Discussion of:

Characteristics *Are* Covariances: A Unified Model of Risk and Return

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Introduction

Main questions/ideas

- What explains the cross-section of equity returns?
  - factors or characteristics?
  - behavioral or rational?
- What are the factors that matter?
  - Methodology: IPCA
- Comparison with other traded factors.
  - CAPM, FF3&5, (Fama and French, 1993, 2015), Carhart (1997)
- Which characteristics matter?

Principal Components

History

- Early CAPM tests
  - Black, Jensen, and Scholes (1972), Fama and MacBeth (1973)
- Roll (1977) critique; Ross (1976)
- Testing the APT using PCA:
  - Chamberlain and Rothschild (1983), Connor and Korajczyk (1986, 1988).
  - From Chamberlain and Rothschild (1983):

    > *[In an approximate APT setting] ... [t]he corresponding K eigenvectors converge and play the role of factor loadings. Hence only a principal component analysis is needed in empirical work.*

  - "testability" questions: Shanken (1982) and others
- …using economically motivated factors:
  - Chen, Roll, and Ross (1986) and others
- …using long-short "factor-portfolios" based on predictive characteristics:
  - Fama and French (1993) and (numerous) others.

Principal Components

PCA Methodology

- (Static) PCA effectively minimizes the sum of squared residuals over the N assets and T periods in the sample:

$$\min_{\beta,F} \sum_{t=1}^{T} (\mathbf{r}_t - \beta \mathbf{f}_{t+1})'(\mathbf{r}_t - \beta \mathbf{f}_{t+1}) \quad \left(= \min \sum_{t=1}^{T} \boldsymbol{\epsilon}_t' \boldsymbol{\epsilon}_t \right)$$

  by choosing:

  1. the (N×K) matrix of time-invariant factor loadings $\boldsymbol{\beta}$
  2. the set of (T) K-vectors $F = \{\mathbf{f}_t\}_{t=1}^{T}$.

  Can be solved using an eigenvalue/vector decomposition of sample cov. matrix.

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- However, there are problems with PCA:
  1. it is "static." (i.e, β is time invariant)
  2. it tends to be unstable
  3. economic interpretation of the factors can be tricky.
     - Chen, Roll, and Ross (1986)
  4. asset "repackaging" will change the principal components
     - Bray (1994)

PCA

Asset Repackaging

- For standard PCA, the weight applied to each squared residual $\epsilon_{i,t}^2$ is equal.
- This means that repackaging influences the estimated PC's.
- Example:
  - As of 5/2017, the Mkt Cap of AAPL was 5,000 times higher than the smallest firms in the Russell 3000.
  - Suppose you were to break Apple (the largest) into 5,000 "mini"-apples, and treat these as separate elements in the covariance matrix.
  - The first few principal components would then look a lot like AAPL

Principal Components

IPCA Methodology

- IPCA again minimizes the sum of squared residuals, over N & T:

$$\min_{\beta,F} \sum_{t=1}^{T} (\mathbf{r}_t - \boldsymbol{\beta}_t \mathbf{f}_{t+1})'(\mathbf{r}_t - \boldsymbol{\beta}_t \mathbf{f}_{t+1})$$

- Now, time variation in the (N$\times$K) matrix of factor loadings is captured with a (N$\times$L) set of instruments $\mathbf{Z}_t$.
- If $\boldsymbol{\beta}_t = \boldsymbol{\Gamma}_\beta \mathbf{Z}_t$, parameter estimation becomes finding the argmin of:

$$\min_{\Gamma_\beta,F} \sum_{t=1}^{T} (\mathbf{r}_t - \boldsymbol{\Gamma}_\beta \mathbf{Z}_t\ \mathbf{f}_{t+1})'(\mathbf{r}_t - \boldsymbol{\Gamma}_\beta \mathbf{Z}_t\ \mathbf{f}_{t+1})$$

  - Γ_β is (L$\times$K) and is the coefficients in the mapping from the L instruments to the K factor loadings.

