Discussion of:

Horizon Pricing

by:
Kamara, Korajczyk, Lou and Sadka

Kent Daniel†

†Columbia University, Graduate School of Business

AFA Annual Meetings
5 January, 2013

Outline – *Empirical Results*

1. Variance-ratios/autocorrelations of candidate factors
   - Are the economic shocks underlying popular factors persistent?
2. Pricing of Long-Horizon Risk
   - Are factor pricing models more successful at longer horizons?
   - Are there "short-horizon" and "long-horizon" factors.
3. Characteristic vs. Covariance tests.
   - Are long-horizon factors priced, conditioning on characteristics?

Outline – *Empirical Results*

1. Variance-ratios/autocorrelations of candidate factors
   - Are the economic shocks underlying popular factors persistent?
2. Pricing of Long-Horizon Risk
   - Are factor pricing models more successful at longer horizons?
   - Are there "short-horizon" and "long-horizon" factors.
3. Characteristic vs. Covariance tests.
   - Are long-horizon factors priced, conditioning on characteristics?

Outline – *Empirical Results*

1. Variance-ratios/autocorrelations of candidate factors
   - Are the economic shocks underlying popular factors persistent?
2. Pricing of Long-Horizon Risk
   - Are factor pricing models more successful at longer horizons?
   - Are there “short-horizon” and “long-horizon” factors.
3. Characteristic vs. Covariance tests.
   - Are long-horizon factors priced, conditioning on characteristics?

Factor Autocorrelations

- The authors note that:

  Factors might exhibit autocorrelation either because they represent persistent economic shocks or because of non-synchronous trading.

Variance Ratios

- The key tool the authors use to assess factor autocorrelation is the variance ratio
  - See Lo and MacKinlay (1988); Lo and MacKinlay (1989).
- The variance ratio for factor mimicking portfolio k at horizon h is defined as:

$$\mathrm{VR}_h = \frac{\mathrm{var}(\tilde{r}_k^h)}{h \cdot \mathrm{var}(\tilde{r}_k)}$$

  where the h superscript denotes an h period factor realization.
- If we take the approximation that the return over h periods is the sum of the h single period innovations (or, alternatively, use logs), we get:

$$\mathrm{VR}_h \approx \frac{\mathrm{var}(\tilde{r}_{k,t} + \tilde{r}_{k,t+1} + \cdots + \tilde{r}_{k,t+h})}{h \cdot \mathrm{var}(\tilde{r}_{k,t})}$$

- The VR is approximately:

$$\mathrm{VR}_h \approx \frac{\mathrm{var}(\tilde{r}_{k,t} + \tilde{r}_{k,t+1} + \cdots + \tilde{r}_{k,t+h})}{h \cdot \mathrm{var}(\tilde{r}_{k,t})}$$

- However, the variance in the numerator simplifies to:

$$\begin{aligned}
\mathrm{var}(\tilde{r}_k^h) &= \mathrm{cov}\left((\tilde{r}_{k,t} + \cdots + \tilde{r}_{k,t+h}), (\tilde{r}_{k,t} + \cdots + \tilde{r}_{k,t+h})\right) \\
&= cov(\tilde{r}_{k,t}, \tilde{r}_{k,t+h}) + 2 \cdot cov(\tilde{r}_{k,t}, \tilde{r}_{k,t+h-1}) + \cdots \\
&\quad + h \cdot var(r_{k,t}) + (h-1) \cdot cov(r_{k,t}, r_{k,t-1}) + \cdots \\
&\quad 2 \cdot cov(\tilde{r}_{k,t+h-1}, \tilde{r}_{k,t}) + cov(\tilde{r}_{k,t+h}, \tilde{r}_{k,t})
\end{aligned}$$

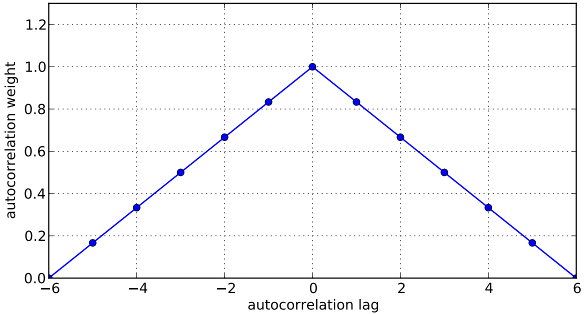
- Collecting terms and simplifying, we get

$$VR_h = 1 + 2 \cdot \sum_{j=1}^{h-1} \left(\frac{h-j}{h}\right) \rho_j$$

where ρ_j is the j-lag return autocorrelation.

