Discussion of:

Risks for the Long Run: Estimation and Inference

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Paper Outline

- Bansal and Yaron (2004) introduced an important framework for thinking about how the effects of "long run risks" on asset return premia.
- Here BKY introduce a framework procedure for estimation of the long run risk model and apply this to size and value sorted portfolios (as in Fama and French (1993)).
  - They show that the size and value premia are explained by covariation with the shocks in the LRR model.
- GMM estimation of the LRR model, applied to a large set of size/value sorted portfolios, yields reasonable (?) estimates of RA of 15-16 and IES of ≈ 0.5, and low pricing errors.
- Robustness checks (with an alternative consumption growth specification) yield higher RA estimates ($= 27.7$), but again yield low pricing errors for these portfolios.

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The Dynamics of log-consumption growth obey:

$$\begin{aligned}\Delta c_{t+1} &= (\mu_C + x_t) + \sigma_t \tilde{\eta}_{t+1} \\ x_{t+1} &= \rho x_t + \phi_e \sigma_t \tilde{e}_{t+1} \\ \sigma^2_{t+1} &= \bar{\sigma}^2 + \nu(\sigma^2_t - \bar{\sigma}^2) + \sigma_w \tilde{w}_{t+1}\end{aligned}$$

where the key feature is that Δc_{t+1} has both time-varying drift and volatility, with:

- "transient" shocks to consumption $\tilde{\eta}$
- persistent fluctuations in growth rate in consumption x_t, with shocks $\tilde{e}$.
- shocks $\tilde{w}$ to both the volatility of consumption growth and the volatility of the growth rate(σ_t) .

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IMRS calculation

BKY show that, with these consumption dynamics, with Epstein and Zin (1989) type preferences, and with their log-linearization, the pricing kernel is given by:

$$\tilde{m}_{t+1} = \boldsymbol{\Gamma}' \begin{bmatrix} 1 \\ x_t \\ \sigma_t^2 \end{bmatrix} - \boldsymbol{\Lambda}' \begin{bmatrix} \sigma_t \tilde{\eta}_{t+1} \\ \sigma_t \tilde{e}_{t+1} \\ \sigma_w \tilde{w}_{t+1} \end{bmatrix} \tag{1}$$

- Thus, given suitable estimates of the state variables and the shocks, they can estimate the vectors $\boldsymbol{\Gamma}$ and $\boldsymbol{\Lambda}$ from the asset returns and test overidentifying restrictions.

State Variable/Shock Estimation

The state variabes and shocks are estimated using the following system applied to annual consumption growth over 1930-2002.

$$\Delta c_{t+1} = \underbrace{\mathbf{b}_x' \begin{bmatrix} 1 \\ log(P_t/D_t) \\ r_{f,t} \end{bmatrix}}_{=x_t} + \sigma_t \tilde{\eta}_{t+1}$$

$$\sigma_t^2 = \mathbf{b}_\sigma' \begin{bmatrix} 1 \\ log(P_t/D_t) \\ r_{f,t} \end{bmatrix}$$

- The drift and volatility innovations are the result of fitting AR(1) processes to $\hat{x}_t$ and $\hat{\sigma}_t^2$.
- Adjusted $R^2 = 35\%$ for annual Δc
  - Significant $+$ and $-$ coefficients on P/D and r_f, respectively

Returns and Betas

In adition to applying GMM to estimate their model, BKY estimate a factor model implied by the structure of the pricing kernel:

$$\tilde{R}_{i,t+1} = E_t[R_{i,t+1}] + \underbrace{\beta_{i,\eta}\tilde{\eta}_{t+1}}_{\text{transient}} + \underbrace{\beta_{i,e}\tilde{e}_{t+1}}_{\text{long run}} + \underbrace{\beta_{i,w}\tilde{w}_{t+1}}_{\text{volatility}} + \tilde{u}_{i,t+1}$$

$$E_t[R_{i,t+1}] = \lambda_0 + \lambda_\eta\beta_{i,\eta} + \lambda_e\beta_{i,e} + \lambda_w\beta_{i,w}$$

- To "*expand the degrees of freedom*", BKY estimate the second equation on 10 size and 10 B/M sorted portfolios, plus the aggregate market.
- *Together, the 3 betas explain about 84% of the cross-sectional variation in mean return.*
- Beta and premium estimates are generally reasonable (though $\lambda_w < 0$).

