Discussion of:

Robust Portfolio Choice

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Basic Idea

- Motivating Question:
  - How can we identify the MVE portfolio (SDF)?
- Discussion Outline:
  1. Some background on the literature
  2. Comparison with Quant-Investment approach
  3. This paper's approach
  4. Suggestions

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Basic Problem

- Since Markowitz (1952) we have known that, with perfect knowledge of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, the vector of weights of the MVE portfolio of risky assets is given by:

$$\mathbf{w}_{MVE} = \gamma^{-1}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$$

giving

$$\begin{aligned}
\mu_{MVE} &= \gamma^{-1}\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \\
\sigma_{MVE} &= \gamma^{-1}\sqrt{\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}} \\
SR^2_{MVE} &= \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \\
R_{MVE,t} &= \mathbf{w}'_{MVE,t}\mathbf{R}_t
\end{aligned}$$

Basic Problem

$$\mathbf{w}_{MVE} = \gamma^{-1}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$$

- The obvious solution suggested by this math is a "plug-in" portfolio:
  1. Calculate the sample-average excess returns $\hat{\boldsymbol{\mu}}$ and sample-covariance matrix $\hat{\boldsymbol{\Sigma}}$ over some estimation period (e.g, 120 months)
  2. Invest over next month in the portfolio

$$\hat{\mathbf{w}}_{MVE} = \gamma^{-1}\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\mu}}$$

- This works really badly for a bunch of reasons:
  - $\hat{\boldsymbol{\mu}}$ is unstable, and only a very noisy estimator of future returns.
  - $\hat{\boldsymbol{\Sigma}}$ is also noisy, and is likely to have a few very small eigenvalues.
  - The resuting portfolio generally line-up with a small eigenvalues that had a high realized return over the sample period.
- Since the 1950s, we've look for solutions to this problem, both based on economic and statistical arguments.

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Factor-model based approaches

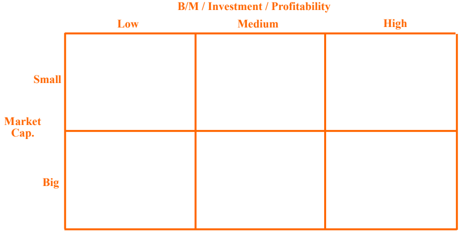
- Inspired by Merton (1973) and Ross (1976), key approaches have focused on factor models.
- Timeline:
  1. Chen, Roll, and Ross (1986) economic factors:
     - Evidence of that there were premia associanted with innovations in macroeconomic variables, but the Sharpe ratios associated with these portfolios were small.
  2. Connor and Korajczyk (1988) statistical factors using PCA:
     - effective in explaining the covariance structure, but all but the first PC—which looks like the market—did not carry much of a premium.
  3. Fama and French (1993) characteristic sorted portfolios:
     - "The 3-factor model does a good job in explaining the cross-section of average returns."

Characteristic Portfolios

- The Fama and French (1993) approach sorting on characteristics to form *characteristic portfolios* (CPs) has become standard in the empirical asset pricing literature.
- That is, find a characteristic that is associated with expected returns, e.g. book-to-market, and create a corresponding characteristic portfolio by sorting on this characteristic.
  - The resulting characteristic portfolio goes long high- and short low-characteristic stocks.
  - *Portfolio construction doesn't directly use any information about the covariance structure.*
- *Examples:* SMB, HML, RMW, CMA; UMD; WML; LIQ; ISU; QMJ, etc.
  - Fama and French (1993, 2015); Carhart (1997); Daniel and Moskowitz (2016); Pástor and Stambaugh (2003); Daniel and Titman (2006); Asness, Frazzini, and Pedersen (2013); Lustig, Roussanov, and Verdelhan (2011)

Fama-French CP construction

- In particular, Fama and French (2015) form five zero-investment portfolios: (1) the market; and portfolios based on:
  - (2) "size" (SMB), (2) book-to-market (HML), (3) investment (CMA), and (5) profitability (RMW)



  - each component portfolios is (1) rebalanced annually, and (2) is VW/buy-and-hold.
    - Thus, it should incur very low t-costs, and have no exposure to ST-reversal (Jegadeesh, 1990; Lehmann, 1990)
  - The SR^2 of the optimal combination of the 5 FF portfolios is 1.17 (1963-2017), vs. 0.19 for the mkt.

Characteristic portfolios are inefficient

- PCA ignores information about expected returns that comes from characteristics
- Characteristic sorts ignore information about the covariance structure that come historical individual firm's return covariances.

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Can characteristic portfolios be improved?

