Discussion of:

When do cross-sectional asset pricing factors span the stochastic discount factor?

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Basic Idea

- Motivating Question:
  - If asset expected returns are linear in characteristics, how should should we construct a set of factor-portfolioss that span the SDF?
- Discussion Outline:
  1. Some background on the literature
  2. factor-portfolio construction choices
  3. Comparison with Quant-Investment approach
  4. Suggestions

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Introduction

- Multifactor models of the sdf posit that:

$$m^* = a + \mathbf{b}'\mathbf{f}^* \quad \text{with} \quad \mathbb{E}[m^* r_i] = 0$$

  for *any* excess return r_i and a set of traded "factors" $\mathbf{f}^*$ that span the MVE portfolio.
- Implying that

$$\mathbb{E}[r_i] = \boldsymbol{\beta}_i \boldsymbol{\lambda}$$

  where $\boldsymbol{\lambda}$ is the price of risk, and $\boldsymbol{\beta}_i$ is (the vector of) projection coefficients of r_i onto $\mathbf{f}^*$.
- ...which is motivation for time series regressions like:

$$(R_{i,t} - R_{f,t}) = \alpha_i + \beta_{i,m} \cdot (R_{m,t} - R_{f,t}) + \beta_{i,SMB} \cdot \text{SMB}_t + \beta_{i,HML} \cdot \text{HML}_t + \epsilon_t$$

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Search for $\mathbf{f}^*$ in in the Space of Returns

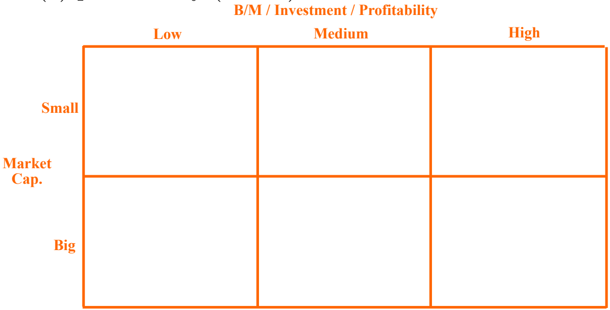
- Search in the space of returns for $\mathbf{f}^*$. But how?
- Timeline:
  1. Chen, Roll, and Ross (1986) economic factors:
     - Evidence of that there were premia associanted with innovations in macroeconomic variables, but the Sharpe ratios associated with these portfolios were small.
  2. Connor and Korajczyk (1988) statistical factors using PCA:
     - effective in explaining the covariance structure, but all but the first PC—which looks like the market—did not carry much of a premium.
  3. Fama and French (1993) characteristic sorted portfolios:
     - "The 3-factor model does a good job in explaining the cross-section of average returns."

Characteristic Portfolios

- As in Fama and French (1993), sorting on characteristics to form *characteristic portfolios* (CPs) has become standard in the empirical asset pricing literature.
- That is, find a characteristic that is associated with expected returns, e.g. book-to-market, and create a corresponding characteristic portfolio by sorting on this characteristic.
    - The resulting characteristic portfolio goes long high- and short low-characteristic stocks.
- *Examples:* SMB, HML, RMW, CMA; UMD; WML; LIQ; ISU; QMJ, etc.
    - Fama and French (1993, 2015); Carhart (1997); Daniel and Moskowitz (2016); Pástor and Stambaugh (2003); Daniel and Titman (2006); Asness, Frazzini, and Pedersen (2013); Lustig, Roussanov, and Verdelhan (2011)

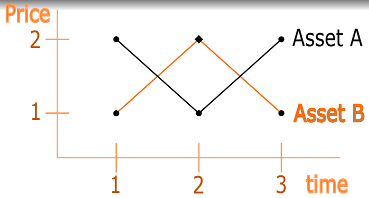
FF-CP construction

- In particular, Fama and French (2015) form five zero-investment portfolios: (1) the market; and portfolios based on:
  - (2) "size" (SMB), (2) book-to-market (HML), (3) investment (CMA), and (5) profitability (RMW)



  - each component portfolios is (1) rebalanced annually, and (2) is VW/buy-and-hold.