IPCA

Choice of Instruments (from Table X)

- Here, L =36 instruments are used:

size	2.18	**	a2me	0.13		e2p	0.03
mom_12_2	1.71	**	s2p	0.12		c	0.03
mom_1_0	0.83	**	pcm	0.10		d2a	0.02
mom_12_7	0.69	*	fc2y	0.08		dpi2a	0.02
beta	0.65	**	roe	0.08		q	0.02
rel_high	0.61	**	roa	0.08		free_cf	0.02
ol	0.49		sga2m	0.07		rna	0.01
at	0.44	*	suv	0.07		investment	0.01
cto	0.39		mom_36_13	0.06	*	prof	0.01
idio_vol	0.21		pm	0.05		lev	0.01
lturnover	0.16		beme	0.03		oa	0.01
spread_mean	0.14		ato	0.03		noa	0.01

IPCA

Managed Portfolio Interpretation

- In Section 2.1.1, KPS provide a very nice interpretation of IPCA in terms of *managed characteristic portfolios.*
- In essence, what IPCA is doing is a K-dimensional PCA of a set of L managed portfolios:

$$\mathbf{x}_{t+1} = \mathbf{Z}_t' \mathbf{r}_{t+1}$$

  - Recall that K is the number of factors, and L is the number of instruments.
- Therefore:

  > *If the first three characteristics are, say, size, value, and momentum, then the first three columns of* **X** *are time series of returns to portfolios managed on the basis of each of these. …Likewise, the estimates of* $\mathbf{f}_{t+1}$ *would be the first K principal components of the managed portfolio panel. (p. 26)*

IPCA

Does IPCA solve all of the problems in the asset pricing literature?

- Lewellen, Nagel, and Shanken (2010) and Daniel and Titman (1997, 2012) argue that characteristic-based factor-models will, mechanically, explain the returns of characteristics sorted portfolios.
  - However, they also argue that the appropriate set of test assets will allow rejection of the factor model.
- The claim here is that:

  > *The asset pricing literature has struggled with the question of which test assets are most appropriate for evaluating models (Lewellen, Nagel, and Shanken, 2010; Daniel and Titman, 2012). IPCA provides a resolution to this dilemma.*

  - I'm going to argue that this statement is a little strong.

Asset Pricing Tests

test of $\mathbf{\Gamma}_\delta = \mathbf{0}_{L\times M}$

- KPS test whether other proposed factors ($\mathbf{g}_{t+1}$, incl., HML, SEM, etc.) help to explain returns
- The extended model is:

$$\mathbf{r}_t = \underbrace{\mathbf{\Gamma}_\beta \mathbf{Z}_t}_{\boldsymbol{\beta}_t} \mathbf{f}_{t+1} + \underbrace{\mathbf{\Gamma}_\delta \mathbf{Z}_t}_{\boldsymbol{\delta}_t} \mathbf{g}_{t+1} + \boldsymbol{\epsilon}_{t+1}$$

  - And the test is a test of whether the restriction $\mathbf{\Gamma}_\delta = \mathbf{0}_{L\times M}$ affects the fit of the model.
  - It does not.
- I don't think this is xsurprising—it is equivalent to testing whether a characteristic managed portfolio based on B/M is explained by HML.

Asset Pricing Tests

test of characteristic model: $\mathbf{\Gamma}_\alpha = \mathbf{0}_{L\times 1}$

- The extended model here is:

$$\mathbf{r}_t = \underbrace{\mathbf{\Gamma}_\alpha \mathbf{Z}_t}_{\boldsymbol{\alpha}_t} + \underbrace{\mathbf{\Gamma}_\beta \mathbf{Z}_t}_{\boldsymbol{\beta}_t} \mathbf{f}_{t+1} + \boldsymbol{\epsilon}_{t+1}$$

- KPS test whether setting $\mathbf{\Gamma}_\alpha = \mathbf{0}_{L\times 1}$ affects the fit of the model.
  - It does not.
  - In other words, after controlling for the PC loadings, the characteristics have no explanatory power.
- However, this doesn't mean that the factor model explains average returns better than the characteristic model.
- I would conjecture that the predictive R^2 is the same for both models $\mathbf{\Gamma}_\alpha = \mathbf{0}_{L\times 1}$ and $\mathbf{\Gamma}_\beta = \mathbf{0}_{L\times K}$
- However, with a different set of test assets (i.e. $\mathbf{Z}^\alpha$s), you could.