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Size and Value Premia

- One of the motivations of this paper is demonstrating that the large size- and value-premia in equity markets are a result of the covariation of the returns of these portfolios with the shocks in the LRR model
- Studying these porfolios has been a focus of the asset pricing literature.
  - This is (probably) because of the high Sharpe ratios of these portfolios, and the correspondingly high implied σ_ms
    - Hansen and Richard (1987), Hansen and Singleton (1982), Hansen and Jagannathan (1991)
  - The VW market Sharpe-ratio is 0.31; for a portfolio with size and value tilts it is 0.80 (annualized, 1968-2004).

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Explaining Size and Value Premia

Paper	Factor(s)	Cond. Vars.
Conditional (C)CAPM Models		
Ferson and Harvey (1999)	VW	S&P 500 Dividend Yield
Lettau and Ludvigson (2001)	VW or Cons Growth	*cay*
Santos and Veronesi (2001)	VW + Labor Income Growth	Labor Income to Cons Ratio (s)
Petkova and Zhang (2005)	VW Index	$E[R_m]$ based on BC Vars
Alternative-Factor Models		
Fama and French (1993)	VW, HML, SMB	
Jagannathan and Wang (1996)	Labor Income Growth	DEF
Heaton and Lucas (2000)	Proprietary Income Growth	
Piazzesi, Schneider, and Tuzel (2003)	Cons Growth $+\Delta$NH Expenditure Ratio ($\Delta log(\alpha)$)	Non-Housing Expenditure Ratio (α)
Lustig and Nieuwerburgh (2002)	Scaled Rental Price Change ($A\Delta log\rho$)	Housing Collateral Ratio
Aït-Sahalia, Parker, and Yogo (2004)	Luxury Good Consumption	
Li, Vassalou, and Xing (2002)	Sector Inv. Growth Rates	
Parker and Juillard (2005)	Innovations in Future Long Horizon Consumption Growth	
Campbell and Vuolteenaho (2004)	CF and DR news	

Return Space Geometry

- It has become standard practice to test asset-pricing models with the size/BM sorted portfolios as in Fama and French (1993).
- These portfolios provide a good spread in average returns. However, they lie in a low-dimensional excess return space.
  - Tests based on these assets do not examine whether variation in factor loading outside of this space is priced.
- Thus, any two factors which span the return space will also explain the returns of size/BM sorted portfolios.
- To provide statistical power, some dispersion in test-asset factor loading independent of Size/BM is necessary.

- Fama and French (1993) (Table 6) run time-series regressions for each of the 25 SZ/BM sorted portfolios:

$$\tilde{R}_{i,t} - RF_t = a + b \cdot (\tilde{R}_{m,t} - RF_t) + h \cdot \widetilde{\mathrm{HML}}_t + s \cdot \widetilde{\mathrm{SMB}}_t + \tilde{\epsilon}_t$$

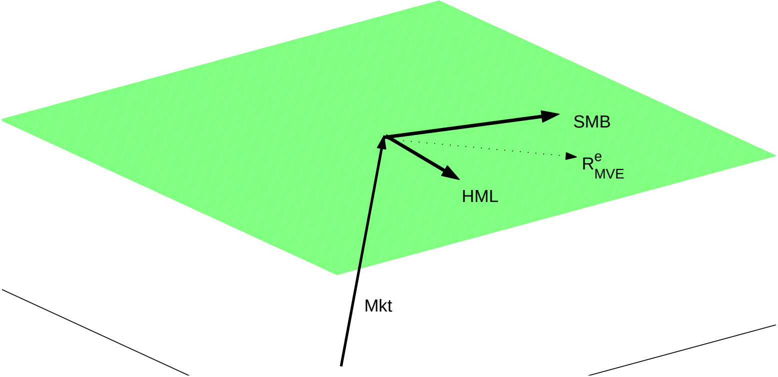
- The R^2s are:

	Low	2	3	4	High
Small	0.94	0.96	0.97	0.97	0.96
2	0.95	0.96	0.95	0.95	0.96
3	0.95	0.94	0.93	0.93	0.93
4	0.94	0.93	0.91	0.89	0.89
Big	0.94	0.92	0.88	0.90	0.83

- In addition, the estimates of b range from 0.91 to 1.18 (std-dev $=$ 0.06).