Weighted Autocorrelation Function

- Graphically, the weight function has a tent shape:



- Again, the key thing is that if serial correlation at all lags is zero, $VR = 1$.

Factor Variance Ratios (Table 1)

Table 1. Factor Variance Ratios

Each traded factor (MKT, SMB, HML, and UMD) represents excess return portfolios. For example, MKT is the market return in excess of the risk free rate; SMB is the return of small firms in excess of big firms. A q-period variance ratio is defined as the ratio of variance of the factor over a q-period horizon and the product of q and the variance at the one-period horizon. $VR(q) = VAR(r^c_{q,t})/[q \cdot Var(r^c_{1,t})]$ where $r^c_{q,t}$ is the continuously compounded excess return for period t over a q-period horizon for traded factors, and unexpected liquidity of horizon q for non-traded factor LIQ. For example, $r^{c,MKT}_{q,t} = \ln[\prod_{i=0}^{q-1}(1 + r_{m,t-i})] - \ln[\prod_{i=0}^{q-1}(1 + r_{f,t-i})]$. The non-traded liquidity factor LIQ of horizon q in month t is the realized market liquidity level in month t, less its expected value at month t-q. To compute the expected value of liquidity level, we estimate an AR(2) model for the level of market liquidity using the entire time series of liquidity level from August 1962 to December 2010, and the expected market liquidity level in month t of horizon q is the q-month-ahead forecasted market liquidity at month t-q. Sample period is 1963 through 2010.

	Panel A: Variance Ratio					Panel B: Pvalue. H0: Variance ratio = 1				
Months (q)	MKT	SMB	HML	UMD	Liq	MKT	SMB	HML	UMD	Liq
1	1	1	1	1	1					
2	1.11	1.07	1.17	1.05	0.50	0.07	0.25	0.00	1.00	0.00
3	1.12	1.11	1.26	1.02	0.35	0.14	0.18	0.00	0.53	0.00
4	1.15	1.10	1.33	1.01	0.27	0.15	0.35	0.00	0.84	0.00
5	1.17	1.08	1.38	1.02	0.21	0.15	0.50	0.00	0.93	0.00
6	1.21	1.08	1.40	1.02	0.18	0.11	0.54	0.00	0.89	0.00
7	1.23	1.10	1.43	1.03	0.15	0.12	0.49	0.00	0.89	0.00
8	1.25	1.13	1.45	1.03	0.13	0.12	0.41	0.00	0.84	0.00
9	1.25	1.15	1.46	1.03	0.12	0.13	0.37	0.01	0.87	0.00
10	1.26	1.17	1.48	1.03	0.11	0.15	0.35	0.01	0.89	0.00
11	1.27	1.18	1.48	1.02	0.10	0.15	0.34	0.01	0.89	0.00
12	1.28	1.19	1.50	1.00	0.09	0.16	0.33	0.01	0.94	0.00

- Since:

$$VR_2 = 1 + 2 \cdot \left(\frac{1}{2}\right) \rho_1$$

  We can easily calculate the first order autocorrelations of the HML and *liq* series:

$$VR_2^{HML} = 1.17 \Rightarrow \rho_1^{HML} = +0.17$$

$$VR_2^{liq} = 0.5 \Rightarrow \rho_1^{liq} = -0.5$$

- The (non-traded) liquidity factor has huge negative serial correlation.
- The (traded) HML factor has very large positive serial correlation at 1 months (note: $\rho_2 = 0.03$)
  - Stambaugh, Yu, and Yuan (2011) note this, and show that most of the persistence comes from the short side of value.