Factor Models

Characterics or Covariances

- As KPS note, absence of arbitrage implies the existence of a factor model that prices all assets/portfolios.

$$\mathbb{E}_t[\tilde{m}_{t+1}\tilde{r}_{p,t+1}] \iff \tilde{r}_{p,t+1} = \beta'_{p,t}\tilde{f}_{t+1} + \tilde{\epsilon}_{p,t+1}$$

- Trivially, it is also true that there exists a characteristic that prices all assets/portfolios.
- In DT(97), we make the point that the right charateristics model will "beat" a bad factor model
- perfect characteristics- and factor-models will both perfectly explain returns.
- Moreover, this will be true whether models are behavioral or rational.
  - See **?** and Daniel, Hirshleifer, and Subrahmanyam (2001)
- However, with a "bad set" of test assets, a test may fail to reject a bad factor model.
  - That is, it will have low statistical power.
  - DT(1997,2012), and LNS(2010).

The Characteristics Model

- Suppose that we iable to dentify a single characteristic **c** ($N \times 1$) that is a perfect proxy for expected excess returns
  - Our logic goes through if there are multiple characteristics
- We can monotonically transform the characteristics so that:

$$\mathbf{c} = \boldsymbol{\mu}$$

- Further, we can form a *characteristic-managed portfolio* (like SMB, HML, or the characteristic managed portfolios here) with weights:

$$\mathbf{w}_c = \kappa \mathbf{c} = \kappa \boldsymbol{\mu}$$

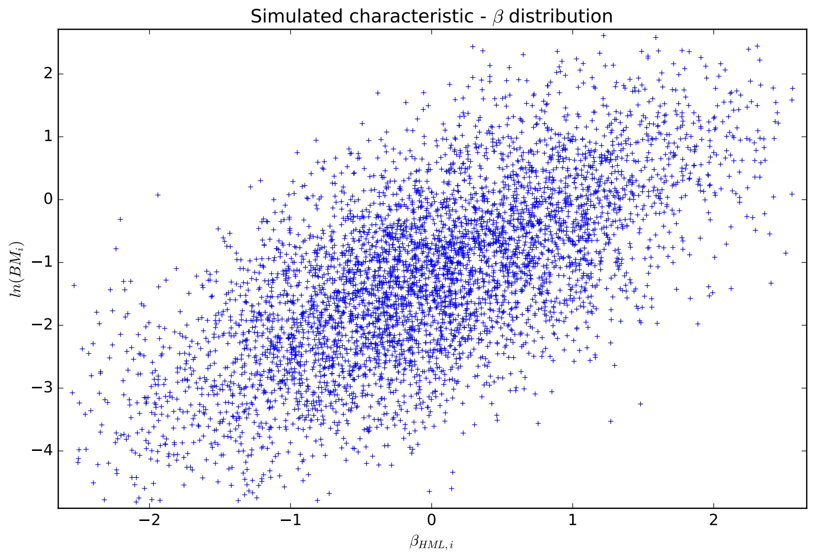
- However note that this portfolio will not *in general* be MVE, in that:

$$\mathbf{w}_{\mathrm{MVE}} = \kappa \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \neq \kappa \boldsymbol{\mu}$$

- the characteristic managed portfolio will, however, explain the cross section of returns for certain sets of test assets.
  - This was the point of DT(1997,2012), and LNS(2010).