- These characteristic portfolios explain the full cross-section of returns iff they span the mean variance efficient (MVE) portfolio
- In Daniel, Mota, Rottke, and Santos (2020, DMRS), we show that characteristics were likely to be correlated with *un*priced factor risk.
  - In this case, the set of characteristics portfolios will not span the MVE portfolio.
- DMRS propose a methodology to hedge *un*priced risk ...
  - The DMRS hedge portfolios are based on the FOC for portfolio optimization (based on a characterstics model)
  - The hedge portfolio are *characteristic-balanced* and use forecasts of the factor loadings based on historical asset covariances with the proposed factor-portfolios.
  - The hedge portfolios are formed annually (in July), and are value-weighted/buy-and-hold.
    - Thus, like the FF portfolios, they should have very low t-costs, and no exposure to short-term reversal.
    - Hedging the unpriced risk in the FF portfolios raises SR^2 from 1.17 to 2.13.

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MAXSER portfolio

- MAXimum-Sharpe-ratio Estimated and sparse Regression (MAXSER) of Ao, Yingying, and Zheng (2019).
- Uses sample-average returns and covariance matrix over 60 months to estimate parameters create candidate MVE portfolio, and evaluates the performance of the portfolio over next month.
    - It performs this exercise for the DJIA 30 stocks (or 100 S&P500 stocks), plus the FF3 portfolios.
- optimization problem is:

$$\mathbf{w}(r_c) \coloneqq \arg\min_{\mathbf{w}} \frac{1}{T} \sum_{t=1}^{T} (r_C - \mathbf{w}'\mathbf{r}_t) \quad \text{s.t.} \quad \|\mathbf{w}\| \leq \lambda$$

- This resulting SR improves with the addition of individual 30/100 stocks.
    - This is a surprising result; past returns for individual stocks aren't (positively) correlated with their future returns

Quant Portfolio Optimization Approaches

- Standard quant portfolio construction approach determines trades by soving:

$$\max_{\mathbf{w}'} \left\{ \mathbf{w}'\mathbf{X}\boldsymbol{\lambda} - \frac{\gamma}{2} \cdot \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} - \tau \cdot tc(\Delta\mathbf{w}) \right\}$$

  subject to portfolio constraints.
- where
  - $\mathbf{X}$ is a matrix of characteristics; $\boldsymbol{\lambda}$ are characteristic premia.
  - $\boldsymbol{\Sigma}$ is the risk model.
  - $tc(\cdot)$ is the transaction cost model that captures both proportional costs and price impact.
  - portfolio contstraints can include leverage constraints, sector constraints. etc.
- The relation between $\mathbf{X}$ and future returns in (somewhat) stable; but individual stock characteristics are unstable.
  - *NB*: some form of momentum and short-term-reversal are in $\mathbf{X}$.

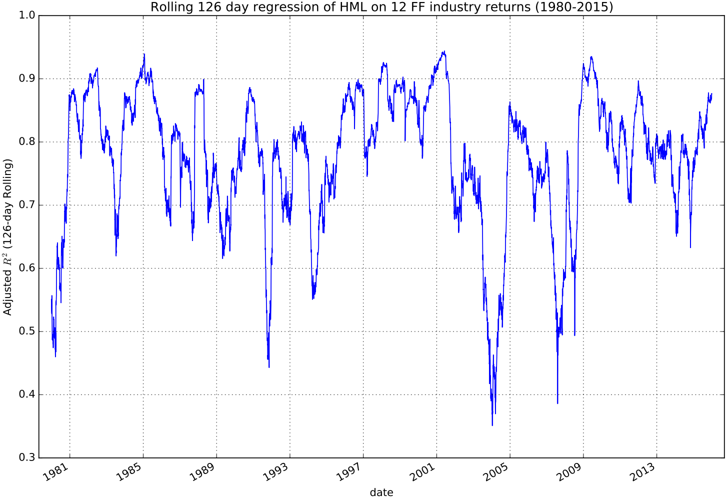
Quant risk models

- It is well known in industry that the use of a full-dimensional (N×N) $\hat{\mathbf{\Sigma}}$ in optimization leads to unstable portfolio weights.
- Quant optimization approaches deal with this in multiple ways:
  1. The use of Black and Litterman (1991) like approaches to shrink $\mathbb{E}[R]$ model estimates towards an equilibrium prior.
  2. Dimensionality reduction methods for $\hat{\Sigma}$ based on a factor model:

$$\hat{\mathbf{\Sigma}} = \mathbf{B}\hat{\mathbf{\Omega}}\mathbf{B}' + \hat{\mathbf{\Delta}}$$

  3. includes priced and unpriced factors; **B** is based on both characteristics and historical covariances.
     - Key suppliers of risk models are BARRA and Axioma.
     - *It is crucial that the* $\mathbf{X}$*'s in the return model are be in the risk model.*
  4. $\hat{\mathbf{\Omega}}$ estimated with historical data; different half-lives for ρ and σ^2 estimation.
  5. diagonal $\hat{\mathbf{\Delta}}$ estimated with historical data.

