Discussion of:
New and Old Sorts: Implications for Asset Pricing

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Columbia Business School

Key Findings

- The authors examine characteristic-sorted portfolios ("factors") for the 56 chars from Freyberger, Neuhierl, and Weber (2017).
- BBT examine whether "new" factors—based on contemporaneous characteristics—can explain returns from "old" portfolios based on characteristics lagged up to 60 months.
- BBT's key finding is that:

  For over one-third of the 56 characteristics we study, the older sorts provide a significantly negative alpha, indicating that average returns decay too fast after portfolio formation relative to the decay in the characteristic spread (p. 2)

- BBT find that, to explain the x-section of returns, you need to incorporate changes in characteristics.
  - suggesting that the characteristics models used so far are misspecified.
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Idiosyncratic Volatility

- There is a very nice paper, Rachwalski and Wen (2016), entitled "Idiosyncratic Risk Innovations and the Idiosyncratic Risk-Return Relation"
- RW examine the idiosyncratic volatility-future return relationship (Ang, Hodrick, Xing, and Zhang, 2006, 2009).
- The measures of the *level* of idiosyncratic volatility they use are the volatility of the residuals from an FF three-factor regression over different horizons:
  - IV_t^1: over the past month ($t-22$ to $t-5$ trading days)
  - IV_t^6: over the past 6 months ($t-126$ to $t-5$ days)
- They do spanning regressions, as is done here, but also do Fama and MacBeth (1973) regressions, which I'll concentrate on.

iVol (2)

- Consistent with Ang, Hodrick, Xing, and Zhang (2006), the FM regression of future R_{t+1} on IV_t^1 yields a coefficient of -0.157 (t =3.49).
  - However, RW concentrate their analysis on IV_t^6 for which the FM coefficient is -0.121 (t =1.92)
    - IV_t^6 doesn't work as well because one-month IV's predictive power falls off very quickly; if you lag by IV_t^1 6-months there is no statistically-significant relation with future returns.
- While the forecast power for future returns falls off quickly, IV is highly autocorrelated:

  The time-series average of the cross-sectional correlation between six-month IVR and the six-month IVR in three, five, and ten years after portfolio formation is 0.56, 0.50, and 0.44 (p. 321)

iVol (3) – RW FM Regression Results

Rachwalski and Wen (2016), Table 4:[1]

FM Regr.	$\mathbf{X}_t$: IV_t^6	IV_{t-6}^6	$IV_t^6 - IV_{t-6}^6$
(1)	-0.121^{*}		
	(0.063)		
(2*)		~ 0	
(3)	-0.350^{***}	0.284^{***}	
	(0.043)	(0.042)	
(4)	-0.066		-0.284^{***}
	(0.068)		(0.042)
(5*)		~ 0	-0.350^{***}
			(0.043)

[1] Regressions 1,3, and 4 are from Rachwalski and Wen (2016), Table 4. The "results" in regressions 2 and 5 are my guesses.

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Results for BM/Sz/Pr/Inv Regressions

Table 1:

	Book-to-market			Size			Profitability			Investment		
	Panel A: Summary statistics											
	Firms	Ret.	t-stat	Firms	Ret.	t-stat	Firms	Ret.	t-stat	Firms	Ret.	t-stat
$R_{X,(t),t+1}$	1155	0.53	1.86	2121	0.31	1.55	1085	0.43	3.28	1343	0.51	3.48
$R_{X,(t-12),t+1}$	1029	0.61	3.12	1882	0.50	2.49	960	0.26	2.13	1193	0.22	1.80
$R_{X,(t-24),t+1}$	917	0.50	2.85	1661	0.41	2.05	848	0.14	1.14	1054	0.09	0.75
$R_{X,(t-36),t+1}$	821	0.49	2.84	1476	0.32	1.71	756	0.02	0.15	937	0.03	0.22
$R_{X,(t-48),t+1}$	738	0.51	3.02	1316	0.37	2.06	677	0.02	0.13	837	0.00	0.03
$R_{X,(t-60),t+1}$	664	0.38	2.25	1176	0.36	1.93	610	-0.01	-0.04	750	-0.06	-0.44
	Panel B: Relative pricing errors across horizons											
	α^u	β^u	α^c	α^u	β^u	α^c	α^u	β^u	α^c	α^u	β^u	α^c
$R_{X,(t-12),t+1}$	0.36	0.47	0.43	0.22	0.90	0.26	-0.08	0.78	-0.08	0.01	0.41	-0.01
	(2.68)	(7.93)	(3.35)	(2.69)	(18.46)	(3.40)	(-1.29)	(15.80)	(-1.43)	(0.05)	(11.86)	(-0.14)
$R_{X,(t-24),t+1}$	0.33	0.32	0.42	0.15	0.84	0.21	-0.13	0.63	-0.12	-0.05	0.28	-0.06
	(2.28)	(6.49)	(3.03)	(1.46)	(16.93)	(2.16)	(-1.55)	(9.84)	(-1.51)	(-0.40)	(6.38)	(-0.51)
$R_{X,(t-36),t+1}$	0.36	0.24	0.43	0.07	0.79	0.13	-0.23	0.58	-0.23	-0.10	0.24	-0.14
	(2.35)	(4.78)	(2.95)	(0.77)	(18.37)	(1.44)	(-2.35)	(9.60)	(-2.59)	(-0.74)	(6.30)	(-1.10)
$R_{X,(t-48),t+1}$	0.39	0.23	0.43	0.14	0.75	0.20	-0.22	0.55	-0.24	-0.18	0.37	-0.23
	(2.56)	(4.79)	(2.97)	(1.43)	(17.78)	(2.13)	(-2.25)	(10.03)	(-2.66)	(-1.42)	(6.97)	(-1.76)
$R_{X,(t-60),t+1}$	0.27	0.21	0.28	0.13	0.74	0.18	-0.23	0.52	-0.25	-0.24	0.35	-0.29
	(1.75)	(4.62)	(1.93)	(1.20)	(14.21)	(1.75)	(-2.19)	(7.26)	(-2.60)	(-1.79)	(5.68)	(-2.22)