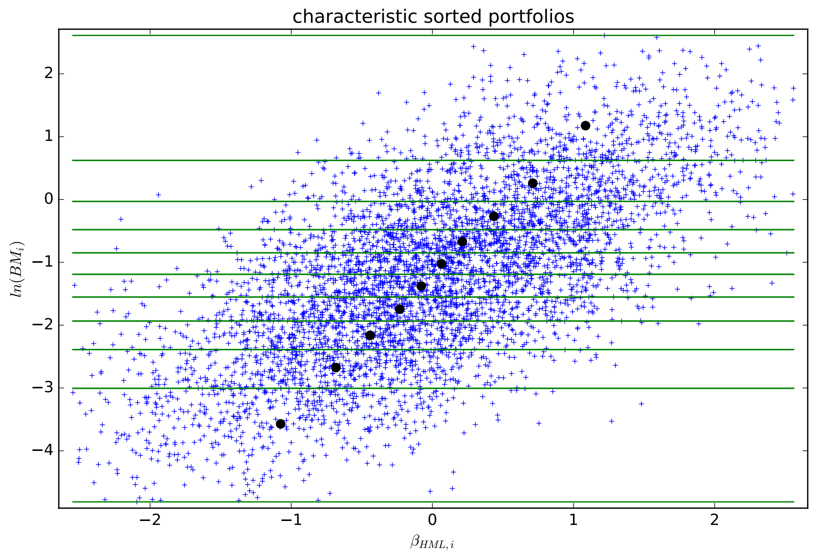
Test Asset Choice

Model of asset distribution



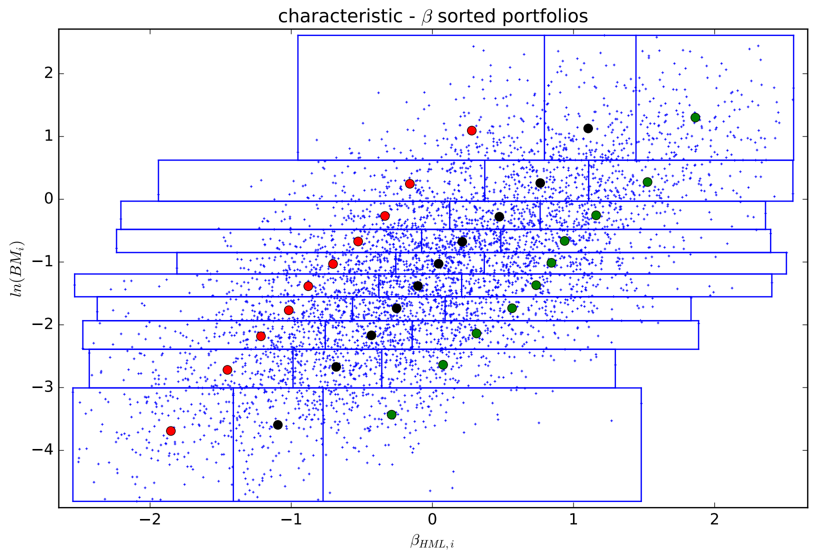
Test Asset Choice

characteristic sorted portfolios



Test Asset Choice

characteristic & beta-sorted portfolios



Test Asset Choice

instrument choice

- The key empirical challenge here is selecting an ex-ante instrument that provides the best possible forecast of future factor-betas, controlling for characteristics.

$$\hat{\beta}_{i,HML} = \beta_{i,HML} + \epsilon_i$$

- *In essence, to have a powerful test, you need instruments that are correlated with the (proposed) factor portfolio, but which are uncorrelated with the characteristic*.

Choice of instruments

- It is possible that the instruments used here are good proxies for the full cross-section of forecast returns
  - i.e., $\boldsymbol{\mu}_t = \gamma_{1 \times L} \mathbf{Z}_t$
- However, there are definitely other characteristics/instruments that explain the covariance structure, but not returns, that aren't included here.
- Unless these additional instruments are included in the estimation, the factor portfolios won't span the MVE portfolio and thus won't price the full cross section:
- Some linear combination of the managed portfolios has to have weights