Industry Loading



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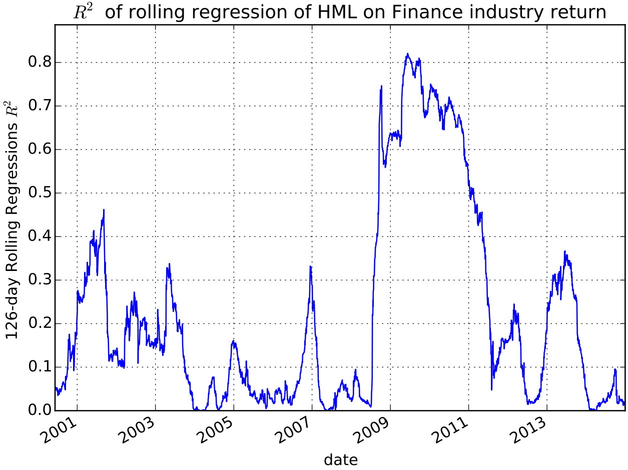

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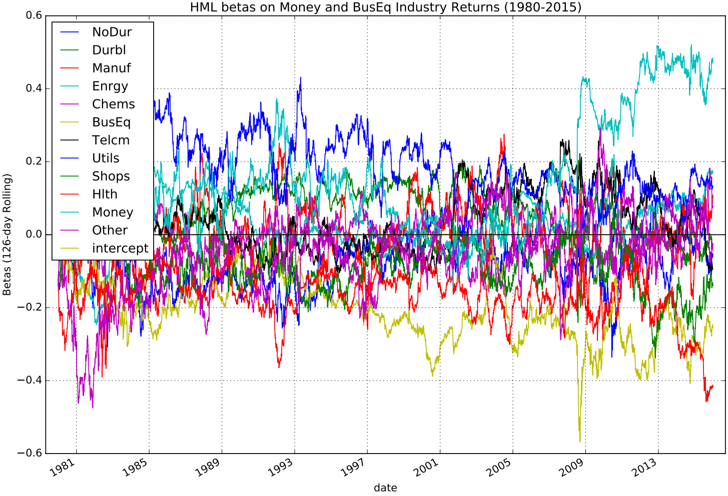

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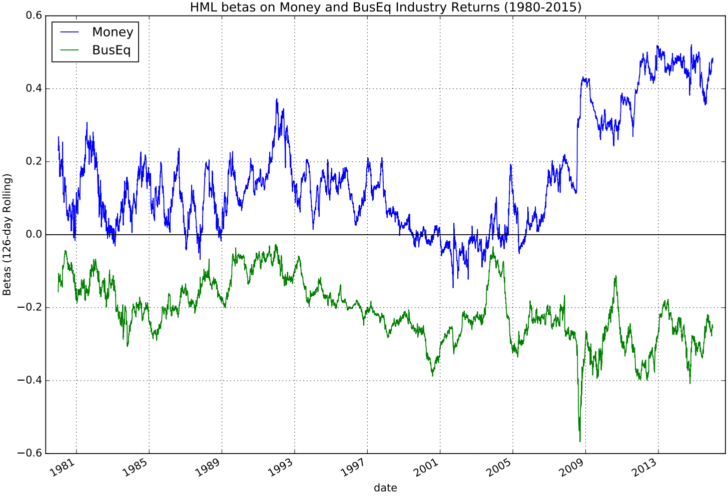
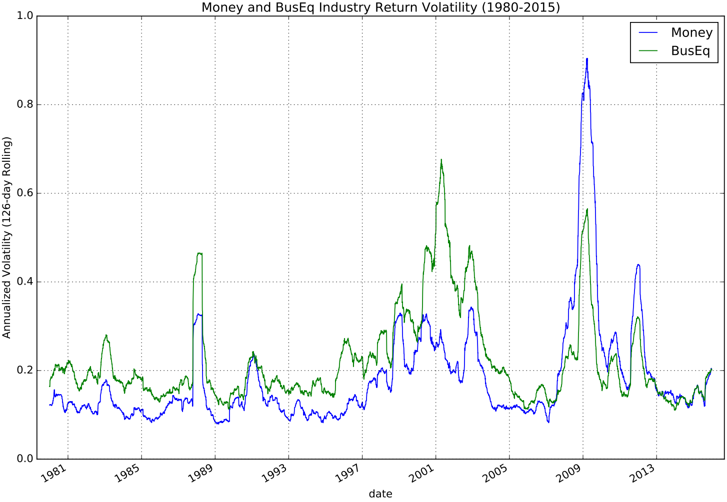


Industry Return Volatility



What this paper does

- Like AYZ, 30 year period.
- 120 month rolling estimation of parameters; 240 month test period
  - *i.e.*, parameters change each month, based on realized individual stock returns over the preceding 10 years, and the returns of the FF3 portfolios.
- Two sets of test assets:
  1. DJIA 30 individual stocks, plus FF 3 factor portfolios
  2. Randomly selected 100 individual stocks from S&P 500, plus FF 3 factor portfolios.