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$R_{X,(t-60),t+1}$	0.27	0.21	0.28	0.13	0.74	0.18	-0.23	0.52	-0.25	-0.24	0.35	-0.29
	(1.75)	(4.62)	(1.93)	(1.20)	(14.21)	(1.75)	(-2.19)	(7.26)	(-2.60)	(-1.79)	(5.68)	(-2.22)

Relation of this paper to DMRS/DT and HMM

- BBT note that they take a different approach than Daniel, Mota, Rottke, and Santos (2020), Daniel and Titman (1997) and Herskovic, Moreira, and Muir (2019):
  [These papers] argue that factors can be traded more profitably by combining a factor ... with an offsetting position in a hedge portfolio. We instead argue that combining the newest portfolio with an older portfolio improves investment opportunities. (p. 15)

- I would frame this differently:
  - DMRS shows that *if* a characteristic proxies for expected return, a portfolio with weights proportional to the characteristics will not be MVE, and hence won't price the cross-section.
  - I presume that combining these portfolios with hedge portfolios could also reduce their risk without affecting their expected returns.

Characteristic Math

- A *characteristic* is measurable quantity that proxies for expected returns.
- If a single vector of characteristics $\mathbf{X}_t$ describes expected excess returns:

$$\boldsymbol{\mu}_t = \mathbb{E}_t\left[\mathbf{R}_{t+1}\right] = \mathbf{X}_t \lambda_c,$$

  where the *characteristic premium* λ_c is a scalar.
- The beta w.r.t. the (conditional) MVE portfolio also describes expected returns:

$$\boldsymbol{\mu}_t = \boldsymbol{\beta}_t \lambda$$

  where the *factor premium* λ is again a scalar
- Therefore:

$$\mathbf{X}_t = \frac{\lambda_c}{\lambda} \cdot \boldsymbol{\beta}_t = \frac{1}{\lambda_c} \cdot \boldsymbol{\mu}_t$$

- Thus, the vectors of (1) characteristics, (2) MVE-portfolio loadings, and (3) expected returns are all the same (to a scalar).

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- Thus, the vectors of (1) characteristics, (2) MVE-portfolio loadings, and (3) expected returns are all the same (to a scalar).

The MVE Portfolio

- The key point in DMRS is that if $\mathbb{E}_t\left[\mathbf{R}_{t+1}\right] = \mathbf{X}_t \lambda_C$, then the weights of the MVE portfolio are:

$$\mathbf{w}_t^* = \gamma^{-1} \Sigma^{-1} \boldsymbol{\mu}_t = \frac{\lambda_c}{\gamma} \Sigma^{-1} \mathbf{X}_t$$

  - It is easy to show that, the vector of asset betas w.r.t. this portfolio is:

$$\boldsymbol{\beta}_t = \frac{\Sigma \mathbf{w}_t^*}{\mathbf{w}_t^{*\top} \Sigma \mathbf{w}_t^*} = \frac{\mathbf{X}_t \lambda_C}{\mathbb{E}_t\left[\mathbf{R}_{t+1}^*\right]}$$

- However, a portfolios with weights $\mathbf{w}_t \propto \mathbf{X}_t$ will not be MVE.
  - That is, a portfolio that goes long high-characteristic stocks and short low-characteristic stocks will generally be inefficient.
    - it will contain both both priced and unpriced risk.
  - To make it efficient you have to hedge out the unpriced risk.

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