Traded Factors and Shock Persistence

- A linear factor model specifies that, for an excess return $\tilde{r}^e_{i,t}$:

$$\tilde{r}^e_{i,t} = \sum_k \beta_{i,k,t-1} \left(\tilde{f}_{k,t} + \lambda_{k,t-1} \right) + \tilde{u}_{i,t}$$

  where

$$\mathbb{E}_{t-1} \left[\tilde{u}_{i,t} \right] = 0 \text{ and } \mathbb{E}_{t-1} \left[\tilde{f}_{k,t} \right] = 0$$

  - Note that:

$$\mathbb{E}_{t-1} \left[\tilde{f}_{k,t} \right] = 0 \Rightarrow cov_{t-1} \left(\tilde{f}_{k,t}, \tilde{f}_{k,t+\tau} \right) \Rightarrow cov \left(\tilde{f}_{k,t}, \tilde{f}_{k,t+\tau} \right)$$

- This means that:

$$\mathbb{E}_{t-1} \left[\tilde{r}^e_{i,t} \right] = \sum_k \beta_{i,k,t-1} \lambda_{k,t-1}$$

Traded Factors and Shock Persistence

- For a traded factor (such as HML) which by definition has $\beta_{HML} = 1$ and $\tilde{u} = 0$:

$$r_{HML,t} = \tilde{f}_{\mathrm{HML},t} + \lambda_{\mathrm{HML},t\text{–}1}$$

  a $\mathrm{VR}_2 > 1$ implies:

$$cov(r_{\mathrm{HML},t}, r_{\mathrm{HML},t+1}) > 0 \Rightarrow cov(\lambda_{HML,t}, \tilde{f}_{HML,t}) > 0$$

- That is, it must be the case that the factor premium next period increases with a positive surprise to HML this period.
  - In this rational expectations framework, the $\tilde{f}_t$ are shocks/innovations, and are by definition uncorrelated (conditionally and unconditionally).
  - They can't be positively or negatively "reinforcing."
  - If the authors are moving away from a rational expectations framework, they should specify how they are doing this.

Pricing Long-Horizon Risk

- The authors next investigate whether long horizon betas are priced.
- Here, they sort individual firms into decile portfolios on the basis of their betas on the 5 factors estimated over the last 60 months.
  - However, the individual firm betas are based on long-horizon regressions (over horizon q).
- They find evidence of pricing at horizons of:

Mkt	-	5 mo. - 1 yr.
HML	-	2-3 years
SMB,UMD	-	Not priced

Pricing of Long-Horizon Risks

Horizon	Market Beta		SMB Beta		HML Beta		UMD Beta		Liq Beta	
(q)	Return	Spread	Return	Spread	Return	Spread	Return	Spread	Return	Spread
1	1.50	[0.58]	-1.20	[-0.35]	1.82	[0.65]	-2.74	[-1.17]	3.35	[1.73]
2	1.32	[0.54]	-1.97	[-0.58]	1.92	[0.70]	-1.20	[-0.52]	2.02	[1.11]
3	1.19	[0.53]	-2.98	[-0.90]	0.90	[0.35]	-1.88	[-0.83]	2.14	[1.04]
4	3.22	[1.52]	-1.45	[-0.45]	2.45	[0.94]	-1.60	[-0.78]	2.96	[1.53]
5	4.38	[2.18]	-1.32	[-0.42]	3.34	[1.32]	0.18	[0.09]	4.78	[2.33]
6	4.42	[2.19]	-1.78	[-0.60]	2.03	[0.83]	0.56	[0.27]	3.88	[1.93]
7	4.90	[2.44]	-1.14	[-0.40]	2.26	[0.92]	0.62	[0.31]	3.85	[2.11]
8	5.42	[2.83]	-1.40	[-0.51]	2.76	[1.19]	0.67	[0.33]	3.36	[1.78]
9	5.10	[2.68]	-1.86	[-0.67]	3.89	[1.69]	0.80	[0.42]	2.11	[1.15]
10	4.02	[2.19]	-2.28	[-0.80]	4.46	[1.89]	0.17	[0.08]	-2.18	[-1.17]
11	2.56	[1.33]	-1.30	[-0.46]	3.73	[1.62]	-0.58	[-0.29]	-1.31	[-0.68]
12	4.45	[2.42]	-1.33	[-0.49]	3.31	[1.47]	-0.66	[-0.33]	-0.64	[-0.36]
13	3.53	[1.92]	-0.78	[-0.28]	4.38	[1.91]	-0.49	[-0.24]	1.26	[0.68]
14	0.73	[0.39]	-1.21	[-0.44]	3.91	[1.74]	-1.38	[-0.68]	1.73	[0.95]
15	0.96	[0.53]	-1.20	[-0.44]	4.50	[1.98]	0.04	[0.02]	-0.01	[-0.01]
16	0.18	[0.10]	-0.42	[-0.16]	5.53	[2.43]	0.19	[0.10]	1.22	[0.69]
17	0.83	[0.44]	0.11	[0.04]	4.00	[1.74]	-0.30	[-0.15]	0.37	[0.21]
18	1.60	[0.85]	1.13	[0.42]	3.10	[1.37]	-0.50	[-0.25]	-0.04	[-0.02]