Why is buy and hold important?



$$R_{EW,2} = (1/2)*(-50\%) + (1/2)*(+100\%) = 25\%$$
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- The est'd rets of non buy-and-hold portfolios will be biased.
- magnitude bias will depend on port. asset liquidity.
- Note that Asness, Frazzini, Israel, Moskowitz, and Pedersen (2018) show that there is no (unconditional) size effect.
  - The size effect was originally demonstrated in Banz (1981) and Keim (1983), who used EW portfolios.

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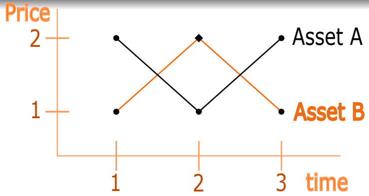

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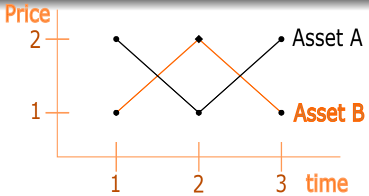

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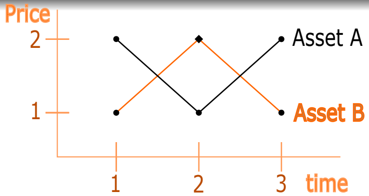

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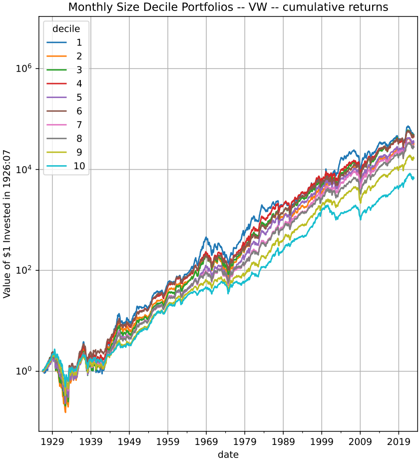
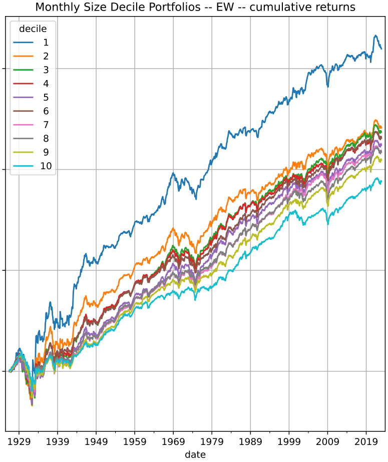
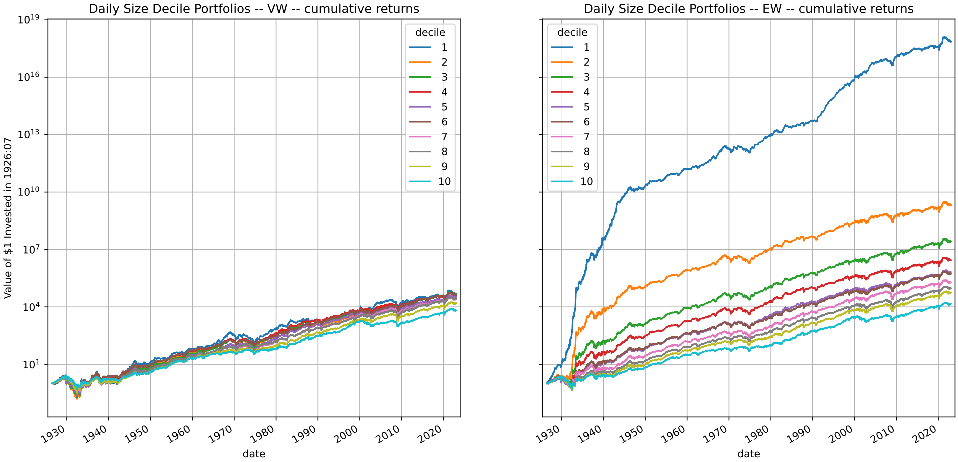
Monthly VW and EW Size Decile Portfolio Returns





Daily VW and EW Size Decile Portfolio Returns



Characteristic portfolios are inefficient

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Can characteristic portfolios be improved?