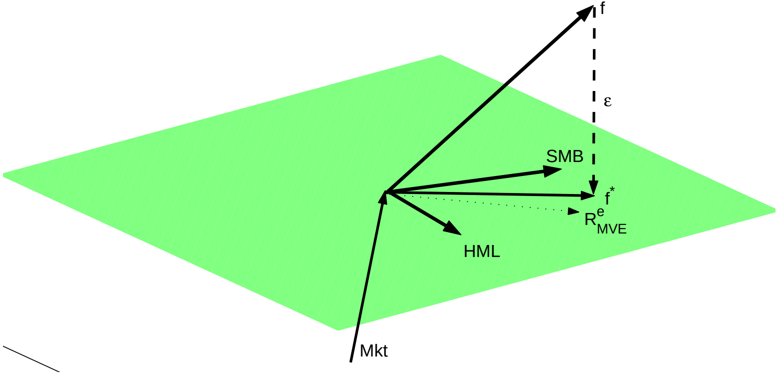
Return Space Geometry

- This means that the returns of these 25 portfolios, net of the market return, lie *approximately* in a 2-dimensional excess return space $\underline{\mathbf{R}}^{e*}$ spanned by HML and SMB:

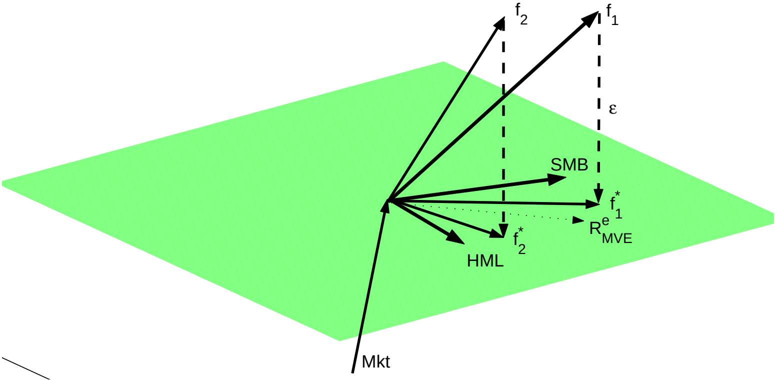


Test Geometry

- In any test where the factor premia (λs) are free parameters, a test of a single-factor model with the 25 FF portfolios is a test of whether $corr(f^*, R^e_{MVE}) = 1$



- However, with two factors, assuming $f_1^* \neq k \cdot f_2^*$, some linear combination of the $\tilde{f}$s will always price the assets.
  - That is, f_1^* and f_2^* form a basis for this return subspace.



A More Powerful Test

- The problem is that *any* $\mathbf{b}'\tilde{\mathbf{f}}$ such that

$$\mathbf{b}'\tilde{\mathbf{f}} = \tilde{R}^e_{MVE} + \tilde{\epsilon} \text{ , for } \epsilon \perp \mathrm{HML}, \mathrm{SMB}$$

  will price size/BM sorted portfolios.
  - Thus, a just about *any* model with two shocks will have low pricing errors.
  - Note that, if you need to price the risk-free asset in addition to size/BM sorted portfolios, you need three factors.
  - One caveat is that counterintuitive factor premia (λs) may be necessary.
- Thus, to fully examine the model (with a powerful test) the test asset space must be augmented in the direction of ϵ.

A More Powerful Test

- To increase test power, the test asset space must be augmented in the direction of ϵ.
  - Intuitively, this means that we need to identify portfolios which covary with $\{\tilde{\eta}, \tilde{e}, \tilde{w}\}$, but which are orthogonal to $\widetilde{\mathrm{HML}}$ and $\widetilde{\mathrm{SMB}}$
- Here, this means adding portfolios for which the returns covary with innovations in the market price-dividend ratio, and with innovations in the risk-free rate, but which have are "balanced" in terms of size/BM.
  - Then, need to test whether these portfolios still have the high return premia consistent with the model.

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