Pricing of Long-Horizon Risks

17	0.83	[0.44]	0.11	[0.04]	4.00	[1.74]	-0.30	[-0.15]	0.37	[0.21]
18	1.60	[0.85]	1.13	[0.42]	3.10	[1.37]	-0.50	[-0.25]	-0.04	[-0.02]
19	1.61	[0.84]	0.90	[0.34]	2.86	[1.23]	-0.50	[-0.25]	0.95	[0.53]
20	0.95	[0.50]	0.53	[0.20]	4.14	[1.83]	0.48	[0.24]	0.58	[0.34]
21	0.34	[0.19]	1.19	[0.46]	5.20	[2.30]	1.02	[0.54]	0.95	[0.53]
22	0.30	[0.15]	1.21	[0.49]	5.64	[2.55]	0.44	[0.23]	1.12	[0.60]
23	-0.12	[-0.06]	2.12	[0.88]	5.42	[2.46]	0.30	[0.16]	0.37	[0.21]
24	1.26	[0.65]	1.52	[0.66]	4.48	[2.03]	0.71	[0.36]	-2.10	[-1.28]
25	1.06	[0.55]	1.52	[0.65]	4.94	[2.24]	0.22	[0.11]	-0.29	[-0.17]
26	0.73	[0.38]	1.32	[0.58]	4.63	[2.14]	0.13	[0.07]	-0.58	[-0.31]
27	1.30	[0.65]	1.67	[0.72]	3.89	[1.81]	-0.23	[-0.13]	-0.98	[-0.52]
28	1.19	[0.62]	1.70	[0.74]	3.62	[1.70]	-1.46	[-0.77]	1.82	[0.93]
29	3.29	[1.65]	0.68	[0.29]	4.21	[1.96]	-1.06	[-0.55]	1.32	[0.69]
30	2.83	[1.45]	-0.18	[-0.07]	4.63	[2.15]	-0.11	[-0.05]	0.11	[0.06]
31	3.65	[1.91]	-0.67	[-0.29]	4.02	[1.95]	-0.85	[-0.42]	1.60	[0.92]
32	3.35	[1.79]	-0.91	[-0.40]	4.08	[1.99]	0.02	[0.01]	1.18	[0.64]
33	1.78	[0.97]	-1.01	[-0.44]	4.64	[2.28]	0.05	[0.02]	0.20	[0.10]
34	1.74	[0.91]	-0.48	[-0.21]	4.79	[2.39]	-0.36	[-0.18]	-0.95	[-0.51]
35	2.27	[1.15]	-0.05	[-0.02]	4.27	[2.07]	-0.73	[-0.37]	0.62	[0.34]
36	1.78	[0.89]	1.18	[0.55]	4.76	[2.21]	-1.10	[-0.56]	1.39	[0.79]

Pricing of Long-Horizon Risks

- This is a multiple-comparison test. This needs to be taken into account in statements of statistical significance.
- This test examines whether historical betas are related to future expected returns.
  - Where individual firm factor loadings are not perisitent, *e.g.* for UMD, this won't tell you muuch about whether the factor is priced.
- What are the long-horizon betas capturing?