- These characteristic portfolios can only explain the cross-section of returns if they span the mean variance efficient (MVE) portfolio
- DMRS argued and showed that characteristics were likely to be correlated with *un*priced factor risk
  - In this case, the set of characteristics portfolios will not span the MVE portfolio.
- DMRS propose a methodology to hedge *un*priced risk . . .
  - The DMRS hedge portfolios are based on the FOC for portfolio optimization (that $\beta \propto X$).
  - They are *characteristic-balanced* and use forecasts of the factor loadings based on historical asset covariances with the proposed factor-portfolios.
  - The hedge portfolios are formed annually (in July), and are value-weighted/buy-and-hold.

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OLS Fama and MacBeth (1973) Portfolios

- A Fama and MacBeth (1973) regression examines the time-series of coefficients from a set of cross-sectional regressions of the form:

$$\tilde{R}_{t+1} = X_t \beta_t + \tilde{u}_{t+1}$$

  where R is N×1, X is N×K and β is K×1.
- The FM x-sectional OLS coefficients are:

$$\hat{\beta}_t = (X'X)^{-1} X' R_{t+1}$$

- These are just returns on K portfolios with (N×K) weights:

$$W_t' = (X_t' X_t)^{-1} X_t'$$

- Since $W_t' X_t = I$, the kth portfolio has:
  1. unit "exposure" to the kth characteristic,
  2. zero exposure to other characteristics,
  3. has weights that are a lineaer combination of the characteristics.

GLS Interpretation

- Suppose also that

$$\mu_t = X_t \phi_t$$

where ϕ is K×1, and that

$$\Sigma_t \equiv E_t[u_{t+1} u'_{t+1}]$$

- Dropping t subscripts, the GLS estimator of β is:

$$\hat{\beta}_{GLS} = \left(X' \Sigma^{-1} X\right)^{-1} X' \Sigma^{-1} R_{t+1}$$

- As with the OLS estimator, $\hat{\beta}_{GLS}$ can be interpreted as the returns on K portfolios with an N×K matrix of portfolio weights:

$$W' = \left(X' \Sigma^{-1} X\right)^{-1} X' \Sigma^{-1}$$

GLS FM Interpretation

- Given the matrix of portfolio weights:

$$W' = \left(X'\Sigma^{-1}X\right)^{-1} X'\Sigma^{-1}$$

  and defining the K GLS portfolio returns as:

$$R_{GLS,t+1} = W_t' R_{t+1},$$

- Since

$$W'X = I$$

  the kth GLS portfolio has the properties that

  1. has unit exposure to the kth characteristic,
  2. and has zero exposure to all other characteristics,
  3. is minimum variance.
  4. is a combination of the univariate minimum variance portfolios with weights:

$$w_k = \kappa\Sigma^{-1}x_k$$

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GLS Interpretation

- The GLS portfolios weights are:

$$W' = \left(X'\Sigma^{-1}X\right)^{-1} X'\Sigma^{-1}$$

- This means that the GLS portfolio returns have means and variances given by:

$$\mathbb{E}_t[R_{GLS,t+1}] = \phi_t$$

and

$$[\mathbb{E}_t[(R_{GLS} - \bar{R}_{GLS})(R_{GLS} - \bar{R}_{GLS})'] = \left(X_t'\Sigma_t^{-1}X_t\right)^{-1}$$

where ϕ is the characteristic premium defined by:

$$\mu_t = X_t\phi_t$$

- Thus, the MVE portfolio return is in the span of the GLS portfolio returns:

$$w_{MVE} = \kappa\Sigma^{-1}X\phi = \kappa\Sigma^{-1}\mu$$

Quant Portfolio Optimization Approaches

- Standard quant portfolio construction approach determines trades by soving:

$$\max_{w'X} \{w' X_t \phi_t - \lambda \cdot w' \Sigma_t w - \tau \cdot tc(\Delta w_t)\}$$

  subject to portfolio constraints.
- where
  - X_t is a vector of characteristics—the expected return model
  - Σ_t is the risk model.
  - $tc(\cdot)$ is the transaction cost model that captures both proportional costs and price impact.
  - portfolio contstraints can include leverage constraints, sector constraints. etc.