  $$\mathbf{w} = \kappa \boldsymbol{\Sigma}_t \boldsymbol{\mu}_t$$

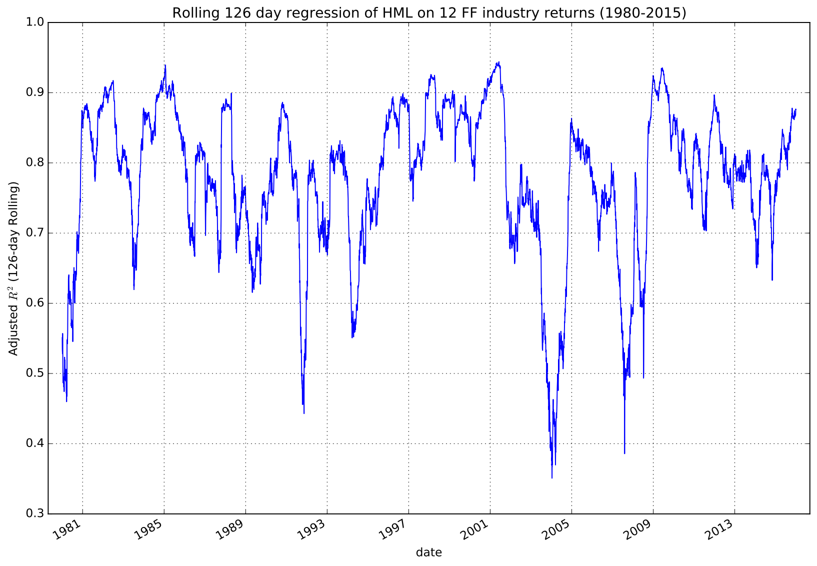
  which won't happen without these additional instruments.

Choice of instruments

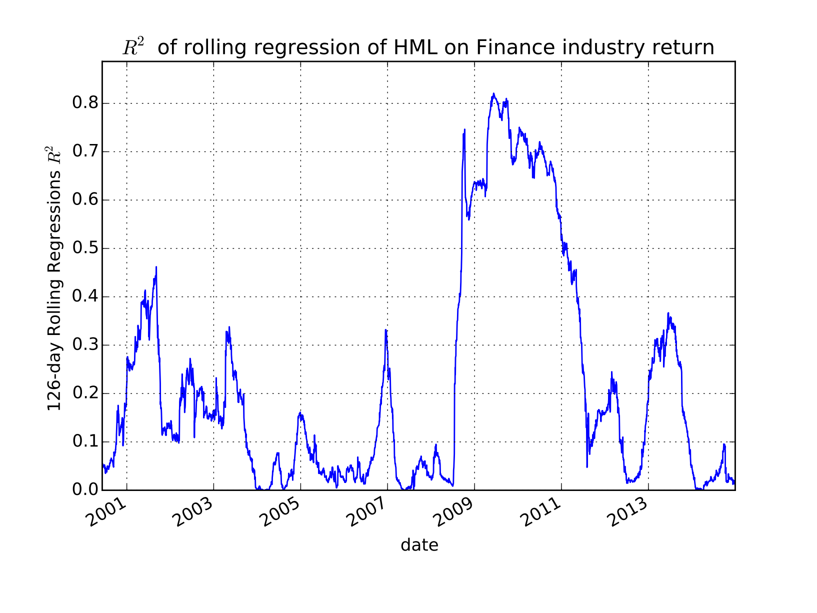
Other ideas

- See Fan, Furger, and Xiu (2016), who argue that, while the FF factors do explain expected returns, they don't explain the covariance matrix well.
  - They argue that industrys explain far more of the covariance structure.
- In Daniel, Mota, Rottke, and Santos (2017) form a set of test assets based on both characteristics and forecast forecast loadings.
  - With these portfolios, we reject the FF5 model at high levels of statistical significance.
  - After hedging out the unpriced risk in the FF5 portfolios, the squared-Sharpe ratio the of the MVE combination of factor portfolio doubles.

