = 0$ and $\alpha = 0$ are equivalent to the high not taking the project.

The Lagrangian for this problem is:

$$\mathcal{L}(\alpha, C, I, \lambda) = H + (1-\alpha)(V_H(I) - I - C)\lambda(\alpha L + (1-\alpha)\beta - L - V_L(I^*) - I^*)$$

The first order conditions for an interior solution result in the following restrictions:

$$\frac{\partial \mathcal{L}}{\partial \alpha} = -(V_H(I) - I - C) - \lambda(L - \beta) = 0 \quad \rightarrow \quad \lambda = \frac{V_H(I) - I - C}{\beta - L} \tag{18}$$

$$\frac{\partial \mathcal{L}}{\partial C} = -(1-\alpha) - \lambda(1-\alpha)\frac{\partial \beta}{\partial C} = 0 \quad \rightarrow \quad \lambda = \frac{-1}{\frac{\partial \beta}{\partial C}} \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial I} = (1-\alpha)(V_H'(I) - 1)\lambda(1-\alpha)\frac{\partial \beta}{\partial I} = 0 \quad \rightarrow \quad \lambda = \frac{V_H'(I) - 1}{\frac{\partial \beta}{\partial I}} \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \alpha L + (1-\alpha)\beta - LV_L(I^*) - I^* = 0 \quad \rightarrow \quad \alpha L + (1-\alpha)\beta = LV_L(I^*) - I^*$$

where

$$\begin{aligned} \frac{\partial \beta}{\partial C} &= I\frac{H_V - L_V}{H_V^2} - 1 \\ \frac{\partial \beta}{\partial I} &= V_L'(I) - \frac{L_V}{H_V} + I\frac{L_V V_H'(I) - H_V V_L'(I)}{H_V^2} \end{aligned}$$

A necessary condition for an interior solution (*i.e.,* for a three signal combination to be optimal) is that all first order conditions be binding. If a two signal combination is optimal then two of the first order conditions (18), (19), or (20) will be satisfied, and if only one signal is optimal then only one of the FOC's (18), (19), or (20) will be satisfied.

[41] Although we assume that the high type underinvests as a signal, our analysis here does not preclude overinvestment. If the full-information investment level for the high type is considerably higher than for the low, and if $V_L(I)$ is sufficiently concave, the high will choose to overinvest as a signal. (see Ambarish, John and Williams (1987)).

Signal Levels:		*First Order Conditions:*		*Second Order Conditions:*	
α	0.3596	λ_α	1.083003385	$\|H_2^B\|$	0.018903489
C	7.891955805	λ_C	1.083003385	$\|H^B\|$	0.0
I	20	λ_I	1.083003385		
Parameters:					
High Firm:		*Low Firm:*		*Derived Values:*	
H	100	L	50	λ	1.083003385
$V_H(I)$	33.0000025	$V_L(I)$	23.02	$\frac{\partial\beta}{\partial C}$	-0.923358148
$V'_H(I)$	1.479425617	$V'_L(I)$	1	$\frac{\partial^2\beta}{\partial C^2}$	0.001225211
$V''_H(I)$	-0.010069594	$V''_L(I)$	0	$\frac{\partial^2\beta}{\partial C\partial I}$	0.00263209
Π_H	103.2711931	I^*	25	$\frac{\partial^2\beta}{\partial I^2}$	-0.003643387
		$V_L(I^*)$	28.0204828	$\frac{\partial\beta}{\partial I}$	0.44268155
		Π_L	53.0204828		

Table 7: Three Signal Example

To verify that the second order conditions are satisfied, the bordered Hessian matrix can be evaluated as described in Varian (1984). The condition that there be an interior solution can be reduced to the following two inequalities:

$$\lambda(\beta - L)^2(1-\alpha)\frac{\partial^2\beta}{\partial C^2} > 0 \tag{21}$$

and

$$\lambda(\beta - L)^2\left(\frac{\partial^2\beta}{\partial C^2}(V''_H(I) - \lambda\frac{\partial^2\beta}{\partial I^2}) + \lambda(\frac{\partial^2\beta}{\partial I\partial C})^2\right) < 0 \tag{22}$$

where:

$$\begin{aligned}
\frac{\partial^2\beta}{\partial C^2} &= 2I\frac{H_V - L_V}{H_V^3} \\
\frac{\partial^2\beta}{\partial I^2} &= \frac{1}{H_V^2}\left[2(1 - I\frac{V'_H(I)}{H_V})\left(L_V V'_H(I) - H_V V'_L(I)\right) + I\left(L_V V''_H(I) - H_V V''_L(I)\right) + H_V^2 V''_L(I)\right] \\
\frac{\partial^2\beta}{\partial I\partial C} &= \frac{1}{H_V^2}\left[(H_V - L_V) + I\left((2\frac{L_V}{H_V} - 1)V'_H(I) - V'_L(I)\right)\right]
\end{aligned}$$

The conditions (21) and (22) are necessary and sufficient condition for the matrix of second derivatives of the Lagrangian to be negative definite subject to the constraint.

Table 7 gives the parameters of a numerical example in which a three signal combination is optimal. The upper part of the table gives the signal levels, the first order conditions corresponding to (18), (19), and (20), and second order conditions corresponding to (21) and (22). For this example, the production function for both high and low firms is defined only in terms of the function's value, first and second derivatives at $I = 20$. For the value of $V''_H(I)$ given, the determinant of the bordered Hessian is zero. For $V''_H(I)$ greater than the tabulated value, the three signal combination is a saddle point, not a true maximum. But when the high type's project's production function is more concave than specified (*i.e.,* when $V''_H(I)$ is less than the specified value), the signal levels specified will be a local maximum and, assuming proper behavior of the production function away from this point, a global maximum. Similar examples show for some set of parameter values, any of the three two-signal combinations can be the most efficient signal.