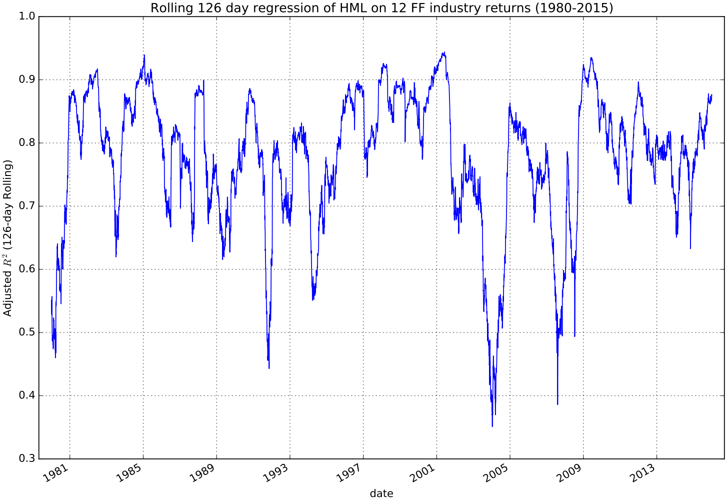
Quant risk models

- The covariance matrix/risk model KN develop has features in common which quant risk models.
- The use of a full-dimensional (N×N) $\hat{\Sigma}$ in optimization leads to unstable portfolio weights.
  - problem is that eigenvectors of $\hat{\Sigma}$ w/ small eigenvalues can align with with $\mathbb{E}[R]$ model premia.
- Quant optimization approaches deal with this in two ways:
  1. The use of Black and Litterman (1991) like approaches to shrink $\mathbb{E}[R]$ model estimates towards an equilibrium prior.
  2. Dimensionality reduction methods for $\hat{\Sigma}$:

$$\hat{\Sigma} = B\hat{\Omega}B' + \Delta$$

     - B includes priced and unpriced factors
     - $\hat{\Omega}$ estimated with historical data; different half-lives for ρ and σ^2 estimation.
     - diagnonal Δ estimated with historical data.

Industry Loading



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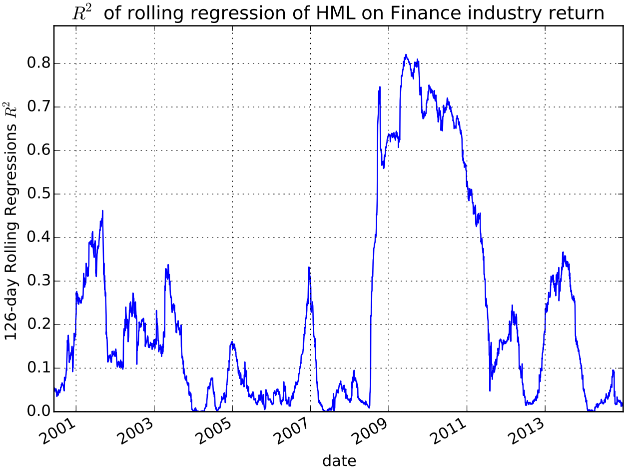

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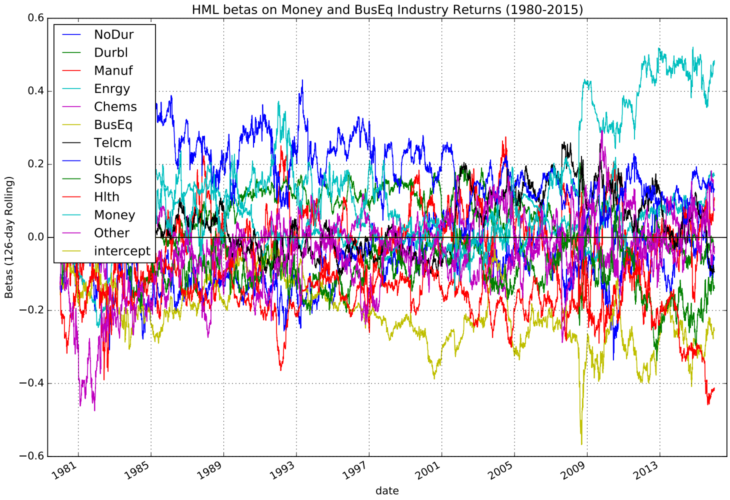