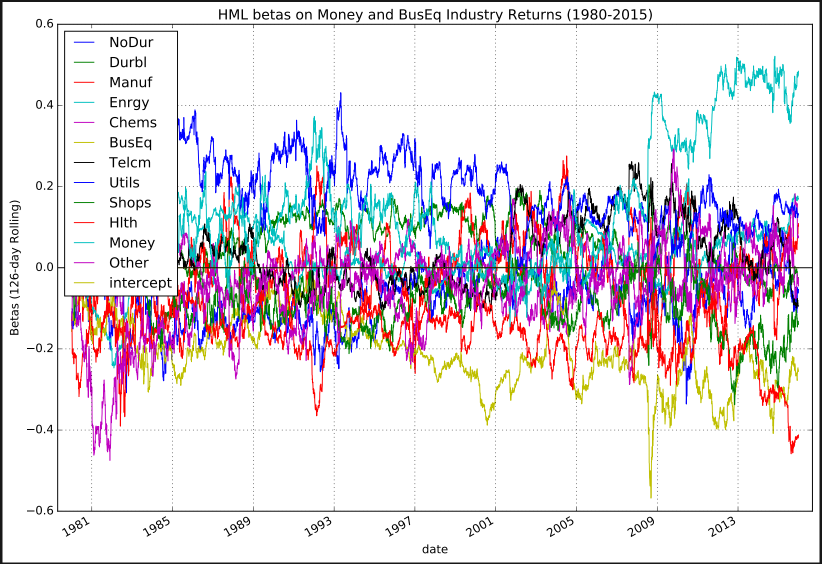
Industry Loading



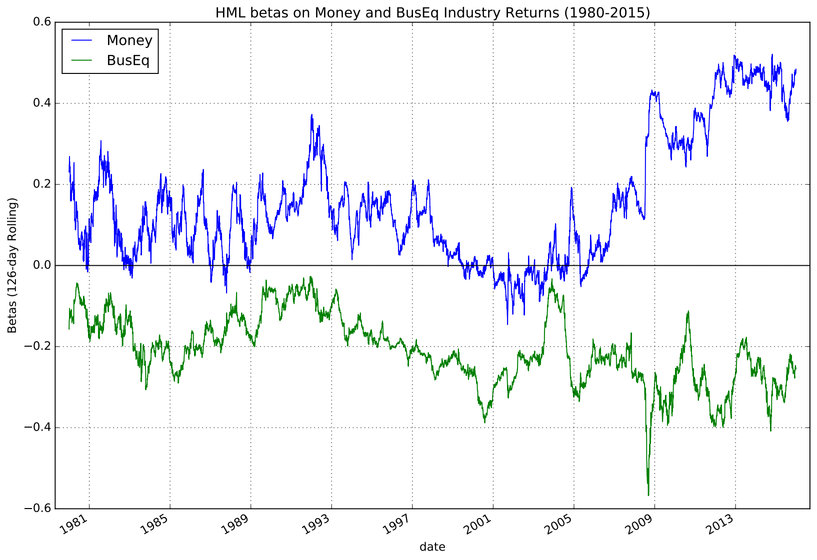
Industry Loading



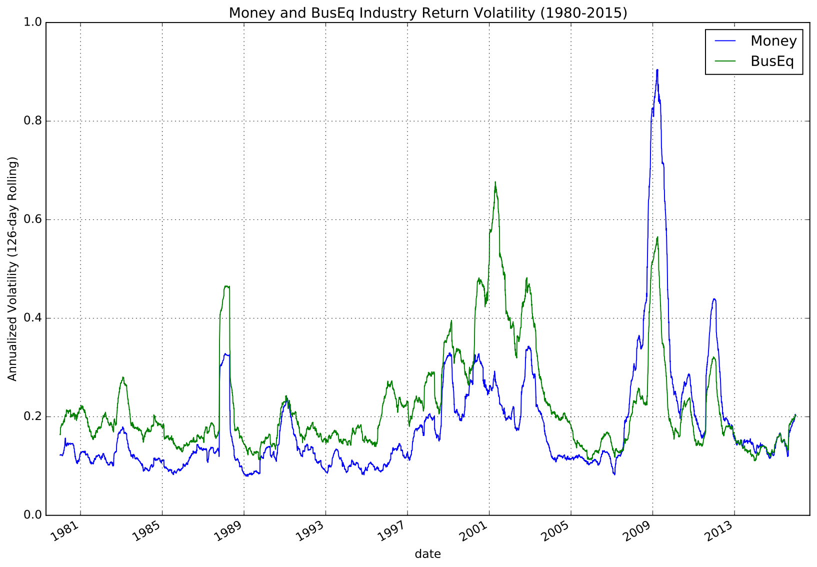
Industry Loadings



Industry Loadings



Industry Return Volatility



Conclusions

- The IPCA technique is interesting and potentiall alleviates some of the difficulties associated with standard PCA.
- However, choice of intstruments is import.
- The instruments need to include both good return forecasters, but also instruments which forecast the covariance structure.

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