What this paper does

- Parameter estimates come from a constrained ML estimation of the process:

$$\mathbf{r}_t = \boldsymbol{\alpha} + \mathbf{B}\mathbf{f}^e_t + \boldsymbol{\epsilon}_t$$

where the $\mathbf{f}^e_t$ are the realized returns of the FF-3 factors and

$$\boldsymbol{\Omega} = \mathbb{E}[\mathbf{f}^e \mathbf{f}^{e'}] \;\text{ and }\quad \boldsymbol{\Sigma} = \mathbb{E}[\boldsymbol{\epsilon}\boldsymbol{\epsilon}']$$

- In the full model that accounts for missing factors

$$\boldsymbol{\alpha} = \mathbf{A}\boldsymbol{\lambda}_{\text{miss}} + \mathbf{a} \;\text{ and }\; \boldsymbol{\Sigma} = \mathbf{A}\mathbf{A}' + \mathbf{C}$$

where $\mathbf{A}$ is N×p, $\boldsymbol{\lambda}_{\text{miss}}$ is p×1, and $\mathbf{C} = \sigma^2\mathbf{I}$ is diagonal.

Estimation Procedure

- parameter estimates come from ML over the 120 month estimation period.

$$\hat{\boldsymbol{\theta}} = \arg\max_{\hat{\boldsymbol{\theta}}} L(\tilde{\boldsymbol{\theta}}) \quad \text{s.t.} \quad \tilde{\mathbf{a}}' \tilde{\boldsymbol{\Sigma}}^{-1} \tilde{\mathbf{a}} \leq \delta_{\text{apt}}$$

  where $\boldsymbol{\theta}$ contains $\boldsymbol{\Omega}, \mathbf{B}, \boldsymbol{\lambda}, diag(\mathbf{C}), \mathbf{a}, \boldsymbol{A}, \boldsymbol{\lambda}_{\text{miss}}$,
- s.t. the constraint that the squared-Sharpe Ratio of the non-factor-linked returns be less than δ_{apt}.
- Estimation each month is iterative, with two steps:
  1. First step estimation doesn't impose the constraint.
  2. Second step uses PCA on $\hat{\boldsymbol{\Sigma}}$ from step 1, to estimate the number of missing factors p, and to estimate the additional factors.

Results

	Mean p.a.	SR p.a.	SR wrt EW	SR wrt MAXSER	t-stat wrt EW	t-stat wrt MAXSER
Panel A: For DJIA 30 constituents						
MV	0.045	0.287	−0.140	−0.326	−0.136	−0.714
GMV-LW	0.030	0.181	−0.458	−0.575	−0.611	−0.855
EW	0.058	0.334	0.000	−0.216	—	−0.303
PCA2	0.026	0.161	−0.517	−0.621	−0.770	−1.179
PCA3	0.013	0.087	−0.739	−0.795	−1.101	−1.510
PCA4	−0.019	−0.121	−1.362	−1.284	−2.030	−2.437
PCA10	−0.032	−0.194	−1.583	−1.457	−2.360	−2.767
MAXSER	0.061	0.426	0.276	0.000	0.303	—
RMV using $\mathbf{V}$	0.080	0.556	0.657	0.298	0.656	0.860
RMV using $\mathbf{V}$: OptComb	0.173	0.872	1.609	1.045	1.949	1.682
RMV using $\mathbf{\Omega}$	0.079	0.576	0.726	0.352	0.729	1.194
RMV using $\mathbf{\Omega}$: OptComb	0.129	0.669	1.003	0.570	0.600	0.519

Conclusions and Suggestions

1. Reconcile the findings here with other findings from the literature.
    - is any of the improvement in SR coming from estimation of $\boldsymbol{\alpha}$ using historical returns?
    - If so, why?
2. Is realized single-stock alpha over the preceding 10 years a good estimator of future asset demand (Koijen and Yogo, 2019) shocks?
    - Asset-demand effects are relatively short-lived ($\ll$ 10 years)
3. An $\boldsymbol{\Omega}$ based on the Fama and French (1993) can probably be improved.
4. Would characteristics/instruments work better than past returns as estimators for $\boldsymbol{\alpha}$?
5. Investigate transaction costs & performace decay (McLean and Pontiff, 2016)

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