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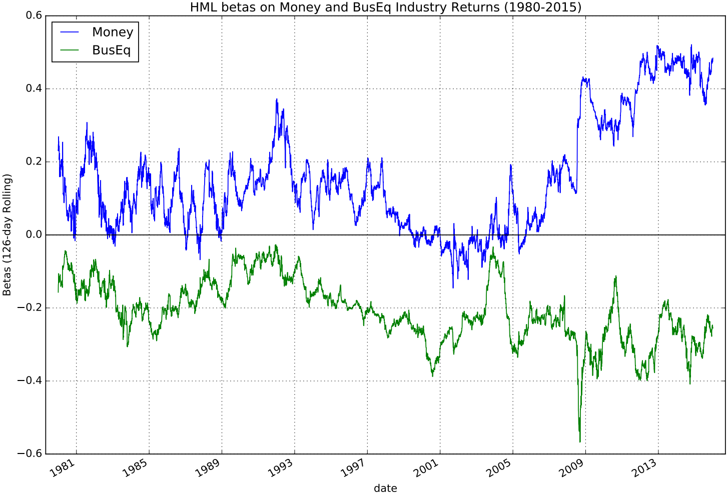
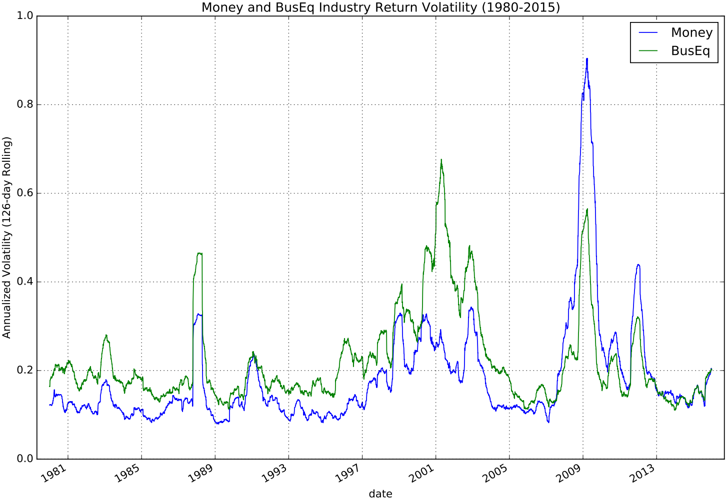


Industry Return Volatility



Conclusions and Suggestions

- This is a really nice and thorough analysis that contributes a lot to this literature
- These are perhaps more suggestion for future efforts than comments on this paper.

Transaction Costs:

- buy-and-hold portfolios, rebalanced once/year.
- Alternatively, directly estimate transaction costs.

Improved $\hat{\Sigma}$ plus "hedging"

- The main idea behind DMRS is based on the FOC that $\beta \propto X$ for optimized portfolios.
- Can hedging improve on optimization with a candidate $\hat{\Sigma}$?
  - If yes, then the $\hat{\Sigma}$ can be improved.
- Iterative hedging is a great idea.

References I

Asness, Clifford, Andrea Frazzini, Ronen Israel, Tobias J Moskowitz, and Lasse H Pedersen, 2018, Size matters, if you control your junk, *Journal of Financial Economics* 129, 479–509.

Asness, Clifford S, Andrea Frazzini, and Lasse H Pedersen, 2013, Quality minus junk, AQR Capital Management working paper.

Banz, Rolf W., 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics* 9, 3–18.

Black, Fischer, and Robert Litterman, 1991, Global asset allocation with equities, bonds, and currencies, Goldman, Sachs & Co. Fixed Income Research Report.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Chen, Nai-Fu, Richard Roll, and Stephen A. Ross, 1986, Economic forces and the stock market, *Journal of Business* 59, 383–403.

Connor, Gregory, and Robert A. Korajczyk, 1988, Risk and return in an equilibrium APT: Application of a new test methodology, *Journal of Financial Economics* 21, 255–289.

Daniel, Kent D., and Tobias J. Moskowitz, 2016, Momentum crashes, *Journal of Financial Economics* 122, 221–247.

Daniel, Kent D., and Sheridan Titman, 2006, Market reactions to tangible and intangible information, *Journal of Finance* 61, 1605–1643.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

References II

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F., and James MacBeth, 1973, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Keim, Donald B., 1983, Size-related anomalies and stock return seasonality: Further evidence, *Journal of Financial Economics* 12, 13–32.

Lustig, Hanno N., Nikolai L. Roussanov, and Adrien Verdelhan, 2011, Common risk factors in currency markets, *Review of Financial Studies* 24, 3731–3777.

Pástor, Ľuboš, and Robert F. Stambaugh, 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* 111, 642–685.