Long Horizon Betas

- Similar math shows that a beta for firm *i* on factor *k* over horizon *h* is:

$$\beta^h_{i,k} = \frac{1}{VR^k_h}\left[\beta^1_{i,k} + \sum_{j=-(h-1)}^{h-1}\left(\frac{h-|j|}{h}\right)\frac{cov(r_{i,t-j},\tilde{f}_{k,t})}{var(\tilde{f}_{k,t})}\right]$$

- Note that:
  - The factor variance ratio (VR^k_h) doesn't affect the relative βs of different firms.
    - Thus, absent any lagged covariances between the factor and the individual firm returns, the individual firm β rankings won't be affected by factor autocorrelation.
    - So, *e.g.*, decile portfolios won't be affected.
  - The only thing that can cause relative betas of firms to change is a lagged beta between the return and the factor.

Long Horizon Betas

- Similar math shows that a beta for firm *i* on factor *k* over horizon *h* is:

$$\beta^h_{i,k} = \frac{1}{VR^k_h}\left[\beta^1_{i,k} + \sum_{j=-(h-1)}^{h-1}\left(\frac{h-|j|}{h}\right)\frac{cov(r_{i,t-j},\tilde{f}_{k,t})}{var(\tilde{f}_{k,t})}\right]$$

- Note that:
  - The factor variance ratio (VR^k_h) doesn't affect the relative βs of different firms.
    - Thus, absent any lagged covariances between the factor and the individual firm returns, the individual firm β rankings won't be affected by factor autocorrelation.
    - So, *e.g.*, decile portfolios won't be affected.
  - The only thing that can cause relative betas of firms to change is a lagged beta between the return and the factor.

Long Horizon Betas

- Similar math shows that a beta for firm i on factor k over horizon h is:

$$\beta_{i,k}^{h} = \frac{1}{VR_h^k}\left[\beta_{i,k}^{1} + \sum_{j=-(h-1)}^{h-1}\left(\frac{h-|j|}{h}\right)\frac{cov(r_{i,t-j},\tilde{f}_{k,t})}{var(\tilde{f}_{k,t})}\right]$$

- Note that:
  - The factor variance ratio (VR_h^k) doesn't affect the relative βs of different firms.
    - Thus, absent any lagged covariances between the factor and the individual firm returns, the individual firm β rankings won't be affected by factor autocorrelation.
    - So, *e.g.*, decile portfolios won't be affected.
  - The only thing that can cause relative betas of firms to change is a lagged beta between the return and the factor.

What does long-horizon investment mean?

- Does it mean that you can't touch your investment for h periods?
  - Then, you arguably want to look at h-period betas.
- If, however, you have a target-portfolio date $t + h$, but are allowed to revise your portfolio each period than you should care about the premium associated with single period betas.
- Finally, what would you need in a model to get different factors, for the same set of assets, being priced at different horizons?

What does long-horizon investment mean?

- Does it mean that you can't touch your investment for h periods?
  - Then, you arguably want to look at h-period betas.
- If, however, you have a target-portfolio date $t+h$, but are allowed to revise your portfolio each period than you should care about the premium associated with single period betas.
- Finally, what would you need in a model to get different factors, for the same set of assets, being priced at different horizons?

Conclusions

- Really interesting, provocative results.
- It would be great to see a model which motivates the horizon pricing results.
  - Could such a model generate pricing of the Market at a horizon of 7 months, and value at 2-3 years?
- Account for multiple comparisons in the statistical significance calculations.
- Generate better forecasts of *ex-post* loadings.
  - For UMD and probably LIQ, the 60-month *ex-ante* betas won't provide a good forecast.

References I

Lo, Andrew W., and A. Craig MacKinlay, 1988, Stock market prices do not follow random walks: Evidence from a simple specification test, *Review of Financial Studies* 1, 41–46.

———, 1989, The size and power of the variance ratio test in finite samples, *Journal of Econometrics* 40, 203–238.

Stambaugh, Robert F., Jianfeng Yu, and Yu Yuan, 2011, The short of it: Investor sentiment and anomalies, *Journal of Financial Economics* 104, 288–302 Wharton School Working Paper.