A Discussion of:

Default Risk Premia and Asset Returns

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Overview

- Links movements in defaultable bond prices with pricing of other assets.
- Uses CDS data in an innovative way
- Very impressive technically – neat empirical methods.
- I think that there are some problems with the empirical methodology which make the current results difficult to interpret.
    - However, these can be rectified.

Discussion Outline

- I'm going to first go through the steps of the empirical methodology
- Then discuss some alternatives to the current methodology

Empirical Methodology - λ^P Estimation:

Step 1 – *Estimate* $\lambda^P_{t,i}$*:*

1. Assume that $\lambda^P_{i,t}$ follows an Ornstein-Uhlenbeck process:

$$d\log(\lambda^P_{i,t}) = \kappa_i\left(\theta_i - \log(\lambda_{i,t})\right)dt + \sigma_i dB^i_t$$

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3. Calculate time series of λ^P_ts using $\Theta^{i,P}$ and EDFs (sampled weekly).

Empirical Methodology - λ^Q Estimation:

Step 2 – *Estimate* $\lambda_{i,t}^{Q}$*:*

1. Also, assume that $\lambda_{i,t}^{Q}$ follows an Ornstein-Uhlenbeck process:

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   - However, BLO impose *no* relation between Θ^P and Θ^Q.
   - Note also, that there is substantial x-sectional variation in Θ_i^Q (and in Θ_i^P), yet specification doesn't allow for any time-series variation.

Empirical Methodology - "Return" Estimation

Step 3A – *Calculate "Premium Returns":*

1. Using $\lambda^Q_{i,t}$ (and r_t), infer the price of constant-maturity, defaultable, zero-coupon bonds with period h issued by firm i:

$$P_{t,h} = E^Q_t[e^{-\int_t^{t+h} r_s ds} \cdot \underbrace{e^{-\int_t^{t+h} \lambda^Q_s ds}}_{p^Q(t,h)}]$$

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2. The "return" (change in the CM bond price) is then the change in price over 1 week:

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- However, the change in a CM bond price is not a return!

Empirical Methodology - "Return" Estimation

Step 3B – *Calculate "Premium Returns":*

1. Using the "return" R_t^i and the "risk-neutral return" R_t^{iP}, calculate the component of (return for firm i) that is due to changes in the premium:

$$R_t^{iu} = R_t^i - R_t^{iP}$$

where R_t^P is what the "return" would have been if investors had been risk-neutral.

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- The goal here is to extract "that portion of R_t^i that is not due to changes in expected default losses or changes in risk-free rates."
- However, R_t^{iu} is *not* a return, and cannot be treated as such in asset-pricing tests.

Empirical Methodology - Estimate Common Component

Step 4 – *Calculate Common Component:*

1. Run the pooled regression:

$$R_t^{u,i} = \alpha^i + \boldsymbol{\beta}^{S,i} \cdot \mathbf{F}_t^S + \delta_t + \epsilon_t^i$$

   for all i, t.

   - $\mathbf{F}_t^S$ includes Mkt, SMB, HML, UMD, DEF, TERM.

2. The the "default risk-premium" factor (DRP) is defined as"

$$DRP(t) \equiv \frac{1}{N} \sum_{i=1}^{N} \left(\hat{\alpha}^i + \hat{\delta}_t \right)$$

Empirical Methodology - Pricing Tests

Step 5 – *Asset Pricing Tests:*

1. Next, BLO run a set of Fama and French (1993) style time-series regressions using DRP_t along with the other 5 FF(93) factors to explain bond, equity and equity-option returns:

$$\begin{aligned} R_t^i - R_t^f &= \alpha^i + \beta_M^i (R_{m,t} - R_t^f) + \beta_{SMB}^i SMB_t + \cdots \\ &\quad + \beta_{DEF}^i DEF_t + \beta_{TERM}^i TERM_t + \beta_{DRP}^i DRP_t + \epsilon_t^i \end{aligned}$$

Note that all of the independent (and dependent) variables in this regression are the returns to zero-investment portfolios, except DRP_t.

TSR Test - Motivation

Starting with the standard set of factor pricing equations:

$$\tilde{R}_{i,t} = E_{t-1}[\tilde{R}_{i,t}] + \sum_k \beta_k^i \tilde{f}_t^k + \tilde{\epsilon}_t^i$$

$$E_{t-1}[\tilde{R}_{i,t}] = \alpha_{t-1}^i + R_t^f + \sum_k \beta_k^i \lambda_{t-1}^k$$

Combining these gives :

$$\tilde{R}_{i,t} - R_t^f = \alpha_{t-1}^i + \sum_k \beta_k^i \underbrace{\left(\lambda_{t-1}^k + \tilde{f}_t^k\right)}_{\substack{\text{zero-inv} \\ \text{port ret}}} + \tilde{\epsilon}_t^i$$

- Given a set of zero investment portfolio returns that span the set of priced factors, a time-series regression can be used to test whether $\alpha = 0$.

Asset Pricing Tests

$$\begin{aligned} \tilde{R}^i_t - R^f_t &= \alpha^i + \beta^i_M(\tilde{R}_{m,t} - R^f_t) + \beta^i_{SMB}\widetilde{SMB}_t + \cdots \\ &\quad + \beta^i_{DEF}\widetilde{DEF}_t + \beta^i_{TERM}\widetilde{TERM}_t + \beta^i_{DRP}\widetilde{DRP}_t + \tilde{\epsilon}^i_t \end{aligned}$$

The problem with this specification is that DRP_t is *not* the return from an implementable strategy:

- $\widetilde{DRP}_t$ captures only the component of $\tilde{f}$ due to changes in the risk premium (the λ).
  - Thus, the $\hat{\beta}^i_{DRP}$ will be mis-estimated, and $\hat{\alpha}^i$ is not interpretable as a pricing error.

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2. However, the EDF data is useful in that it can provide a direct estimate of expected returns at each time t.
3. This is particularly valuable in an econometric setting like this, with only 4 1/2 years worth of (CDS) return data:
   - With this amount of data, realized returns are a very noisy estimate of expected returns.

What Restrictions can be tested?

1. If we assume that λ_t^{iP} is the true default intensity for firm i, and $L_{i,t}$ the loss-given-default for firm i, then, for a defaultable bond with price $P_{i,t}$, the expected excess return can be inferred from λ_t^{iQ} and λ_t^{iP}:

$$\left(\lambda_t^{iQ} - \lambda_t^{iP}\right) L_{i,t}\, dt = E\left[\frac{dP_{i,t}}{P_{i,t}} - r_f dt\right]$$

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2. Then, given a pricing kernel, one can test the restriction that the expected return is equal to covariance with the pricing kernel:

$$\left(\lambda_t^{iQ} - \lambda_t^{iP}\right) L_{i,t}\, dt \stackrel{?}{=} -E_t\left(\frac{dP_i}{P_i}, \frac{dM}{M}\right)$$

Tests using EDF data

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- Of course, the results from all of this are only as good as the EDF data!

References I

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.